Inaugural address

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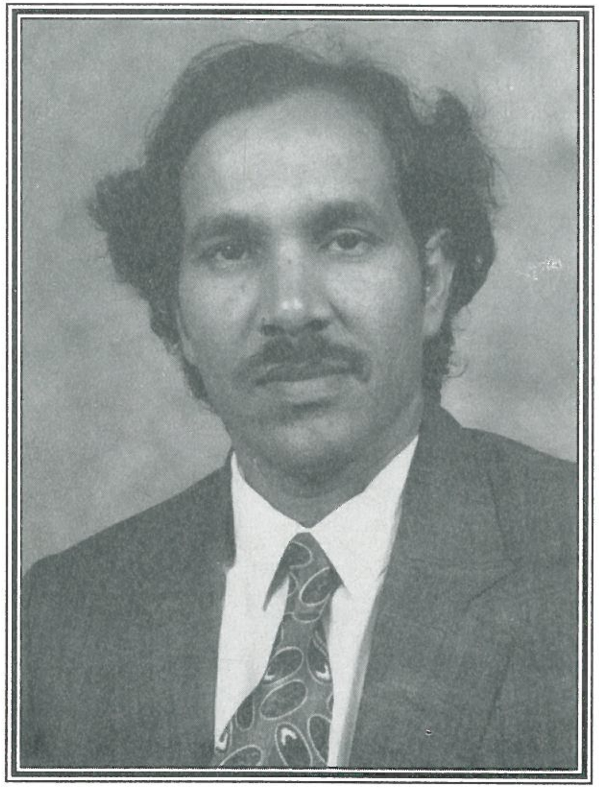

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STATISTICS & THE FOUNDATION OF GOOD THINKING

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Introduction

Mr Chairman (the Vice-Chancellor),
Mr Deputy Vice-Chancellor, Mr Dean, Honoured Guests, my Academic colleagues, Students, Ladies & Gentlemen: My grateful thanks to you all for coming to listen without any imposition of a decree - whose task is more arduous? yours of listening or mine of delivering this lecture. I leave this to be decided later.

It was earlier decreed by our Senate that professors of this university must prove worthy by open declaration of their sentiments, beliefs etc. This in my opinion, is a noble custom.

The topic is of my choice and in my own discipline of which I know something. So here is what I wish to say to you:

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Civilization is sustained by degrees of mutual trust & belief among nations, between & within communities inhabiting & sharing some of the common resources of our planet earth. Once this trust & belief, basic ingredients of peace & happiness are eroded through fear or folly, chaos takes over often resulting in long lasting hostility, hatred, distrust & violence - whose end products are innocent victims of dread of life, driving them out of their own countries to become refugees (often long term) depending upon the mercies of others for survival.

It's therefore, often of great value to establish some measure of trust & belief in order to enhance it or expand it. It's often of paramount importance for individual that given various circumstances, to decide rationally. I must hasten to add here that our methods of making rational decisions should not depend on whether we are statisticians. Consistency, however, is important. A few people think there is danger that too much stress upon consistency may retard the progress of science.

This danger, I do not think is serious. The resolution of inconsistencies will always remain an essential method in

science & in cross examinations. There are occasions when it is best to behave irrationally, but whether there are should be decided rationally.

It's worth looking for unity in the methods of statistics, science & rational thought & behaviour; first in order to encourage a scientific approach to non-scientific matters, second to suggest new statistical ideas by analogy with ordinary ideas, and third because the unity is extremely aesthetically pleasing.

In most subjects people usually try to understand what other people mean, but in Philosophy & Mathematics (near Philosophy) they do not usually try so hard.

SCIENTIFIC THEORIES

No Scientific theory may be deemed really satisfactory until it has the following structure:

1. There should be a very precise set of axioms from which a purely abstract theory can be rigorously

deduced. In this abstract theory some of the words or symbols may remain undefined; e.g. in projective geometry it is not necessary to know what points, lines and planes are in order to verify the correctness of the theorems in terms of the axioms.

2. There should be precisely stated rules of application of abstract theory which give meaning to the undefined words and symbols.

3. There should be suggestions for using the theory, these suggestions belong to the technique rather than to the theory.

The suggestions may not be as precisely formulated as the axioms and rules.

The adequacy of the abstract theory cannot be judged until the rules of applications have been formulated. These rules within themselves, contain indications of what the undefined words and symbols of the abstract theory are all about, but the indications will not be complete. It is the theory as a whole, i.e. the axioms and rules combined, which gives

meaning to the undefined words and symbols. It is mainly for this reason that a beginner finds difficulty in understanding a scientific theory.

It follows from this account that a scientific theory represents a decision and a recommendation to use language & symbolism in a special way, and possibly also to think and act in a particular way. Let us take for example, the fundamental principle of conservation of energy, or energy and matter. Apparent exceptions to this principle have been patched up by extending the idea of energy, to potential energy for example. Nevertheless the principle is not entirely tautological. Some theoreticians formulate theories without specifying the rules of application, so that the theories cannot be grasped at all without a lot of experience. Such formulations are philosophically unsatisfactory.

In the empirical sciences the selection of the theories depends much more on experience. The theory of probability occupies an intermediate position between logic and empirical sciences. Some regard any typical theory of probability as self evident and many others say it depends on experience. The fact is that, as in many philosophical disputes, it is a question of degree: the theory of probability does depend on experience, but does not require much more experience than does ordinary logic. There are a number of different ways of making the theory seem nearly tautological by more or less a priori arguments. The two main methods are those of "equally probable cases" and of limiting frequencies. Both methods depend upon idealizations, but it would be extremely surprising if either method could be proved to lead to inconsistencies. In estimating probabilities, most of us use both methods. It may be possible to trace back all probability estimates to individual experiences of frequencies, but this has not been attempted yet - as far as I know. Two examples in which beliefs do not depend in any obvious way on frequencies are (i) the estimations of the probability that a particular card will be drawn from a well-shuffled pack of 117 cards; (ii) the belief which newly-born calves appear to have that it is a good thing to walk round mother cow's leg to reach her nipples. This example is cited for the benefit of those who interpret a belief as a tendency to act.

DEGREES OF BELIEF & PROBABILITY

We may define the theory of probability as the logic of degrees of belief. Therefore, it's often essential to introduce degrees of belief, either subjective or objective. According to Keynes degrees of belief are assumed to be partially ordered only, ie. some pairs of beliefs may not be comparable at all.

A simple statistical hypothesis H is an idealized proposition such that for some E, p(E/H) is a credibility with a specified value. Such probabilities may be called "tautological probabilities".

RATIONAL BEHAVIOUR

Once the theory of probability is taken for granted, the principle of maximising the expected utility per unit time (or rather its integral over the future) is the only fundamental principle of rational behaviour. It teaches us, for example, that the older we become the more important it is to use what we already know rather than to learn more. In the applications of the principle of rational behaviour some complications arise, such as:

1. We must balance the expected time for doing mathematical & statistical calculations against the expected utility of these calculations. Obviously, at times less good methods may therefore be preferred. For example, in an emergency, a quick random decision is (usually) better than no decision.

2. We must allow for the need to convince other people in some situations. So if other people use theoretically inferior methods we may be encouraged to follow suit. It was for this reason that Newton translated his calculus arguments into a geometrical form in his Principia. Fashions in modern statistics occur partly due to the same reason.

3. We may seem to defy the principle of rational action when we insure articles of fairly small value against postal loss. It is possible to justify such insurances on the grounds that we are purchasing peace of mind,

knowing that we are liable to lapse into an irrational state of worry.

4. Similarly we may take on bets of negative financial utility because the act of gambling has a utility of its own.

5. Because of a lack of precision in our judgement of probabilities, utilities, expected utilities and "weights of evidence", we may often find that there is nothing to choose between alternative courses of action, i.e., we may not be able to say which of them has the larger expected utility. Both courses of action may seem reasonable and a decision may then be arrived at by the operation known as "making up one's mind". Decisions reached in this way are often irreversible, owing to the negative utility of vacillation. People who attach too great a value to the negative utility of vacillation are often known as "obstinate".

6. Public and private utilities may not always coincide. This often leads to ethical problems.

Here is an example: An invention is submitted to a scientific adviser of a firm. The adviser makes the following judgements:

a. The probability that the invention will work is 'p' $(0 < p \leq 1)$.

b. The value to the firm if the invention is adopted and works is V .

c. The loss to the firm if the invention is adopted and fails to works is L .

d. The value to the adviser personally if he advises the adoption of the invention and it works is v.

e. The loss to the adviser if he advises the adoption of the invention and it does not work is 'l'.

f. The losses to the firm and to the adviser if he recommends the inventions to be rejected are both negligible, because neither the firm nor the adviser have any rivals.

Then the firm's expected gain if the invention is accepted is $p V - (1 - p) L$ and the adviser's expected gain in the same circumstances is $p v - (1 - p) l$. The firm has positive expected gain if $p/(1-p) > L/V$, and the adviser has positive expected gain if $p/(1-p) > l/v$. If now $l/v > p/(1-p) > L/V$, the adviser will be faced with an ethical problem, i.e. he will be tempted to act against the interests of the firm.

Of course, real life is more complicated than this, but the difficulty obviously arises. In an ideal society public and private expected utility gains would always be of the same sign.

What can the firm do to prevent this sort of temptation from arising? A suggestion: The firm should ask the adviser for his own estimates of p, V, and L and should take the onus of actual decision on its own shoulder. In other words, leaders of industry should become more probability conscious.

If leaders of industry did become probability conscious there would be quite a reaction on statisticians. For they would have to specify probabilities of hypotheses instead of merely giving advice.

FAIR FEES

The above example raises the question of how a firm can encourage its experts to give a fair estimates of probabilities. In general this is a complicated problem. We can consider here only a simple case and provide only a tentative solution.

Suppose that the expert is asked the probability of an event E in circumstances where it will fairly soon be known whether E is true or false, e.g. in weather forecasts.

It is convenient at first to imagine that there are two experts A and B whose estimates of the probability of E are $p_1 = p_1(E)$, $p_2 = p_2(E)$. We imagine also that their objective probabilities are denoted by p. We introduce hypotheses H_1 and H_2 where H_1 (or H_2) is the hypothesis that A (or B) has objective judgement.
Then $p_1 = p(E/H_1)$; $p_2 = p(E/H_2)$.

Therefore, taking "H_1 or H_2" for granted, the factor in favour of H_i (i.e. the ratio of its final to initial odds) if E happens is p_1 / p_2. Such factors are multiplicative if a series of independent experiments are performed. By taking logs we

obtain an additive measure of the difference in the merits of A and B, namely $\log p_1 - \log p_2$ if E occurs or $\log (1-p_1) - \log (1-p_2)$ if E does not. By itself $\log p_1$ (or $\log (1-p_1)$) is a measure of the merit of a probability estimate, when it is theoretically possible to make a correct prediction with certainty. It is never positive and represents the amount of information lost through not knowing with certainty what will happen.

A reasonable fee to pay an expert who has estimated a probability as p_1 is $k \log(2p_1)$ if the event occurs, and $k \log(2-2p_1)$ if it does not. If $p_1 > 1/2$ the latter payment is really a fine. (k is independent of p_1 but may depend on the utilities. It is assumed to be positive). It can be easily verified that its expectation is maximized if $p_1 = p$ the true probability, so that it is in the expert's own interest to give an objective estimate.

It is also in his interest to collect as much evidence as possible. Note that no fee is paid if $p_1 = 1/2$. The justification of this is that if a larger fee were paid the expert would have a positive expected gain by saying that $p_1 = 1/2$ with looking at the evidence at all. If the class of problems put to the expert have the property that the average value of p is x, then the factor 2 in the above formula should be replaced by $x^{-x} (1 - x)^{-(1-x)} = b$. (For more than two alternatives the corresponding formula for b is $\log b = \Sigma -x_i \log x_i$, the initial entropy. Another modification of the formula should be made in order to allow for the diminishing utility of money (as a function of the amount, rather than as a function of time). In fact if Daniel Bernoulli's logarithmic formula for the utility of money is assumed, the expression for the fee ceases to contain a logarithm & becomes

$$c \{(bp_1)^{k-1}\} \quad \text{or} \quad -c \{1 - (b - bp_1)^k\}$$

where c is the initial capital of the expert.

This method could be used for introducing piece work into the Meterological office. The weather forecaster would lose money whenever he made an incorrect forecast.

Let use talk now about statistics, the applied cousin of probability which has grown in stature and utility so much so that it's now taught to almost all undergraduates all over the world, as a subject in its own right.

From its precarious beginning it has now attained an enviable a respectability. Recalling its past which is not so ancient, one is reminded of the discussions around the questions " What is Statistics?" "Is it a Science?"

In 1856, it is reported from an International Statistical Congress in Paris that " Statistics are to Politics and to the art of governing, what the observation of the star is to "Astronomy". The same year, however, the British Association, still doubting the scientific worth of the subject, enlarged the merit of its statistical section to include all economic sciences.

In 1865 Guy takes up the question with a paper "on the original and acquired meaning of the term statistics". He decides that "there is 'Science of Statistics. . . worthy of respect, encouragement and support."

In 1877, the British Association tries to abolish its statistical section as not being properly scientific, but the attack was repelled. From then on the question seems to die away. The Statisticians continue doing it, & worry less about what "it may be". In 1911, however a book review of Yule's book (later to become Yule & Kendall) claims that doubts . . . whether there be a science of statistics . . . are now definitely resolved . . .", Mr Yule has left the scientific nature of statistical theory plain to the most hardened doubter but, by then, statistical theory was different in kind from anything that would have been envisaged by the founders.

STATISTICS TO-DAY

According to Professor G A Barnard a distinguished British Mathematical Statistician (now Professor Emeritus of Sussex) Statistics is an Applied Mathematical Science which derives its maximum inspiration from Pure Mathematics.

Here, however, is another view of Statistics expressed by Professor David S Moore of Purdue University, USA: Statistics is a mathematical science, but it is not a branch of Mathematics. Statistics is a Methodological discipline, but it is not a collection of methods appended to economics, or Psychology or quality engineering. The historical roots of statistics lie in many of the disciplines that deal with data; its development owes much to mathematical tools, especially

probability theory. But by mid-twentieth century Statistics had clearly emerged as a discipline in its own right, with characteristics modes of thinking that are more fundamental than either specific methods or mathematical theory. We can summarize the core elements of Statistical thinking as follows:

1. The omnipresence of <u>variation</u> in processes. Individuals are variables: repeated measurements on the same individual are variables. The domain of a strict determinism in nature and in human affairs is circumscribed .

2. The need for <u>data</u> about processes Statistics is steadfastly empirical rather than speculative. Hence looking at the data has the first priority.

3. The design of <u>data production</u> with variation in mind. Aware of sources of uncontrolled variation, we avoid self selected samples and insist on comparison in experimental studies. We introduce planned variation into data production by use of randomization.

4. The quantification of variation. Random variation is described mathematically by probability.

5. The explanation of variation. Statistical analysis seeks the systematic effects behind the variability of individuals and measurements.

Let me add as a postscript to this inaugural lecture the following:

The higher goal of teaching Statistics is to build the ability of students to deal intelligently with variation whatever our audience, whether we are focusing on theory or on methods, we ought not to lose sight of that goal.

It has been a daunting task to deliver this lecture and yet both pleasurable and enjoyable. One learns here to lecture to an audience so very different from our students in many ways. Many thanks for your patience & perseverance. If I have been able to convey to you the spirit of my discipline, which I hold dear, I shall be amply rewarded for my efforts.

To my students, I wish to say " Forge ahead with determination to acquire the habit of thinking which will not leave you once acquired, in your future career".

Mr Chairman, I am honoured to accept the Chair of Professor in the Department of Statistics & Operations Research.

Thank you one and all.