EXPLORING A GROUP OF LIMPOPO PROVINCE'S SENIOR PHASE MATHEMATICS TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF ALGEBRA USING CONTENT REPRESENTATIONS

by

MAMOGOBO EUNICCA BOPAPE

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SUPERVISOR: Dr M. K. Chuene

CO-SUPERVISOR: Dr R.S. Maoto

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DECLARATION

I declare that the dissertation hereby submitted to the University of Limpopo for the degree of Master of Education in Mathematics has not previously been submitted by me for a degree at this or any other at any other university, that this is all my work in design an in execution, and that all material contained herein has been duly acknowledged.

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Bopape, M. E. (Mrs)

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ABSTRACT

This research explored the pedagogical content knowledge (PCK) of algebra as held by a group of senior phase teachers in Limpopo Province, South Africa. Sixty-one teachers from different districts in the province participated in this study. This qualitative study used a case study design. Data was collected using a test and content representation (CoRe) matrix.

The results were analysed through analysis of narratives. The study revealed that most of the teachers could not identify the main concepts that are taught in algebra. It was discovered that some could not differentiate between algebra and arithmetic. The findings also point out that the teachers had inadequate knowledge of algebra subject matter. The participants were able to identify some of the concepts that learners were supposed to do in the next grades. The findings revealed that most of the participants did not know the importance of teaching algebra. It was revealed that most participants could not identify the learners' difficulties; they could not specify the procedures they followed when teaching the identified main concepts in algebra, or give reasons for using those procedures. Furthermore, they could not specify factors that influenced their teaching. However, they were able to indicate different methods they used to assess learners' understanding. It was concluded that these teachers had inadequate PCK of senior phase algebra. The study recommends development programmes for both subject matter knowledge and PCK for practising teachers.

Keywords: pedagogical content knowledge, content representations (CoRes), algebra, senior phase mathematics teachers.

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CHAPTER 1: INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

Mathematics is thought to be a key player in many spheres of life, yet it is one of the subjects that learners are mostly struggling with (Anthony & Walshaw, 2009). It has been seen that South African learners, as measured by the Annual National Assessment, have persistently been shown to be poor performers in mathematics (Department of Basic Education [DBE], 2014). In comparison with learners in other African countries that spend far less on education, South African learners' performance is dismal. This poor performance cannot therefore be attributed to lack of material resources only. It is thought that ineffective teaching methods and poor subject matter (content) knowledge are likely the cause for this poor performance (Departments of Basic Education and Higher Education and Training, 2011). Shakir and Lodhi (2016) state that "the quality of education depends to a great extent on the quality of teacher and quality of teacher without having teaching skills seems impossible" (p. 114).

Shulman (1986) identifies effective teaching as the result of the ability to integrate subject matter knowledge and pedagogical knowledge. He coins such integrated knowledge as pedagogical content knowledge (PCK). He further observes that PCK develops with experience and that it is topic-specific. In support of this, Khan (2012) maintains that effective teaching requires a good relationship between subject matter knowledge and the knowledge of how the subject matter should be imparted to learners.

It is apparent that learning and teaching practices are informed by effective teachers' PCK (Garritz, 2015; Ijeh & Nkopodi, 2013). This follows from the assertion made by Shulman (1986), that PCK "also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and background bring with them to the learning" (p. 9). Consequently, this understanding helps effective teachers to employ different strategies to represent subject matter in such a way that it will be understood by learners (Shulman, 1986). Brijall, Basilal and Moore-Russo (2012) also indicate that if teachers in South Africa can employ the use of representations in mathematics classrooms, then democratic values can be promoted through supporting development of classroom environments.

Shakir and Lodhi (2016) state that "in order to educate in the 21st century, teachers are required to cultivate and maintain the students' interest in the material by showing how this knowledge applies in the real world" (p. 114). This is shared by Ijeh and Nkopodi (2013) who state that the results of teachers' effective PCK will be seen when learners are able to "communicate, reason, apply and transfer classroom content in various facets of their environments and other disciplines in the school curriculum" (p. 474). Also, when teachers have a well-developed PCK performance of learners will improve (DoE, 2007; Desimone, Porter, Garet, Yoon & Birman, 2002; Garritz, 2013; Hill, et al., 2008).

1.2 RESEARCH PROBLEM

1.2.1 Source of the problem

DoE (2007), through The National Policy Framework for Teacher Education and Development in South Africa acknowledges the role of PCK by observing that "both conceptual and content knowledge and pedagogical knowledge are necessary for effective teaching. ...These attributes need to be integrated..." (p. 16). On the other hand, The White Paper Notice 196 on Education and Training of 1995 indicates that theory and practice were treated separately pre-1994, when South Africa became a democracy, as a result, it stipulates the integration. Deducing from the white paper and observation, it can be concluded that the department used to view knowledge of subject matter and pedagogical knowledge as separate primary requirements for a qualification in teaching a subject. However, qualification in a subject does not automatically result in effective teaching of that particular subject (Kind, 2009; Shulman, 1986). Hence, acquisition of PCK is found to be of benefit for effective teaching (Garritz, 2013, 2015; Ijeh & Nkopodi, 2013; Kind, 2009; Shulman, 1986).

1.2.2 Background to the problem

Research reveals that effective teaching is one of the important factors contributing to performance (Department of Education [DoE], 2007; Desimone et al., 2002; Garritz, 2013; Hill, Ball, & Schilling, 2008; Hume & Berry, 2011). Hence, different qualities associated with effective teaching are listed in the literature. For example, amongst these, good knowledge of subject matter (content) and pedagogical knowledge are also listed as important components of teaching and learning (Ball, Thames, & Phelps, 2008; Garritz, 2015; Ijeh & Nkopodi, 2013; Shulman, 1986).

Therefore, DoE (2007) through the National Policy Framework for Teacher Education and Development in South Africa specifies that “all programmes developed as a result of this policy, must emphasise the integrated development of learning area or subject knowledge content and pedagogical skills” (p. 29). Despite this prescription, the present researcher observed from the professional developmental programmes, which were arranged for Limpopo FET mathematics curriculum advisors, that much emphasis was put on the development of subject matter knowledge only. As a result, when district curriculum advisors arrange professional developmental programmes for mathematics teachers, they also pay more attention to the development of subject matter knowledge only, thus failing to contribute to mathematics teachers’ growth in PCK. Therefore, it can be asserted that, no time is scheduled to either explore or develop mathematics PCK for teachers.

1.2.3 Statement of the research problem

PCK of teachers of mathematics in Limpopo Province was hardly known because it was not documented through empirical evidence. Hence this exploratory study was designed to document senior phase mathematics teachers' PCK of algebra in the province.

1.3 LITERATURE REVIEW

The literature reveals that PCK is a major contributory factor of effective teaching (Ijeh & Nkopodi, 2013; Kanyongo & Brown, 2013; Shulman, 1986, 1987). Although some researchers such as Shulman (1986, 1987) view subject matter as a component of PCK, researchers like Ball et al. (2008) view subject matter as separate from PCK. Although there are different views about what PCK is composed of, there is agreement that it is the knowledge of practice that develops with time and which promotes effective teaching. However, a consensus was reached in a PCK summit held in USA in 2012 (Fernandez, 2014; Garritz, 2015). In the agreed model, subject matter is one of the components of PCK.

Studies also reveal that there are different methods used to explore PCK. The methods include test, questionnaire, interview, lesson observation, document analysis, CoRes and pedagogical and professional experience repertoirs (PaP-eRs), and concept mapping (Depaepe, Verschaffel, & Kelchtermans, 2013; Fernandez, 2014). On the other hand, CoRes and (PaP-eRs) are also used to develop PCK (Hume & Berry, 2011; Williams & Lockley, 2012).

1.4 PURPOSE OF THE STUDY

The purpose of the study was to explore algebra pedagogical content knowledge of a group of senior phase mathematics teachers in Limpopo Province using a content representation (CoRe).

1.5 RESEARCH QUESTION

The research question that was answered in the study to address its purpose was: What PCK of algebra do senior phase teachers hold?

1.6 RESEARCH METHODOLOGY

This study, which explored senior phase teachers' algebra PCK, used the qualitative approach. This approach was chosen because the main purpose of the study was to seek the in-depth understanding of algebra PCK that the teachers held. Therefore, quantitative study, which aims at generalizing the results to a larger population (Marshall, 1996) would not be suitable for this study.

1.6.1 Research Design

In order to get a deeper understanding of the nature of PCK that the algebra teachers held, this study adopted a case. A case study was a better choice among the other possible designs for these three reasons: (i) the study was dependent on the knowledge of the teachers at that time of data collection, unlike in histories that depend on past phenomenon; (ii) I did not have control over variables that were at play as is the case with experiments and (iii) the study needed a few participants to get an in-depth PCK that the teachers held, which would not suffice for a survey (Noor, 2008; Rowley, 2002; Yin, 2003).

1.6.2 Sampling of participants

The participants in the study were purposively selected in accordance with the needs of the study (Cohen, Manion, & Morrison, 2005) – exploration of algebra PCK inherent in teachers. Practising senior phase mathematics teachers who had applied successfully for training programme at MASTEC Institute were sampled for this study. About 80 teachers from the five districts (i.e. 16 teachers per district) in

Limpopo Province were selected to take part in the programme. However, due to some logistical problems in the districts, 61 teachers attended the programme. Then all those teachers agreed to form part of this study.

1.6.3 Data collection

Of the recommended data collection methods for case studies, written responses to a test (see Appendix A) and compilation of CoRe (see Appendix B) were used in the study. The test was developed through the help of some experts from the University of Limpopo. The first question of Part 1 of the test and all other questions in Part 2 and Part 3 of the test were adapted from mathematics module 4 for upper primary and junior secondary school teachers developed by the Southern African Development Community (SADC) in partnership with The Commonwealth of Learning (2001). The other remaining questions in Part 1 of the test were adapted from the guide used for training GET senior phase educators in Limpopo at MASTEC. The participants wrote the test as individuals. However, the CoRe matrix, which the participants compiled in groups, was adapted from Loughran, Mulhall, & Berry, 2004).This was to determine the PCK of individuals and also the PCK of grade groups since CoRes are social constructs (Loughran et al., 2004).

1.6.4 Data analysis

All collected data were configured through analysis of narratives (Polkinghorne, 1995). That is, data with the same theme were classified and different categories were developed according to the themes identified. Then the ideas from the CoRe were applied to determine how these ideas were demonstrated in the collected data.

1.6.5 Rigour of the study

In qualitative studies credibility and dependability show the truth of the results, while transferability has to do with the experiences of the participants and confirmability should reflect the adequacy of the information Rolfe (2006).

The study used data triangulation, i.e., the use of different data sources Bitsch (2005) through its 61 participants who wrote the same test and compiled CoRes on the same topic to ensure credibility of the results. Also, adequate time was allocated for the writing of the test and the completion of the CoRe matrix. Furthermore, confirmability of data was addressed through availing configured data to the participants for verification of results – member checking (Creswell & Miller, 2000). In addition to all these, for possible transferability of results, a full description of the participants' mathematics teaching experience and educational background were provided.

1.6.6 Bias

Even though the study had earmarked the group that was selected to participate in a development programme, the grade groups that were used for the study were due to the participants grouping themselves. This move was to reduce bias.

1.7 ETHICAL CONSIDERATIONS

To get access to the teachers who were selected to be trained in one of the continuing professional development (CPD) centres, I first wrote a letter to the head of the institution where the training was going to take place. Then the head advised me to address the letter to the Head of the Department of Education (HoD) in Limpopo Province. The letter is attached in Appendix C. I then got permission from the HoD of Education in Limpopo Province, which is attached in Appendix D. Furthermore, I also asked for consent from the teachers who were to be trained to be part of my study by first informing them about the purpose of the study. The consent form is attached in Appendix E. In addition, I assured them that they would be trained even if they did not want to take part in the study. I did not put pressure on them to participate. I also assured them about the confidentiality of all the information obtained from them.

1.8 SIGNIFICANCE OF THE REPORTED STUDY

In order to improve the quality of teaching in our schools, emphasis should be laid on effective teaching. Since PCK was identified as one of the major contributory factors to effective teaching, then this study was aimed at exploring that knowledge. Therefore, the study added to the existing body of knowledge about PCK and the use of CoRe matrix to identify in Limpopo Province’s mathematics teachers’ algebra PCK. Such studies were non-existent in the literature reviewed. It is hoped that service providers will take necessary steps to further identify and develop mathematics teachers’ PCK on other topics in this province.

1.9 CONCLUSION

This chapter, discussed the background of the study where the problem of the study originated, the literature reviewed, the purpose of conducting this study, the question that was answered by the study, the methods which were used to conduct the research, steps which were followed for ethical purposes, and the significance which emphasised the need of this study. The next chapter presents literature reviewed in this study.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

The literature reviewed in this section discusses: pedagogical content knowledge (PCK), components of PCK, development of PCK, measurement of PCK and the theoretical framework of the study.

2.2 PEDAGOGICAL CONTENT KNOWLEDGE

To a large extent, learning and teaching are determined by teachers' knowledge (Fernandez, 2014, Shakir & Lodhi, 2016). By teachers' knowledge I mean all the knowledge that is required for teachers to be effective in class. I therefore assert that a learner-centred approach, which includes enriching the knowledge that learners already have and addressing their difficulties, will result in effective teaching (Shulman 1986). This is supported by Ball et al. (2008) who affirm that learners understand more when teaching focuses on their misconceptions or their difficulties. Also, Shakir and Lodhi (2016) maintain that "teachers are expected to use the best practices and strategies to meet challenging demand of their career, which involves imparting knowledge and developing essential skills and attitude in the students" (p. 115).

In the light of the above, Shulman (1987) outlines what he calls knowledge base for effective teaching. This knowledge base is composed of content knowledge, general pedagogical knowledge, curricular knowledge, pedagogical content knowledge (PCK), knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of purposes, educational purposes and educational values and their philosophical and historical bases. This implies that acquisition of this knowledge base will improve teaching and learning.

Among all the components of knowledge base, Shulman (1986, 1987) distinguishes pedagogical content knowledge (PCK) as a special kind of content knowledge that he claims is central to effective teaching. He distinguished PCK after realising that the knowledge of subject matter only does not translate into effective teaching. It is how well one conducts learning that can translate into effective teaching (Ball et al., 2008; Fernandez, 2014; Ijeh, 2013; Shulman, 1986). Shulman (1986, 1987) discovered that effective teachers have knowledge of blending subject matter knowledge and pedagogical knowledge. He named this ‘blending’ of subject matter and pedagogical knowledge PCK. He indicates that during this blending, subject matter knowledge is transformed in such a way that the knowledge to be transferred will be understandable to learners. This is supported by Ball and Bass (2000) who point out that teachers should know definitions of concepts and be able to use these definitions to address learners’ difficulties. Shulman (1986, 1987) further reveals that PCK is specific to the subject or topic being taught, unique to each teacher, and develops with experience of teaching. Furthermore, he indicates that PCK differentiates expert teachers from subject specialists. Again, he reveals that PCK “goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching” (Shuman, 1986, p. 9).

Shulman’s notion of PCK is supported by many researchers, who amongst others include Hurrell (2013), Ijeh and Nkopodi (2013), Kanyongo and Brown (2013), Loughran et al., (2004), Van Driel, Verloop, and De Vos (1998), and Plotz, Froneman & Nieuwoudt (2012). These researchers also view PCK as knowledge for practice which develops with time. To complement this, Hurrell (2013) regards PCK as some form of knowledge of teaching and learning which is practical and which is dependent on the contextual knowledge of the setting of a particular classroom. In view of all these, Cankoy (2010) summarises PCK as ‘special attributes’ that a teacher uses to impart knowledge to learners. Hence, Van Driel et al. (1998) regard PCK as some form of craft knowledge that determines teachers’ actions in the way that they use subject matter. On the other hand, Hashweh (2005) defines PCK as:

> the set or repertoire of private and personal contents specific general event-based as well as story-based pedagogical constructions that the experienced teacher has developed as a result of repeated planning and teaching of, and reflection on the teaching of, the most regularly taught topics. (p.77)

Seemingly, all these researchers accept Shulman's concept of PCK as the knowledge of practice which develops with time as teachers regularly teach the same topic. It is apparent that these researchers also view PCK as the knowledge of practice which makes teaching to be effective.

Some researchers expand the concept and others try to modify it. Among these are Cochran, DeRuiter, and King (1993) and Mishra and Koehler (2006). Cochran et al. (1993) claim that PCK as conceptualized by Shulman is fixed. They define PCK as the knowledge of "using the understandings of subject matter concepts, learning processes, and strategies for teaching the specific content of a discipline in a way that enables students to construct their own knowledge effectively in a given context" (p.12). Hence they suggest that pedagogical content knowledge (PCK) be changed to pedagogical content knowing (PCKg) to show that it is a process. On the other hand, Mishra and Koehler (2006) indicate that when teaching and learning are supported by the use of technology, then, the blending of content knowledge, pedagogy and technology occurs. For that reason, they assert that technology should be added to Shulman's (1986) notion to form technological pedagogical content knowledge (TPCK). However, other researchers such as Garritz, Labastida-Piña, Espinosa-Bueno and Padilla (2010) name it according to the nature of PCK that is explored. Hence, they name the knowledge that is required to teach through inquiry, pedagogical inquiry/content knowledge (PICK). Also, Kwong, Joseph, Eric, and Khoh (2007) investigated pedagogical content knowledge of mathematics and they called it MPCK.

2.3 THE COMPONENTS OF PCK

What constitutes PCK is what is debated in the literature. According to Shulman (1986), PCK is composed of

> the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations- in a word, the ways of representing and formulating the subject that make it comprehensible to others... also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9)

Cochran et al. (1993) include environmental context to Shulman's components to make four components of PCK. As a result, their components are subject area knowledge, pedagogical knowledge, knowledge of students and knowledge of the environmental context. On the other hand, Magnusson, Krajcik and Borko (1999) reveal that PCK is made up of five components that help to improve both teachers' and learners' classroom performance. Their components include, orientation towards teaching, knowledge of curriculum on what and when to teach, knowledge on why, what, and how to assess, knowledge of students' understanding of the subject, and knowledge of instructional strategies. The addition of knowledge of assessment makes this model to differ from Cochran's model.

Gess-Newsome (1999) argues that the elements of knowledge base as identified by Shulman (1987) are interrelated. Hence, she comes to the conclusion that knowledge of specific content, curriculum knowledge, assessment procedures, pedagogical knowledge, knowledge of learners and learning, knowledge of specific contexts are all related to PCK. However, Hashweh (2005) views Gess-Newsome's concept differently. He claims that when PCK interrelates with all other components of knowledge base and beliefs, its initial meaning and its development change. He therefore suggests that, in that case, PCK should be given a different name, which is teacher pedagogical constructions (TPC), to suit the new role.

Ball et al. (2008) on the other hand view PCK not as subject matter for teaching, but as a component of subject matter for teaching, which is separate from subject matter knowledge (See Figure 2.1 below). They view PCK as composed of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC). They argue that one can teach effectively without using PCK, but using subject matter only. Their components of subject matter knowledge are common content knowledge (CCK), specialised content knowledge (SCK) and knowledge at the mathematical horizon. According to their model, subject matter knowledge is viewed as separate component from PCK.

The following figure is *adapted from Journal of Teacher Education* (Ball et al., 2008, p. 403).

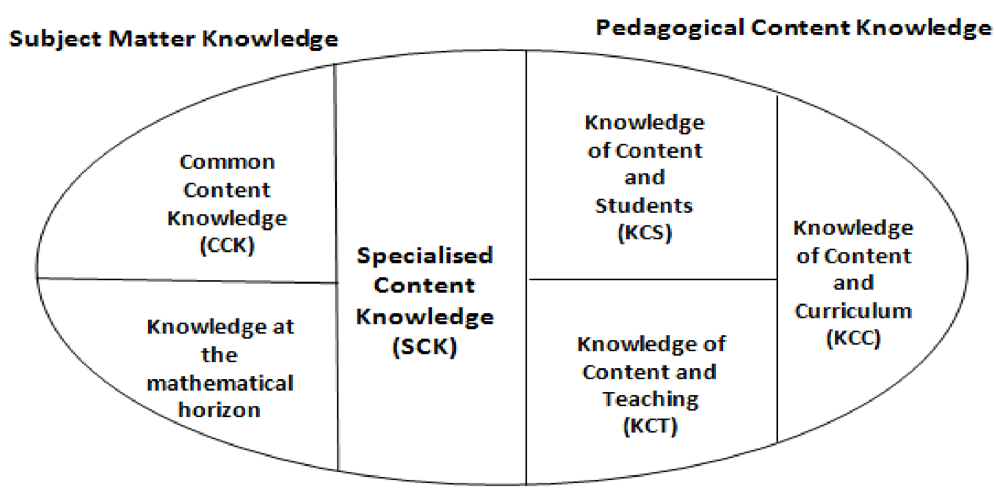


Figure 2.1: Domains of mathematical knowledge for teaching.

Contrary to Ball et al.'s (2008) model, is the assertion made by Shulman (1986) that PCK is the transformation of subject matter knowledge into subject matter for teaching. This view is shared by Hashweh (2005), who maintains that PCK is neither a component of subject matter for teaching nor a component of general pedagogy. Even though Ball et al. (2008) view PCK as different from subject-matter-for-teaching, the difference between the two is not easily identifiable. This follows from their assertion that SCK is a unique knowledge possessed only by teachers, and does not form part of PCK. Furthermore, they indicate that mathematics that teachers need to teach is different from the knowledge of mathematics that any mathematician has. The same distinction has been raised by Shulman (1986) when comparing PCK and subject matter knowledge. For this reason, one can argue that SCK is basically part of PCK.

Further views against Ball et al.'s (2008) model were made by researchers like Hurrell (2013), who indicates that there is no connection between subject matter knowledge and PCK in Ball et al.'s model. They also argue that by definition of PCK, there must be an integration of subject matter knowledge and pedagogical knowledge. This led to Hurrell's refinement of Ball's model of Mathematical Knowledge for Teaching.

Hurrell's revised model of mathematical knowledge for teaching shows the interaction between subject matter knowledge and pedagogical knowledge. This model shows that knowledge of content and students, knowledge of content and curriculum knowledge of content and teaching, common content knowledge, specialised content knowledge and knowledge at the mathematical horizon are components of mathematical knowledge for teaching. In short, this new mathematical knowledge for teaching formed by integrating subject matter knowledge and pedagogical knowledge basically forms PCK. Hurrell's model is illustrated in Figure 2.2 below.

The following figure is adapted from Australian Journal of Teacher Education (Hurrell, 2013, p. 61).

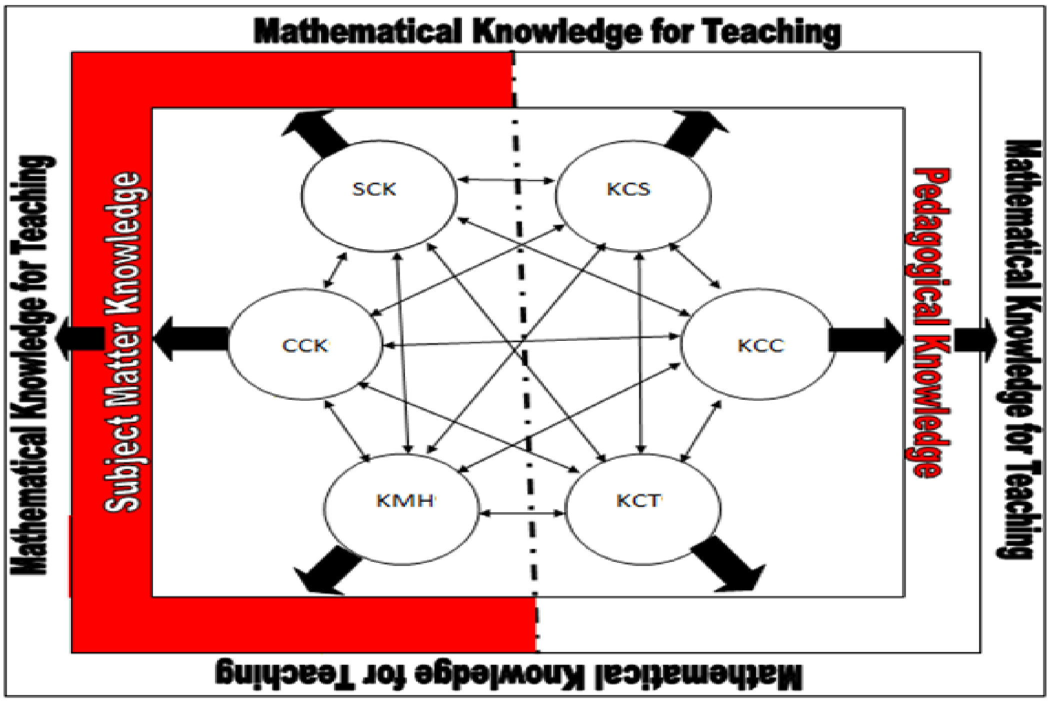


Figure 2.2: Revised model for Mathematical Knowledge for Teaching.

After realising the different interpretations of PCK, a summit was held in Colorado (USA) in 2012 with "an interest in understanding the development and measurement of pedagogical content knowledge (PCK)" (Carlson, Stokes, Helms, Gess-Newsome, & Gardner, 2015, p. 14). During this summit, a consensus was reached about its definition and composition (Fernandez, 2014; Garritz, 2015; Lehane & Bertram, 2016). The following Figure 2.3 represents the agreed model of PCK at the summit.

*This figure is adapted from Inverness Research Inc. (*Helms & Stokes, 2013, p. 17).

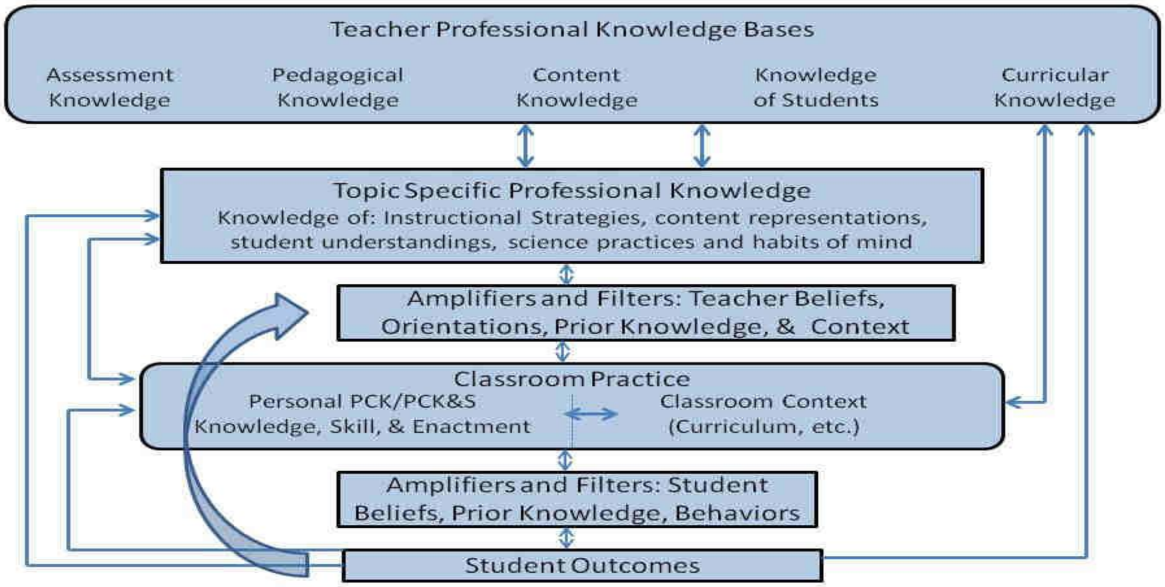


Figure 2.3: Consensus PCK model.

This model identifies five teacher knowledge bases which are: assessment knowledge, pedagogical knowledge, content knowledge, knowledge of students and curricular knowledge. These are referred to as teacher professional knowledge bases. According to this model, professional knowledge bases inform and they are informed by topic specific professional knowledge. This topic specific professional knowledge is made up of instructional strategies, content representations, student understanding, science practices and habits of mind. The model also shows that topic specific professional knowledge is transformed into personal PCK during classroom practice and it is assessed through student outcomes. For it to be transformed, it has to go through filters: which are teacher beliefs and orientation, and also include contexts. However, PCK also filters through student beliefs, prior knowledge and behaviours.

2.4 DEVELOPMENT OF PCK

As studies reveal, PCK is not an in-borne knowledge, it develops with experience (Cochran et al., 1993; Kind, 2009; Shulman, 1986). Therefore, in this section, I discuss the development of PCK under subject matter knowledge, teaching experience and professional development, which seem to be the contributory factors in the development.

2.4.1 Subject matter knowledge

Studies reveal that a good knowledge of subject matter is required for the development of PCK (Karaman, 2012; Kind, 2009; Plotz, et al., 2012; Shulman, 1986; Van Driel & Berry, 2010). Hence, Baumert et al. (2010), Van Driel and Berry (2010), and Van Driel, Verloop and De Vos (1998) emphasise that inadequate subject matter knowledge and misconceptions retard PCK development. Unfortunately, there are arguments that teachers who possess a good qualification in a subject do not guarantee that they will teach the subject well (Ijeh, 2013; Karaman, 2012; Kind, 2009). This is underscored by Shulman (1986) who views knowledge of content only as useless if there are no skills accompanying its application. This view is also shared by Brijlall (2014) who also indicates that teaching is more that the knowledge of subject matter. This is more so if teaching is viewed as the "ability to adapt, adjust and make appropriate professional judgments [and thus making it] crucial to shaping the manner in which teachers teach and respond to their students' learning" (Loughran, Berry & Mulhall, 2012, p. 2). Although this is true, good subject matter knowledge remains the heart of development of PCK (Karami, 2016; Plotz et al., 2012).

The Department of Basic Education (DBE) requires all teachers of mathematics to teach all topics covered in The Curriculum and Assessment Policy and Statement (CAPS) document in classes that they teach. Algebra content knowledge that is required to be taught in the senior phase (Grades 7-9) covers expressions, patterns, equations, graphs (limited to the drawing of linear graphs), functions and relationships. This implies that teachers of mathematics in this phase should know the definition of algebraic terms; be able to identify parts of an equation; determine input values, output values or rules for patterns and relationships; determine a relationship in different ways; be able to simplify algebraic expressions; factorise algebraic expressions; interpret graphs; draw graphs from given descriptions of a problem situation, and also, draw linear graphs from given equations. As indicated earlier, a good knowledge of these concepts is the first step towards the development of PCK of algebra in this phase.

When teachers have knowledge gaps, their learners will also have knowledge gaps. This is supported by Brijall and Maharaj (2014), who after realising that the quality of students they were receiving at their institutions was not satisfactory, started programmes for supporting teachers. Their participants were required to do certain tasks in algebra, trigonometry and calculus to assess their knowledge. It was found that some of the teachers performed below 60%. This indicates that some teachers are struggling with the very subject matter they are required to teach to learners.

The importance of knowledge of subject matter is shown in a study conducted by Stylianides and Stylianides (2006). In their study, an episode of a Grade 3 mathematics lesson where learners were investigating what happens when two odd numbers are added was analysed. It was observed that the teacher was able to engage her learners to come up with a formulation of a conjecture. That is, learners had to make sense of the mathematics involved so that they could establish the truth. Therefore, the teacher focused her learners' attention to the definition of concepts and was able to base the proofs on definitions. As a result, the learners had to give reason and prove their findings. Based on their observations, the researchers were able to conclude that the teacher was able to analyse tasks so that she could make appropriate representations. It can be seen that this lesson clearly reflects the integration of subject matter, the knowledge of learners' conceptions and the knowledge of teaching strategies. On that note, one can safely say that without the knowledge of subject matter, guiding learners to formulation of conjectures and making appropriate representations would not be easy if not impossible.

2.4.2 Experience of teaching

Experience proves to be another essential factor in the development of PCK. As numerous studies indicate, PCK develops with time as teachers gain experience of teaching (Ball et al., 2008; Cochran et al., 1993; Kind, 2009; Shulman, 1986; Van Driel et al., 1998; Williams & Lockley, 2012). This means, teachers without any experience of teaching a subject or with a few years of teaching experience will have no PCK or will have a little PCK (Mulhall et al., 2003; Shulman, 1986; Van Driel & Berry, 2010). This is illustrated in the study conducted by Kanyango and Brown (2013). In their study, these researchers discovered that teachers' classroom practices are related to their level of content knowledge. This follows from their observations, which indicate that teachers with high content knowledge of mathematics employed individual approaches for students in solving problems. Furthermore, they discovered that such teachers used examples that were related to practical situations than teachers who were less knowledgeable in the subject. As Flyvbjerg (2006) puts it, "It is only because of experience ... that one can at all move from being a beginner to being an expert" (p. 222).

Kwong et al. (2007), Lannin et al. (2013) and Wood (2003), whose studies also show the effect of experience on the development of PCK, supported the assertion by Flyvbjerg (2006), in a two-year study conducted by Lannin et al. (2013), which focused on two teachers who had just started teaching. The aim of the study was to explore the kind of knowledge that pre-service and novice teachers bring to class. In addition to the exploration, they studied how that knowledge develops over a course of time. It was found that in the beginning of the study, one teacher lacked knowledge of students' understanding of mathematics and the other teacher lacked knowledge of instructional strategies. As time progressed, it was found that both teachers' knowledge improved. Also, in a different case study conducted by Wood (2003), experience proved to play part in the development of a beginner's PCK. After teaching for the first semester, the teacher studied all the lessons that she taught and with the knowledge and the experience she gained in the first semester, she was able to modify her lessons.

Lack of practice, on the other hand, could result in ineffective teaching. This is shown in the study conducted by Johnson and Larsen (2012). In their study, the researchers analysed three classroom episodes of a mathematician who was implementing an inquiry-oriented abstract algebra. Their aim was to explore the kind of knowledge needed by the mathematician as she implemented the curriculum. It was observed that the students tried hard to understand the content at hand and on the other hand, the mathematician could not understand the students' difficulties. In the first episode, it was observed that when the students asked questions, the mathematician was unable to make sense of them because she could not understand the way the students thought. Also, in the second episode, the mathematician tried to make a counter example of the answer that the students repeatedly arrived at, but she was unable to deduce that the counter example was not convincing to the students and some students did not even understand it. Furthermore, in the last episode, the mathematician responded to the students' questions by making an argument that did not make mathematical sense to the students. Considering these observations, it can be concluded that lack of teaching skills gained through experience could compromise learning.

When teachers teach different learners, and as circumstances change, some of the experiences they gain from learners are different, requiring development of different strategies to meet learners' needs (Hurrell, 2013). As Shulman (1986, 1987) indicates, PCK is continuously developing. This continuous development is possible when teachers follow a cycle of activities which Shulman (1987) regards as key to PCK for effective teaching. These activities are: (a) comprehension (when a teacher understands what he teaches), (b) transformation (when a teacher can transform what he has learned in a way that it can be taught), (c) instruction (when a teacher observes teaching tasks, e.g. giving work and checking it),

(d) evaluation (when a teacher assesses formally or informally), and (e) reflection (when a teacher looks back at what has been achieved or what needs to be improved for effective teaching to take place) and new comprehension (when a teacher is able come to the new understanding after discussing, analysing and developing some strategies of the ideas taught). This implies that going through this cycle guarantees PCK development. These activities are illustrated in Figure 2.4 below.

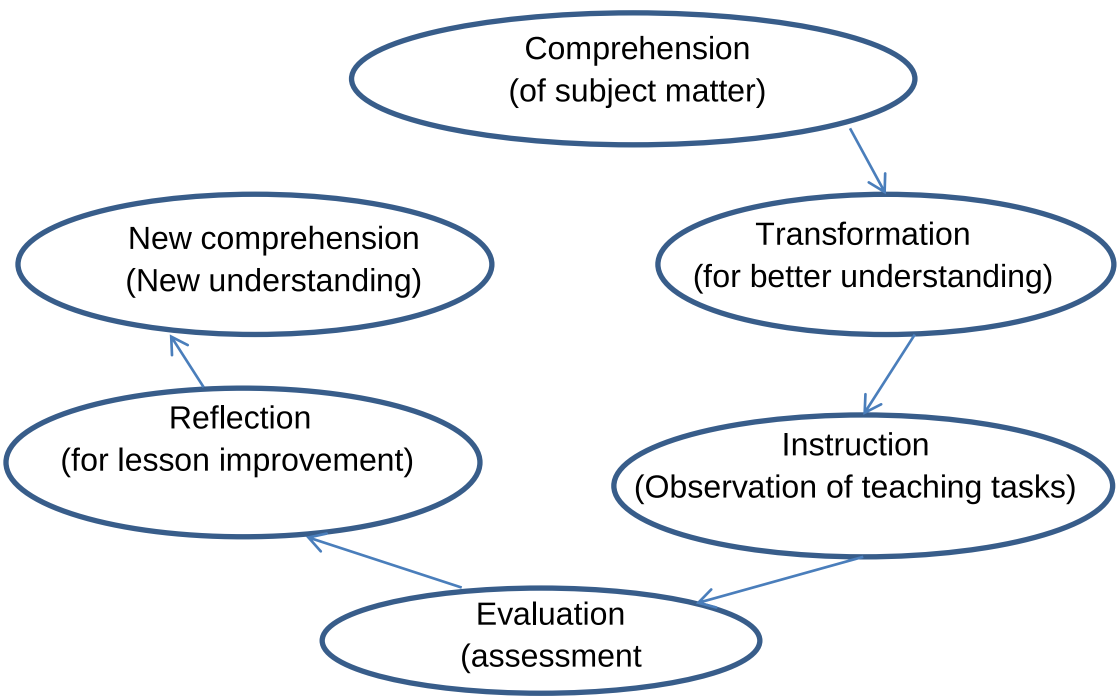


Figure 2.4: Shulman's PCK developmental cycle of activities

2.4.3 Professional development

Professional development programmes and pre-service institutions can also be used for developing pedagogical content knowledge of teachers (Ijeh, 2013). This is supported by Desimone et al. (2002) who indicate that "Professional development is considered an essential mechanism for deepening teachers' content knowledge and developing their teaching practices" (p. 81). Davis and Simmt (2006) and Kaino and Moalosi (2013) add to this view by asserting that for mathematics teachers to develop in conceptual knowledge needed for teaching, teacher education programmes should integrate matters associated with PCK development in mathematics courses. By so doing, chances of teachers who can communicate mathematics effectively will improve (Faulkner & Cain, 2013). This view is also shared by Ball (2000) who makes a call for pre-service and professional development programmes to find some ways of bridging the gap between content knowledge and practice. Ball further emphasises that teachers should be developed in such a way that they would reach the level of understanding of learners. To reach this goal, she suggests that firstly, the content knowledge that is required for teaching should be identified. Secondly, there should be an understanding of how to hold the identified knowledge. Thirdly, teachers should learn how to apply the identified knowledge in practice. To add to these suggestions, Hurrell (2013) emphasises that teachers should be given opportunities to reflect on their teaching during professional development programmes. With such practices, teachers are likely to apply what they learnt during their development in their classrooms (Ball et al., 2008).

Another essential point is that documented practice of good teachers can be used as a starting point in training teachers without experience (Fernandez, 2014). This point is proven by studies that show that one of the effective ways of developing teachers' PCK is by constructing content representations (CoRes) (Hume & Berry, 2011; Lougran et al., 2003; Williams & Lockley, 2012). This follows from the assertion that “CoRes attempt to portray holistic overviews of teachers’ PCK related to the teaching of a particular science topic, e.g., chemical reactions, to make the tacit nature of this expert PCK explicit to others” (Hume & Berry, 2011, p. 344). A summary of these studies is made in the study by Lehane and Bertram (2015).

CoRes were used in Australia and some other parts of the world to develop PCK of pre-service science teachers (Lehane, O’Reilly, & Simmie, 2013; Williams & Lockley, 2012). For instance, in the exploratory study conducted by Hume and Berry (2011), CoRes developed by expert science teachers were studied by pre-service science teachers and then, they later developed their own in different topics. The study shows that initially the students found it difficult to complete the task because they did not have classroom experience. In the year that followed, learning was scaffold for these pre-service teachers to get the information needed before the construction of CoRe. After scaffolding, the process of developing CoRe showed a potential for developing PCK.

In another study conducted by Williams and Lockely (2012), early career science and technology teachers, an expert scientist, an expert technologist, experienced science and technology researchers, worked together to construct science and technology CoRes matrix. The teams’ construction of CoRes helped the early career teachers to explore more about the topic and they also learnt new strategies of delivering the content. As a result, the early career teacher’s PCK was developed. Further related studies were conducted to include technology and geography (Williams & Lockley, 2012).

In further developments of PCK, Garritz (2015) indicates that PCK is either personal or canonical. He indicates that “canonical PCK can be shared and applied by many teachers, and personal PCK is substantiated by personal experience and beliefs/orientations of a single teacher” (p. 77). It emerges that personal PCK develops in an individual through practice, while canonical PCK can be developed by a group of teachers. This is also illustrated in the summit model in Figure 2.3. Also, these points are shown in the studies of PCK that were already discussed in this study. For example, in the study by Hume and Berry (2011), where a group of teachers developed a CoRe together, which in turn helped to develop PCK of others.

2.5 MEASUREMENT OF PCK

Different methods are used to explore PCK. These methods include: test, questionnaire, interview, lesson observation (through field notes, video- or audiotaping), observation of meetings (e.g., meetings with different (preservice) teachers, workshops, intervention courses), document analysis (e.g., lesson plans, portfolios, logbooks, messages on electronic discussion boards), concept mapping, CoRes and pedagogical and professional experience repertoirs (PaP-eRs) (Depaepe et al., 2013; Fernandez, 2014). To add to this list, online teacher PCK analyser (OTPA) was developed to assess programming PCK of teachers of informatics education in the study conducted by Saeli, Perrenet, Jochems, and Zwaneveld (2012). But in the literature that I reviewed, most studies conducted in mathematics use a combination of interview, observation, and a test to explore teachers' PCK. Such studies include those conducted by Ball et al. (2008), Kim (2014), Shakoor and Azeem (2011), Turnuklu and Yesildere (2007).

In this study, I used the combination of test and CoRe to explore algebra PCK of senior phase teachers. CoRe was chosen because I wanted to uncover PCK that could not be observed in one lesson or that could not be obtained from conducting one interview (Mulhall et al., 2003). CoRes were first developed in Australia by Loughran et al. (2004) in their longitudinal study to uncover PCK of experienced science teachers. Initially these researchers used observations and interviews for the exploration. After realising that the methods they used would not help them to document their results in such a way that they could be represented to others, they developed CoRe. Hence the use of CoRe is observed mostly in science. To add, Lehane and Bertram (2016) conducted a study of different studies that used CoRes. None of these studies was conducted in mathematics. This shows that the literature seems to be silent on the use of CoRe matrix to either identify or develop mathematics teachers' PCK. Therefore, this study is also an attempt to break this silence. An individual written test was chosen because of its popular use in mathematics and also to complement the group compiled CoRe.

Studies that employed the tools that I used for the measurement of PCK include a study conducted by Turnuklu and Yesildere (2007). In this study, these researchers used a test to determine the competency of pre-service primary mathematics teachers in pedagogical content knowledge of mathematics. The questions used in the study were open-ended. Through this test, it was found that the teachers had a good knowledge of subject matter. Their knowledge of mathematics subject matter proved to be insufficient for the teaching of mathematics. Also, in another study conducted by Kwong et al. (2007), a test was used to measure the development of mathematical pedagogical content knowledge (MPCK) of student teachers in Post Graduate Diploma in Education (PGDE) programme. The test was administered at the beginning and at the end of the programme. It was found that at the beginning of the programme, the students were weak in MPCK, but there was a huge improvement at the end of the programme. As indicated earlier, the use of CoRes is mainly in science studies. Some of these studies are summarised in the study conducted by Lehane and Bertram (2016). These studies include the one that was conducted by Chapoo, Thathong, and Halim (2013) to explore the PCK of a biology teacher in Thailand. The CoRe completed by the teacher showed he had good assessment practices. Hence, in the present study, CoRe was used to reveal algebra PCK that the participants had.

2.6 THEORETICAL FRAMEWORK

CoRe was used as a framework to document the algebra PCK of senior phase teachers in this study. CoRes as originally constructed, were used to capture, document, and portray PCK of science teachers who were regarded as expert in the field (Loughran, Mulhall, & Berry, 2004; Mulhall et al., 2003). Hume and Berry (2011) and Mulhall et al. (2003) clearly indicate that CoRe help one to understand a general view of PCK of teachers for a specific topic. They further emphasise that CoRe is about teaching to a specific group of students, a specific topic. In addition, they assert that CoRe help to link content knowledge, pedagogical knowledge and learners' conceptions when teaching a specific topic. For this purpose, Mulhall et al. (2003) constructed eight questions which are written in a matrix. In addition to the questions, the CoRe requires a list of important concepts which are connected to the topic to be taught. These concepts are specific for the grade or the level at which the topic is taught. For this study, the original science CoRe was modified to suit algebra PCK. The only modification made was to substitute the word science with the word algebra and the word idea with the word concept. The following are the CoRe questions that form the framework of this study.

2.6.1 The Core

- *The big concepts of algebra in a particular grade:* The first step is to identify the main concepts that make learners to understand what algebra is composed of in a particular grade.
- *What you intend the learners to learn about this concept:* Teachers should be specific about what the particular learners should learn about the particular concept in a particular grade.
- *Why it is important for the learners to know this:* Teachers should indicate the importance of a particular concept in learners' lives. Also,

they should be able to indicate how a particular concept relates to other subjects.

- *What else you might know about this concept (that you don't intend learners to know yet).* Teachers should have knowledge of what particular learners will learn about a particular concept that will be taught in the next grades. This will help teachers to prepare learners for what they will learn in future. .
- *Difficulties/ limitations connected with teaching this concept:* Effective teachers should be able to note the difficulties they encounter as teachers when teaching a particular concept.
- *Knowledge about learners' thinking that influences your teaching of this concept:* Experienced teachers should know what pre-knowledge learners should have about the identified concept and how learners generally respond those particular concepts. In addition they should be able to identify difficulties that learners have when learning a particular concept.

Learners' conception or misconceptions play a vital role in their learning, which in turn affect their "progress and achievement" (Sarwadi & Shahrill, 2014, p. 1). There are studies that have identified difficulties that learners have when learning algebra. These include studies by Booth (1988), Booth, Barbieri, Eyer, and Paré-Blagoev (2014), Kuchemann (1978) and Tirosh, Even and Robinson (1998). Their identified difficulties are summarised in Table 2.2 below.

Table 2.1

Summary of Learners' Algebra Common Errors

Error category	Error type	Example
Variable	Learners regard letters as abbreviations of objects	An expression: $2b + 3g$ is interpreted as 2 boys and 3 girls.
	Learners think different letters always stand for different values	$x + y + z$ can never be equal to $x + y + w$ because z and w are different letters.
	Combining unlike terms	Learners do not accept $2b + 3g$ as an answer they tend to write $5bg$ to complete their answer.
Expressions	Learners have difficulties in understanding that answers can be in the form of an expression or a general statements.	Do not accept answers like $k + 3$ or $A = 4s$, they think answers should always be numbers, they substitute the variables by values of their own.
Equality	When solving equations, learners think they just subtract a term on both sides if they want to remove it or they move a term without changing its sign.	Given: $y - 3 = 7$; then $y = 7 - 3$.

- *Other factors that influence your teaching of this concept:* Teachers should be able to indicate the contextual factors that influence the way they teach a particular concept.
- *Teaching procedures (and particular reasons for using these to engage with this concept)* Teachers should indicate the procedures they follow when teaching a particular concept and why they use those particular procedures.
- *Specific ways of ascertaining learners' understanding or confusion around this concept:* Teachers should indicate the methods they use to monitor the progress that learners make when learning a particular concept.

According to Mulhall et al. (2003), when teachers discuss CoRe as a group, they relook their way of practice. As a result, they modify their practice, and in turn, their PCK improves. As they construct their CoRe, they also realise the importance of teaching experience, but most importantly, the importance of planning for teaching of every lesson. This follows from the fact that construction of CoRe is based on reflection of lessons already taught (Shulman, 1986; Wood, 2003).

A CoRe framework was used by Chordnork, Yuenyong and Hume (2012) in their enquiry study in Thailand. The aim of their study was to explore the PCK of primary science teachers when teaching global warming. Their study showed that the resultant CoRe showed that teachers did not understand child-centred learning, strategies of teaching, assessment, and students' understanding of global warming. Also, a CoRe framework was used by Garritz et al. (2010) in their study to document and assess pedagogical inquiry/content knowledge (PICK) of science teachers. In their study, the researchers developed about seven inquiry activities which were answered by five experienced science teachers. For each activity given, the teachers had to answer questions of the inquiry content representation (I-CoRe). It was found that the teachers used inquiry to improve their way of teaching. Also, Lehane et al. (2013) used CoRe in their two-year study to develop pre-service teachers' PCK focusing on inquiry. It was found that when the students shared ideas during the development of CoRe, they also looked into their ways of practice. As a result, they developed in that regard.

In view of all the findings about PCK, one is compelled to agree that PCK is essential for effective teaching, which is teaching for understanding. The importance of PCK in teaching is summarised in a corollary by Garritz (2013) which says:

> Asking ourselves on the PCK necessary to face up to a particular topic, group of students, reasons for, and way of teaching, we enter in a productive exercise, where we have to think on: our teaching objectives; the knowledge of alternative conceptions of students; their learning difficulties; our own teaching difficulties, what is the appropriate sequencing of topics, the correct use of analogies and examples, ways to address the central ideas, experiments, projects and problems during the class; and the

> ingenious ways of evaluating student progress and understanding. … complete our preparation of the class. (p. 464)

2.7 CONCLUSION

In this chapter, the concept of pedagogical content knowledge was discussed with reference to other studies. The chapter started by discussing different views on the definition of the concept, and then views on what PCK is composed of. Following the components was the discussion on how PCK develops and how it can be measured. The chapter ended by discussing the theoretical framework. The framework is made up of the CoRe questions, which were used in the collection of data and data analysis. The next chapter presents the methodology of this study.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter explains the procedure that was used to explore the algebra PCK of senior phase mathematics teachers. Attention was paid to the research methods used in the study, research design, and the procedure to identify the participants, ethical issues, rigour of the study, analysis and the tools used to collect data.

3.2 RESEARCH METHOD

In choosing a research method, the problem and the purpose of the study are used as reference (Creswell, 2012; Noor, 2008). As a result, a study can employ either a quantitative or qualitative approach or even both.

Quantitative research on the one hand is employed when observable phenomena are systematically and empirically investigated using numerical methods (Sharts-Hopko, 2002). However, statistical techniques could not be used in addressing the research question of the reported study. Furthermore, quantitative research aims at generalising the results to a larger population (Marshall, 1996). This view of research was not applicable to the reported study as the focus was on understanding algebra PCK of senior phase teachers in-depth.

Qualitative research on the other hand, sought for in-depth understanding of the main phenomenon through exploration and analysis of data using text analysis (Bassey, 1999; Bitsch, 2008; Creswell, 2012). Correspondingly, the reported study sought for in-depth understanding of algebra PCK of senior phase teachers. In addition, qualitative approach allows for a purposive selection of information-rich participants in order to fulfil the purpose of the research (Bassey, 1999; Bitsch, 2008; Creswell, 2012; Frankel & Devers, 2000). Thus, this view of research resonated with the reported study since the participants were also purposively selected; hence, a qualitative research approach was adopted.

3.3 RESEARCH DESIGN

Within the qualitative research approach, there are different research designs which can be adopted depending on the purpose of the study. For instance, there are histories, teaching experiments, survey, and case studies (Bassey, 1999; Bitsch, 2008; Creswell, 2012; Merriam, 1998; Noor, 2008; Yin, 2003). Among these designs, the reported study used a case study. The choice of design depended on the knowledge of the teachers at the time when data was collected, unlike in histories that depend on the past phenomenon. Also, the relevant knowledge that the teachers possessed could not be manipulated like in experiments where a researcher has control over variables that are at play. Furthermore, to be able to study PCK of teachers closely, the study needed a small number of participants, which would not suffice for a survey. Hence, a case study was a better option for the study [Noor, 2008; Rowley, 2002; Yin, 2003].

Yin (2003) defines a case study as: “An empirical inquiry which investigates a contemporary phenomenon within its real-life, when the boundaries between phenomenon and context are not clearly evident and using multiple sources of evidence” (p. 13). On the other hand, Merriam (1998) views a case as “a thing, a single entity, a unit around which there are boundaries” (p. 27). This differs a little from Yin’s definition because Merriam specifies that the boundaries of a case must be evident. She points out that if the boundaries around the phenomenon are not visible, then the phenomenon does not qualify to be a case. For instance, she indicates that when there is no limit to the number of participants or when there is no time limit for data collection, then the phenomenon is not adequately bounded. While that is the case, Baxter and Jack (2008) define case study as an approach which facilitates exploration of a phenomenon within its context without mentioning any boundaries of the phenomenon. They also indicate that this exploration uses a variety of sources to collect data.

This study however, concurs with case studies as defined by Merriam (1998) for various reasons. First reason, the purpose of this study was to get a full understanding of algebra PCK as held by senior phase teachers. The focus was limited to algebra as a branch of mathematics, and also limited to senior phase mathematics teachers. Secondly, this study used content representation (CoRe) to explore the PCK. CoRes are topic specific, and they are only designed to suit a specific group of people (Mulhall et al., 2003). As a result, the CoRes were compiled according to the grades that the teachers taught. In addition, the study was focused only on the algebra PCK of a small number of senior phase teachers. Lastly, data was collected at a specific time. Hence, I can assert that the phenomenon is bounded.

Merriam (1998) further classifies case studies into particularistic, descriptive or heuristic. But this study took a descriptive form, which she regards as holistic, lifelike, grounded, and exploratory. This is supported by the nature of this study which required the participants to reveal their PCK which was inherent by reflecting on their subject matter knowledge and their practice through a written test and compilation of CoRe.

Case studies were also used in some studies to explore teachers' mathematical PCK or mathematical understanding. These studies include one by Ijeh (2013) who used this design to explore the teachers' PCK of statistics. Also, Moru, Qhobela, Poka and Nchejane (2014) used this design to investigate teacher knowledge of error analysis in differential calculus. Adding to the list was Nillas (2010), who also used this design to investigate the characterisation of pre-service teachers' mathematical understanding. Finally, Atasoy, Uzun, and Aygun (2016) used case study to investigate the technological pedagogical content knowledge (TPACK) of prospective mathematics teachers with regard to the process of assessment and evaluation. Therefore, the use of exploratory case studies for investigating teachers' PCK of algebra does not deviate from reported literature.

3.4 SAMPLING

Unlike qualitative studies, the quality of quantitative studies is determined by the sample used through which inferences about a population are made (Yin, 2003). Such inferences are not recommended for exploratory studies because that is not where the strength in case studies lie (Baxter & Jack, 2008; Merriam, 1998; Noor 2008; Rowley, 2002; Salvador, 2016*;* Yin, 2003). Instead, analytic generalisations based on the naturalistic nature of exploratory case studies are recommendable (Yin, 2003). This kind of generalisation is meant to expand and to generalise theories which are already developed (Rowley, 2002; Yin, 2003). It can therefore be concluded that using small samples for exploratory case studies is recommended since they do not represent a population (Merriam, 1998; Rowley, 2002; Salvador, 2016; Seawright & Gerring, 2008; Yin, 2003). This is acceptable provided that sampling is done on the basis of what is expected about the content of information that will be provided (Flyvbjerg, 2006). Putting it differently, and with particular reference to this study, by purposively selecting participants, a deeper understanding of algebra PCK that the teachers held was ensured (Baxter & Jack, 2008; Cohen, Manion & Morrison, 2000; Coyne, 1997; Creswell, 2012; Flyvberg, 2006; Salvador, 2016).

The participants were a group of practising senior phase mathematics teachers who had applied to participate in the development programme at one of the professional development institutes in Limpopo Province. For teachers to participate in the programme, an invitation to schools was sent through the districts. The circulars were sent six months in advance before the start of the programme. The teachers had to apply three months prior the start of the programme. The invitation had stated that only the teachers who taught mathematics in the senior phase should apply. Then the principals of the schools, where the interested teachers taught, had to sign and attach the schools' stamps on the application forms as the way of approving that the applicants taught mathematics in the senior phase. Thereafter, the applications were sent to the institute through the districts. Initially, the plan of the institute was to train 80 teachers, but due to certain logistical problems, the number was reduced to 75. Then, from the applicants sent by the districts, 15 teachers from each district were selected. The criterion used by the institute to select these 15 teachers from each district was not specified. However, out of the 75 teachers who were selected, only 61 managed to attend. There were nine teachers from Capricorn District, 15 teachers from Mopani District, 13 teachers from Sekhukhune District, 15 teachers from Vhembe District and nine teachers from Waterberg District. About 14 teachers could not attend the training because their districts could not transport them.

All the 61 teachers who attended the programme agreed to participate in the study. Since CoRes are subject specific and grade specific, before compilation, the teachers grouped themselves according to the grades that they felt comfortable to teach. This was done because some of those teachers taught more than one grade. The process resulted in a group of 26 teachers for Grade 7, 13 teachers for Grade 8, and 22 teachers for Grade 9.

From the literature reviewed, it was revealed that subject matter knowledge plays a major role in the development of PCK (Baumert, et al., 2010; Shulman, 1986; Van Driel & Berry, 2010; Van Driel, Verloop & De Vos, 1998). As a result, the mathematics qualifications of the participants were taken into consideration in the study. The information obtained from the profile form (Appendix F) completed by the participants, showed that their qualifications in mathematics ranged from Standard 8 (Grade 10) to B. Ed (Hons). Three of the teachers had only Grade 10 mathematics and one had Grade12 (Standard 10) mathematics. Further, ten teachers had SPTD only; eleven had SPTD and ACE; one had SPTD and BA; one had JSTC and B. Ed (Hons); nine had STD and ACE; nine had STD only; two had STD and HED; nine had B. Ed. (Hons); one had HED and ACE; one had JPTD and FDE; one had NPDE and two had B. Sc and PGEC.

Literature also reveals that PCK develops as a teacher gains experience in teaching the same subject (Ball et al., 2008; Shulman, 1986; Van Driel et al., 1998; Williams & Lockley, 2012). Hence in this study, the participants' experience in the teaching of mathematics played a very important part. From the information given, it was found that their teaching experience in the grades that they were teaching ranged from one year to 24 years. About 38% (23 teachers) of the teachers had two or less years teaching experience, while 62% (38 teachers) had three or more years teaching experience. However, about 45% of those teachers with less than three years teaching experience in Grade 8 and Grade 9 had more than four years teaching experience in FET band. Also, it was found that the highest experience of those teachers was 26 years. It can be concluded that all the participants with the exception of only one who only had standard 8 and was teaching the grade for the first time, had at least two years' experience.

3.5 DATA COLLECTION

Case studies are known for their being rich in information because of their use of multiple sources of data (Bassey, 1999; Frechtling, 2010; Rowley, 2002; Salvador, 2016*;* Yin, 2003). These different sources of data include observations, interviews, questionnaires, document reading, archival records, physical artifacts, and tests. In the literature that I reviewed, generally for this kind of study, many used observations, interviews and tests. For lesson observations, one would need to visit a site for more than once to gather enough information to make necessary conclusions. Also, for those who used interviews, they were conducted after lesson observations or after the writing of tests. For this study, to really understand the

PCK that the teachers held, I used a test (see Appendix A) for the collection of data. I did not use observations because I wanted the participants to reflect on what was inherent in them at that particular time that they could not reflect in one lesson. In addition to the test, I used a CoRe matrix (see Appendix B) for the collection of data because of its successful use in science which is summarised in the study by Lehame and Bertram (2016). Further, Hume and Berry (2011) indicate that "CoRes attempt to portray holistic overviews of teachers' PCK related to the teaching of a particular science topic", p. 344. Therefore, for this study, CoRe was used to portray algebra PCK of senior phase mathematics teachers. It should be noted that CoRes are social constructs because when they were originally developed, they were compiled by participants in groups since some aspects are complementary (Lougran et al., 2004).

3.5.1 The test

There are different types of tests for different purposes. There are also different ways of developing those tests. One would develop a test in any way that would satisfy one's goals.

Types of testing: According to Cohen et al. (2005) and Frechtling (2010), a test can be parametric or non-parametric and norm-referenced or criterion-referenced. The test used in this study was non-parametric because it was taken by a few individuals and did not make any assumption to a wider population. The test was also criterion-referenced because it was not meant to compare individuals, but to determine their algebra PCK as inherent in them.

Development of the test: The test was divided into three parts namely, part 1: Algebra, part 2: The difficulties that the learners have when learning algebra and part 3: Teaching of algebra. The first question of Part 1 of the test, and all other questions in Part 2 and Part 3 of the test were compiled from mathematics module

4 for upper primary and junior secondary school teachers developed by the Southern African Development Community (SADC) in partnership with The Commonwealth of Learning (2001). The other questions in Part 1 of the test, which needed general Algebra content knowledge, were developed from the guide used for training GET senior phase mathematics educators in Limpopo.

Part 1 of the test assessed the main ideas or concepts taught in algebra in a particular grade, variable, identifying and defining parts of an equation, and writing a word sentence in different ways. In short, this part assessed the knowledge of algebra and the knowledge of the curriculum. Part 2 of the test was about the ability to identify learners' difficulties. Part 3 was about identifying the difficulties that the participants encountered when teaching algebra and to give reasons why it is important to teach algebra.

Studies that use testing: Faulkner and Cain (2013) used a multiple choice test to investigate teachers' knowledge of mathematics teaching. Also, Hill et al. (2008) used a multiple choice test in their study to unpack the PCK of experienced teachers. A test was also used by Nillas (2010) in a qualitative study to investigate how pre-service teachers understood mathematics. Also, in another study conducted by Kwong et al. (2007), a test was used to measure the development of mathematical pedagogical content knowledge (MPCK) of student teachers in PGDE programme.

3.5.2 The content representation matrix

A CoRe, according to Mulhall et al. (2003), the developers of the tool, "represents pedagogical content knowledge because of the reasons it provides which link the how, why and what of the content to be taught with the students who are to learn that content" (p.6). As indicated earlier in the literature review, the CoRe was originally used for documenting PCK of expert science teachers in Australia

(Loughran, et al., 2004; Mulhall et al., 2003, Williams & Lockley, 2012). The results of these studies developed interest such that CoRes are now used successfully to develop PCK of pre-service teachers in Australia and some other parts of the world (Hume, 2010; Hume & Berry, 2011; Lehane & Bertram, 2016; Williams & Lockley, 2012). However, in the literature that I went through, no study was done in mathematics. Hence, in this study, whose purpose is to explore algebra PCK of practising senior phase teachers, I adopted CoRe to document algebra PCK of those teachers.

Development of the CoRe matrix: The CoRe matrix was adapted from Loughran et al. (2004). As mentioned earlier, PCK is topic specific (Lougran et al., 2003; Shulman, 1986). Therefore, it follows naturally that CoRes, since they represent PCK and hence topic specific, they are also grade specific. The matrix consisted of eight questions which were adapted with minor alterations from Loughran et al. (2004). The only alteration done was substitution of the word ‘science’ with the word ‘algebra’ and ‘students’ with ‘learners’.

To complete the matrix, first, the first column in the second row was provided for the grade that the teachers taught. The second, the third, the fourth columns, etc., of the second raw were provided for the main ideas/concepts that participants would identify as taught in that particular grade. Lastly, the questions in the first column had to be answered in connection with the identified main ideas/concepts listed above each column. Table 3.1 below shows the matrix that was completed by the participants.

Studies that use CoRe: In the literature reviewed, most of the studies that used CoRe were in science and none in mathematics. In studies by Loughran et al. (2004), CoRes were used to explore PCK of science teachers. However, in studies by Hume (2010), Hume & Berry (2011) and Williams and Lockley (2012), CoRes were used in the development of PCK of pre-service science teachers.

Table 3.1

CoRe on Senior Phase Algebra

GRADE	**IMPORTANT ALGEBRAIC CONCEPTS**			
	Big Idea 1	Big Idea 2	Big Idea 3	Other
What you intend the learners to learn about this idea.				
Why it is important for learners to learn this.				
What else you know about the idea (that you do not intend learners to know yet).				
Difficulties/limitations connected with teaching this idea.				
Knowledge about learners' thinking which influences your teaching of this idea				
Other factors which influence this idea				
Teaching procedures (and particular reasons for using these to engage with this idea)				
Specific ways of ascertaining learners' understanding or confusion around this idea				

Note. From "In search of pedagogical content knowledge in science: developing ways of articulating and documenting professional practice," by J. Loughran, A. Berry and P. Mulhall, 2004, Journal Of Research In Science Teaching, 41(4), p. 376.

3.5.3 Data collection process

The study was allocated the first day of developmental training for the collection of data. This was due to the fact that the study was aimed at exploring the PCK that was inherent in the participants before they were developed. The development was aimed at improving the teachers' knowledge. The test was written first. It was allocated one and a half hours to complete. Most importantly, the teachers wrote the test individually and without any references. After the completion of the test, the participants were given 30 minutes break.

After the break, the CoRe matrices were completed. The participants grouped themselves into grades that they taught. For those who taught more than one grade, they could choose any grade that they felt comfortable in teaching. Each group was then subdivided into subgroups for brainstorming before the whole grade group's final CoRe matrix completion. The CoRe was first explained to them before compilation. The process of subgroups completion of matrices took one and half hours. After an hour lunch, the subgroups merged and compiled one CoRe matrix for the grade with the exception of grade 9 group, which was subdivided into two groups which compiled two matrices. The process of the completion of the final CoRe took two and a half hours. The teachers were allowed to refer to whatever materials they thought would be of benefit to the compilation of their CoRe matrices. Only the groups' final CoRes were used in the study.

3.6 DATA ANALYSIS

Data was configured through analysis of narratives (Polkinghorne, 1995). That is, data with the same theme was classified and different categories were developed according to the themes identified. The categories were then tabulated (Rowley, 2002). Then the ideas from CoRe were applied to determine how they were demonstrated in the collected data.

3.6.1 Analysing data from the test

The test consisted of different questions. Each question was analysed separately. Responses for each question with the same theme were first categorised and then tabulated. Frequencies were indicated, and in some cases, the percentages were included. The analysis was based on the CoRe questions.

The first step of the CoRe was to identify

- *Big ideas/concepts:*

Teachers were tested on their ability to identify all the main concepts that are taught in algebra and in the grades that they taught. The question in the test that addressed this CoRe question was:

- ➢ Well, you have been teaching algebra for some time now. What are the main ideas that are taught in algebra?

The first CoRe question addressed in the test was:

- *What you intend the learners to learn about this idea.*

In this case, instead of teachers indicating what they wanted learners to learn about their identified ideas, the following questions were asked.

- ➢ One of the ideas we teach in algebra is to solve for 'what we do not know yet'. Like for instance, what is the missing number in the **equation** $\square + 7 = 13$? But we all know that we no longer write the equation in that way. It is written as $x + 7 = 13$, where x is called an **unknown** or a **variable**. Why do we write it in that way, i.e. with a variable?
- ➢ Can we also write it as $y + 7 = 13$? Also give a reason for your answer.
- ➢ In algebra, information can be represented in different ways, i.e. verbally, in flow diagrams, in tables, by formulae, by equations, by graphs, etc. Select any three ways to represent the following information:
- ➢ A certain car rental company charges R 300 per day plus R 1, 50 per kilometre for their five-seater cars.
- ➢ In the equation, $3x + 4 = 19$, which part is the following term?
  coefficient, variable, constant, algebraic operator, term, expression.

Generally, these questions tested knowledge of variables, equations and interpreting word problems.

Then, the second CoRe question was:

- *Why it is important for the learners to know this.*

In the test the question was asked in this way:

- Do you think it is important to teach algebra? Give reasons for your answer.

This question tested the teachers' knowledge of the importance of algebra. Then, the next CoRe question addressed in the test was:

- *Knowledge about learners' thinking that influences your teaching this idea.*

In this case, teachers were asked to explain with examples, the difficulties that learners have when learning algebra. They were also given statements of learners where they were to detect the learners' difficulties. Their ability to diagnose learners' difficulties was tested in this section. The ability to detect learners' errors helps teachers to address these errors when teaching. To address the CoRe question, these questions were framed this way:

- Sometimes when you teach learners, you realise that they have difficulties in learning algebra. Now describe any five difficulties that learners have in the learning of algebra. Use examples in your descriptions.

- Study the following statements that were taken from some learners at a school in Seshego. Identify the statements that are true and the ones that are false.

  a) You can't add $5c, 5b$ and $5t$ because they are like 5 cabbages, 5 beetroots and 5 tomatoes.
  b) $3x + 4 = 19$ and $3y + 4 = 19$ are not the same equations because they have different letters.
  c) If you add three onto $4p$ you get $7p$.
  d) You can't do $P + Q = 10$ because there isn't an answer.
  e) In this school there are three times as many girls as boys, so if b stands for the number boys and g for the number of girls, then in this school, $b = 3g$.

f) $-x$ is a negative number.

The last CoRe question was about,

- *Difficulties in teaching*

Teachers had to identify difficulties that they face as teachers when imparting algebra knowledge to learners. The question addressing this CoRe question was phrased this way:

> ➢ Sometimes when teaching mathematics, you realise that you have difficulties with teaching algebra. What difficulties have you encountered when teaching algebra?

3.6.2 Analysing data from the CoRe Matrix

Since the CoRe was the framework of the study, the responses supplied by the teachers were automatically categorised according to the questions in the matrix. Therefore the analysis of their responses was based on the relevancy and the richness of the information supplied. Therefore, responses were analysed based on the following CoRe questions:

- *Grade level*: this required the grade that the CoRe was intended.
- *Big ideas/concepts:* all the main ideas/concepts that are taught in algebra and in this particular grade were to be identified.
- *What you intend the learners to learn about this idea?* Teachers were to be specific about what they intended learner to learn about the ideas they identified in that particular grade.
- *Why it is important for the learners to know this.* Teachers had to show how knowledgeable they were about the importance of algebra.
- *What else you know about this idea (that you don't intend learners to know yet.* Teachers had to show that they knew more about the ideas that learners were not supposed to do in that particular grade but were to do in the next grades.

- *Difficulties/ limitations connected with teaching this idea.* Teachers were to reflect on the problems they had as teachers when they presented the ideas to the learners.
- *Knowledge about learners' thinking that influences your teaching this idea.* Teachers had to indicate what knowledge learners bring to class, and also, the general responses of learners about the idea that made them change their way of teaching. This includes being able to identify the difficulties that learners have when they learn the ideas.
- *Other factors that influence your teaching this idea.* Teachers had to indicate the contextual factors that influence their teaching.
- *Teaching procedures (and particular reasons for using these to engage with this idea).* Teachers had to indicate which procedures they used when teaching this idea.
- *Specific ways of ascertaining learners' understanding or confusion around this idea.* Teachers had to indicate how they assess that learners have understood the idea.

3.7 ETHICAL CONSIDERATIONS

3.7.1 Permission to collect data

As permission to collect data is one of the ethical requirements in research (Creswell, 2012), I sought permission through the Head of the Department (HOD) of Education of Limpopo Province. I had earmarked senior phase mathematics teachers who were to attend a developmental programme at one of the institutes in the province. I started by writing a letter to the Head of the Institute, but the head referred me to the province where the permission was granted.

3.7.2 Informed consent

One of the important aspects of ethics is to obtain an informed consent from the participants (Cohen et al., 2000; Creswell, 2012; Frechtling, 2010). In order to do this, I then designed a consent form for the teachers. Through the guidance of literature (Cohen et al., 2000; Creswell, 2012; Frechtling, 2010), the form clearly stated the purpose of the study and the tasks they (the teachers) were to do in the research. It also indicated that they were not in any way obliged to participate in the study and that they were free to withdraw at any time without any fear of affecting their relationship with the facilitator or the Institute. In addition, it indicated that their names would not be associated with the research findings in any way, and only I would know their identity. Lastly, they were also informed that there were no known risks or discomforts associated with the study. Then I read all the information on the form to them. Thereafter, I circulated the form for them to append their signatures before the commencement of the research to show that they agreed to be part of my study. To ensure anonymity, no names were written on the answer books, but each participant was given a coded name instead. The codes were written on the answer book. Their codes were R1 to R26 for Grade 7 teachers, T1 to T13 for Grade 8 teachers and S1 to S22 for Grade 7 teachers.

3.7.3 Selection bias

Even though I had purposively selected teachers who had applied and were selected to attend a developmental programme, their grouping into the grades that they taught was done by the teachers themselves.

3.8 RIGOUR OF THE STUDY

Research shows that case studies have been criticised for their lack of scientific rigour and that they do not address scientific generalisability (Noor, 2008; Rowley, 2002). However, rigour of case studies is evaluated by their credibility, transferability, and confirmability (Bitsch, 2005; Creswell & Miller, 2000; Rolfe, 2006; Rowley, 2002; Seale & Silverman, 1997; Sharts-Hopko, 2002).

3.8.1 Credibility

Credibility refers to the trustworthiness of a study (Creswell & Miller, 2000, Seale & Silverman, 1997, Rolfe, 2006). This implies, credibility determines whether the results of a study are valid or not. Therefore, to ensure credibility of this study, I used triangulation of data. That is, I used a test and CoRe matrix for the collection of data. Also, I allocated adequate time for the writing of the test and the completion of the CoRe matrix.

3.8.2 Transferability

Transferability refers to the extent to which the findings can be applied in other situations with different participants (Bitsch, 2005). Therefore, I have indicated the purpose of the study. In addition, I have explained the procedure that I followed to get the sample I used in the study. Also, I have indicated the age of the participants, their mathematics teaching experiences, their qualifications and the locality of the participants. Most importantly, I have fully described the method, the design of the study, and the instruments used to collect data.

3.8.3 Confirmability

Confirmability refers to the extent to which the findings can be confirmed (Bitsch, 2005; Creswell & Miller, 2000). That is, the sufficiency of all the information that is required to trace the findings. Therefore, to ensure confirmability, all the materials used for the development of this report are safely kept. Furthermore, all the sources cited were fully referenced.

3.4 CONCLUSION

The chapter started by giving a detailed explanation of the methods and the approach of the study. In addition, the instruments and the process of collecting data were fully explained. Furthermore, the procedure that was followed to get the sample was detailed. Also, the process of ethical considerations was detailed from the start of data collection to the end. The chapter concluded by giving account of rigour of the study. The next chapter presents findings of this study.

CHAPTER 4: FINDINGS FROM ANALYSIS

4.1 INTRODUCTION

This chapter reports on the findings from the analysis of algebra pedagogical content knowledge (PCK) of senior phase teachers. The findings are based on the test which the teachers wrote as individuals and the Content Representation (CoRe) which the teachers compiled in groups. All the teachers wrote the same test, but they completed different CoRes according to the grades that they taught. This follows from the fact that CoRe is topic specific and also grade specific (Mulhall et al., 2003). Also, the subject matter knowledge required for teaching the senior phase is the same, but the experience gained was different because the teachers taught different grades. Out of the 61 teachers who wrote the test, 26 of them taught in Grade 7, 13 teachers taught in Grade 8, and 22 teachers taught in Grade 9. However, the Grade 9 teachers compiled two matrices. Then the analysis of the test is reported first, and then followed by the CoRe analysis.

4.2 DATA ANALYSIS

As indicated earlier in this report, content representation (CoRe) represents pedagogical content knowledge (PCK) because of its link of subject matter knowledge, teaching strategies and knowledge of students (Mulhall et al., 2003). Therefore, the analysis of the teachers' responses to determine their algebra PCK was based on the following CoRe questions which were adapted from Mulhall et al. (2003).

- **Big algebra ideas/concepts**: All the main ideas that are taught in algebra in this particular grade according to the CAPS should be listed. These are the core concepts that are taught in algebra in this particular grade.
- **What you intend learners to learn about this idea:** Teachers should be specific about what learners should learn about the idea in this particular grade.

- **Why it is important for learners to learn this:** Teachers should indicate the importance of algebra in their learners' lives.
- **What else you know about this idea (that you don't intend learners to know yet):** Teachers had to indicate what they knew about the idea that would be taught in the next grades but not in the grade they were teaching.
- **Difficulties/ limitations connected with teaching this idea.** Teachers should be able to identify the difficulties they encounter when teaching the ideas.
- **Knowledge about learners' thinking that influences your teaching of this idea.** Teacher had to indicate what prior knowledge learners bring to class. This should include the difficulties that learners have when learning the idea. In addition, they had to indicate the general reaction of learners when learning about a particular idea.
- **Other factors that influence your teaching this idea.** Teachers had to indicate the contextual factors about the learners or any factors that influenced their teaching.
- **Teaching procedures (and particular reasons for using these to engage with this idea).** Teachers had to indicate which procedures they used when teaching an idea.
- **Specific ways of ascertaining learners' understanding or confusion around this idea*.*** Teachers had to indicate how they assess learners to ensure their understanding.

Further, in the analysis, the teachers were identified by their coded names. That is, R1 to R26 were the names given to Grade 7 group of teachers, T1 to T13 were given to Grade 8 group, and S1 to S22 were given to Grade 9 teachers.

4.2.1 Analysis of data from the test

Each question of the test was analysed individually. Then, data were categorised according to the themes derived from the participants' responses per question and then organised in tables. Further, each table reflects the category of responses, the codes of teachers in each category, and the frequencies (f). In some cases, percentages were calculated. Furthermore, each category of responses was also analysed. No response in all the tables shows that the teachers indicated, did not respond to the question.

> Question 1 of the test required teachers to think of the main ideas that they teach in algebra. It was phrased in this way:

> Well, you have been teaching algebra for some time now. What are the main ideas that are taught in algebra?

Eight categories were identified from the responses that the teachers gave. They are: Operations, expressions, equations, patterns, variables, representations, numbers, and measurement. The results are summarised in Table 4.1 below.

Table 4.1

Important Algebraic Concepts (N = 61)

Main Ideas/concepts	Resposes			No responses		
	Codes	f	%	Codes	f	%
Operations	T2; T3; T4; T5; T7; T8; T9;S1; S4; S8; S15; S18; R2; R3; R8; R9; R21	17	28	R1; R4; R11; R12; R13; R14; R20; S2; S5; S7	10	16
Expressions	T5; T10; T11; T12; T13; S3; S8; S9; S11; S15; S16; S20;R2; R3; R6; R7; R10; R23; R24	19	31			
Equations	T1; T5; T6; T7; T10; T13; S3; S6; S8; S11; S12; S13; S14; S18; S20; S21; S22; R6; R7; R10; R16; R17; R18; R23; R24; R25; R26	27	44			
Patterns	R23	1	2			
Variables	S1; S10; S16; S17; S19; R2; R3; R7; R8; R15	10	16			
Representations	R5; R6; R10; R17; R23; R25; S1; S15	8	13			
Numbers	T7; S1; S4; S8; S15; S18; R8; R19; R21; R22	10	16			
Measurement	S11	1	2			

Table 4.1 shows that 16% of the teachers did not respond to the question, none of which was from Grade 8 group. Most of those who did not respond to this question, taught in Grade 7. This implies that those teachers, who did not respond to this question, did not know the main concepts of algebra that are taught in the grades that they taught. Also, it suggests that they did not know the algebra curriculum that they taught.

The categories given are mutually inclusive. Hence, there are responses that fall in different categories. For instance, responses given by S1, S8, and S15 fall in four different categories, responses by T5, T7, R3, R6 and R23 are found in three categories, etc. However, more than 50% of the teachers gave either only one concept or more than one concept falling in the same category.

It should be noted in the table that most teachers viewed concepts taught in algebra as equations, followed by expressions and then operations according to the popularity of the concepts given. Only one teacher (R23) identified patterns as the main concept taught in algebra. Also, only 13% of the teachers viewed representations as the main concept that is taught in algebra, and 75% of those teachers, taught in Grade 7. It suggests that most teachers who taught in grades 8 and 9 do not view representations as part of algebra. Another point is that only 16% of the teachers viewed variables as the main concept, and none of those was from the Grade 8 group. Also, one teacher from Grade 9 viewed measurement as the main concept that is taught in algebra.

Twenty eight per cent of the responses given by the teachers were classified in the first category as ‘operations’. These are the procedures or commands in mathematical statements which specify what should be done in an algebraic statement. These are: Addition, subtraction, multiplication, division, order of operation and properties of operations. The responses were as follows:

- Operations (T3, T4, T7, T8, S1, S8, S15, R2, R3 and R21).
- Addition, subtraction, multiplication, division (T3).
- Order of operations (BODMAS rule) (T2, T5, T9, S4, S18 and R8).
- Properties of operation (R9).

T3 identified addition, subtraction, multiplication and division and operations as main concepts. It implies that the teacher viewed addition, subtraction, multiplication and division as different from operations. Further, about 8% of the teachers (T2, T3, T4, T8 and T9) who all taught in Grade 8, saw mathematical operations as the only concept that is taught in algebra. This suggests that these teachers did not have knowledge of algebra curriculum because operations are not the main or the only concepts taught in algebra.

In the second category, expressions, 31% of the teachers identified ideas which are presented below.

- Expressions (R3, R6, R10, R23 and R24).
- Types of polynomials or names of expressions according to number of terms (T5 and S9).
- Identify the coefficient, constant, term (R2, R7, T13, S16, S17 and S20).
- Finding the value of if $x = 2$ find $2x^2 + 4$ (T5).
- Grouping of like and unlike terms (T11, T12, T13 and S16).
- Simplifying expressions/things (adding, dividing and multiplying algebraic expressions/ polynomials (T10, T11, T13, S3, S8 and S9).
- Exponents (S15).
- Factorisation/ factorising expressions (T10, S9 and S11).
- Expand things (S3).

Only 24% of the teachers (R3, R6, R10, R23 and R24) who are classified in this category were able to identify expressions as the main idea. More importantly, all those teachers who identified this concept taught in Grade 7. Then, the remaining teachers identified one or two concepts falling under expressions as the main concepts. For example, most teachers in this category identified ‘simplifying expressions’, which is a concept falling under expressions. Therefore, none of these responses can be regarded as the main concept because they do not embrace the other concepts dealing with expressions. In addition, some of the responses show lack of mathematical terminology. For instance, expressions are referred to as ‘things’ as indicated by S3. In view of all the given responses in this category, it can be concluded that most teachers in this category did not regard expressions as the main concept. This shows that they had inadequate knowledge of what is covered under expressions in the senior phase because most identified only one concept on expressions.

About 44% of teachers in the third category, equations, expressed their responses through the following answers:

- Equations (T7, R6, R10 and R23).
- Equations with 2 terms, 3 terms, are called binomials and trinomials (T10).
- Simultaneous equations (T10).
- Solve for unknowns or solving equations (T1, T5, T6, T10, T13, S3; S4, S8, S11, S12, S13, S21, S22, R10, R16, R17 and R25).
- Apply formulae (R17).
- Solving first and second order operations (S18).
- Solving word problems by using numbers (T9 and S20).
- Number sentences (R24 and R25).
- Formulate equations (R25).
- Problem solving techniques (S14 and R18).
- Knowing the omitted values of the expression (S6).

It shows that only T7, R6, R10 and R23 were able to identify ‘equations’ as the main idea. It implies further that more than 80% of the teachers, who gave responses in this category, identified ten ideas falling under equations as the main ideas. For example, more than 60% of these teachers who identified ideas in this category identified ‘solving equations’ or ‘solving for unknown as the main idea’, which is part, but not all of what is taught about equations. On the other hand, T10 identified simultaneous equations, which does not form part of concepts that are taught in the senior phase algebra. That aside, some teachers’ responses showed inadequate knowledge of mathematical terminology. For example, T10 regarded expressions as “equations with 2 terms, 3 terms, are called binomials and trinomials”. Also, S18 regarded equations as operations. He indicated “Solving first and second order operations” as the main concept. This shows inadequate knowledge of algebraic terminology. Therefore, it can be concluded that more than

80% of the teachers in this category had inadequate knowledge of the main ideas that are covered in the senior phase.

In the fourth category on variables, 16% of the teachers identified variables or operations of symbols as the main idea. It shows that these teachers, R2, R3, R7, R8, R15, S1, S10, S16, S17, and S19, know that variables are the basis of algebra.

In the fifth category, different representations were identified as the main ideas. The representations that were identified are indicated below.

- Graphs (R6, R23 and S1).
- Table (R23).
- Functions (R23, S15, and R5).
- Flow diagram, input-output (R5, R6, R10, R17 and R25).

In this category, only functions and graphs are regarded as the main ideas that are taught in algebra. Tables and flow diagrams are dependent on functions or equations. Hence they cannot be regarded as main concepts. Only five teachers viewed functions and graphs as the main concepts, with only R23 identifying them both as the main concepts. It can be seen that none of the Grade 8 teachers regarded functions and graphs as part of algebra. Generally, most of the teachers did not regard the two concepts as part of algebra. As a result, it can be concluded that most teachers had inadequate knowledge of algebra curriculum in the senior phase.

The seventh category, numbers, was identified by 16% of the teachers whose responses are presented below.

- Numbers (T7 and R21).
- Real numbers (S8 and S15).
- Number and relationships (R19).
- Sets of whole and rational numbers (S18 and R8).
- Changing mixed fractions to improper fractions (S4).

- Simplifying fractions (S4).
- Operation of numbers (R19).
- Calculation, fractions, counting, numbering (R22).
- Finding sum, difference, product, quotient (T5 and R15).

Only two of the teachers (T7 and R21) in this category identified ‘numbers’ only as the main idea. This implies that most of the teachers in this category identified other ideas which fall under numbers as the main concepts. For example, “changing mixed fractions to improper fractions” and “simplifying fractions” by S4, are some of the concepts studied under numbers, but they are not the main concepts. However, it can be concluded that all the teachers in this category, confuse algebra curriculum with arithmetic curriculum. This follows from the teachers identifying numbers as the main concept taught in algebra, while numbers form a foundation of arithmetic.

In the last category, measurement, S11 indicated “calculating certain concepts, e.g. area, volume, perimeter, distance and speed” is the main concept that is taught in algebra. These sections, apply algebra in their calculations, but they do not fall under algebra. This shows uncertainty in the knowledge of algebra curriculum.

In summary, the expected main concepts that are taught in senior phase algebra are: variable, expressions, patterns, equations, graphs, functions and relationships. However, less than 50% of the teachers were able to identify at least one main concept. Most of those teachers taught in Grade 7. Furthermore, only 10% of the teachers (T7, R2, R6, R10, S1 and S5) were able to identify at least two different main ideas. The results also show that only 3% (R3 and R23) were able to identify a maximum of three main concepts. This suggests that most of the teachers did not have knowledge of the main concepts that they taught in algebra in the senior phase.

Question 2 indicated that one of the ideas that is taught in algebra is to solve for 'what we do not know yet'. Like for instance what is the missing number in the equation $\square + 7 = 13$? But we all know that we no longer write the equation in that way. It is written as $x + 7 = 13$, where x is called an unknown or a variable. Then the question is:

Why do we write it in that way, i.e. with a variable?

Reasons given were classified into five categories. The categories identified were: easy use, solution of equations, substitution, checking and representation. The results are summarised in Table 4.2.

Table 4.2

Reasons for Using a letter in an Equation Instead of a Placeholder (N=61)

	Responses			No responses		
Reasons given	Codes	f	%	Codes	f	%
Easy use	S5; S20	2	3	R1; R4; R7; R12; R13; R14; R18; R20	8	13
Solution of equations	T1; T2; T3; T4; T9; T10; T11; T12; T13; S1; S3; S4; S8; S9; S11; S12; S14; S15; S17; S18; S19; S21; R3; R5; R6; R8; R9; R16; R19; R21; R22; R23; R24; R26	34	56			
Substitution	T5; T6; S2; S6; S7; S13; S16; R2; R10; R11; R15; R17; R25	13	21			
Checking	T7; T8	2	3			
Representation	S22	1	2			

Table 4.2 categorises teachers according to the reasons which they gave. Their frequencies and percentages are also reflected in the table. The table also shows that 13% of the teachers did not respond to this question. All of these teachers who did not answer the question, taught in Grade 7. This suggests that these teachers did not know the reasons why the letters are now used as variables in place of

placeholders. Also, the table shows that most of the reasons given were about solution of equations.

In the first category, the table shows that only S5 and S20 were able to indicate that it is easier or quicker to use a letter than to use a placeholder. These responses show that these teachers had knowledge that letters and placeholders represent unknown values.

Some teachers interpreted letters and placeholders differently, and hence, gave different reasons. The reasons given by the teachers in the second category, solution of equations are listed below.

- To be able to calculate the unknown number (T1, T9, T10, S4, S9, S11, S12, S13, S14, S19, S21, R8, R9, R11, R19, R24 and R26).
- To be able to get the correct answer (T11 and S8).
- We do not know the number it represents (T4, S6, S15, S18, R3 and R6).
- Sometimes the box need a partial answers as number and the unknown part (R23).
- To know the values of x (T3, T12, S3, R16, R21 and R22).
- To indicate that something is missing (S1).
- We want to verify if the left hand side is equal to the right hand side (T2 T13 and S17).
- Because it can be exploited (R5).

All these reasons imply that we no longer use a placeholder because we want to solve for the unknown. The indication in these statements is that a placeholder is not used in place of an unknown. On the contrary, learners in the lower grades where a placeholder is commonly used, use mental calculations to solve for the unknown instead of writing procedures. These responses suggest that these teachers did not know that both the letter and the placeholder stand in the place of an unknown value. As a result, the real reason why a placeholder is no longer in use was not answered. This shows that these teachers could not associate a placeholder with a variable.

The third category of reasons classified as substitution, which were given by 21% of teachers are indicated below.

- Any number can be put without solving, so a variable is used to be specific (T5).
- It is not a number, it is a variable (T6).
- Because x can stand for any number (S2, S6, S7, S16, R2 and, R11).
- Variable substitute a number we do not know (R10, R15 and S13).
- So that learners could be able to use letters in place of number (R17).
- So that we can differentiate variable from other numbers (R25).

These responses imply that only 'x' stand for the unknown. This implies that a placeholder does not substitute an unknown value according to the responses. That is, the teachers did not see any connection between the letters and the placeholders. For this reason, it can be concluded that these teachers did not understand why placeholders are no longer in use in the senior phase. This shows inadequate knowledge of representations of the unknown. Also, lack of the knowledge of why certain representations of unknown are used in certain stages is shown in these responses.

The reasons that were given by 3% of the teachers in the fourth category on checking are:

- We have to check if the learners can apply what they were taught (T7).
- To check which side is the major problem (T8).

These reasons do not answer the question why a placeholder is no longer in use. It suggests, the teachers did not connect the letters and placeholders used in place of unknowns. Still, this shows that these teachers had inadequate knowledge of representations of the unknown.

In the last category on representations, S22 indicated that the reason for no longer using a placeholder is "because a square or a rectangle was used previously it is now studied on its own in Grd 9". In this case, the teacher was able to associate placeholders and letters, although the statement is incomplete. However, the teacher did not give the reason why letters replace placeholders in the senior phase.

In summary, the question tested why placeholders are no longer in use in senior phase algebra. It is apparent that almost all the teachers did not know the reason why a placeholder is no longer in use in the senior phase. However, placeholders are still in use in the grades lower than the senior phase. Most importantly, letters are used as variables in the senior phase when learners start working with complicated expressions and equations where it is difficult to use placeholders. Hence the placeholders are replaced by letters. Only 3% of these teachers were able to indicate that it is easier using letters than using a placeholder when solving for the unknown value. It has been noted that all of these teachers, with the exception of three teachers, were unable to link letters and placeholders as variables. This suggests that these teachers had limited knowledge of different representations for the variables. Therefore, it can be concluded that these teachers had inadequate knowledge of variables as placeholders which must be used as pre-knowledge to the use of letters.

> In Question 3, which is a follow-up of Question 2, the question is: Can we also write it $[x + 7 = 13]$ as $y + 7 = 13$? Also, give a reason for your answer.

All the teachers indicated ‘yes’ to the first part of the question. Although T7 did not indicate that the answer is a ‘yes’, he wrote: $y + 7 = 13;\ y = 13 - 7;\ \ y = 6$, which implies that x can be substituted by y in the equation. However, the reasons for the given answer are different. The results show that about 66% (40 teachers) of teachers (T1, T2, T3, T4, T5, T8, T10, T11, T13, S3, S4, S5, S7, S8, S9, S10, S11, S13, S14, S15, S16, S18, S19, S20, S21, S22, R4, R6, R8, R9, R10, R11, R15, R16, R17, R21, R22, R23, R24 and R25) indicated that a number can be substituted by any letter, which was the expected answer. The unexpected reasons given by other teachers are:

- Because is Equation (T6).
- Because we want to know which number can we write in place of unknown and add with 7 (T7 and S2).
- Because y indicate the unknown (T9, S6 and S17).
- Because if you want the value of y, you can solve it (T12).
- Variable stand for a certain number/unknown (S1).
- Because a variable is not consistent (R2).
- Yes y will be our variable it makes calculations to be unforgotten (R19).

These responses did not give the real reason for exchanging the variables. This suggests that the teachers did not know the real reason.

R3 did not give any reason for the answer given. Also, about 15% of the teachers (R1, R5, R7, R12, R13, R14, R18, R20 and R26) did not respond to this question. All these teachers taught in Grade 7. In view of the unexpected given reasons and the failure to answer the question, it can be concluded that these 34% of teachers had inadequate knowledge of representation of variables. This is however, the knowledge which the learners need to learn, that any unknown can be substituted by any variable.

> Question 4 needed the teachers to identify and define the parts of the equation: $3x + 4 = 19$. The parts are: a coefficient, variable, constant, algebraic operator, a term and an expression.

The results of this question are summarised in Table 4.3 and Table 4.4 Table 4.3 shows the results for identifying parts of the equation and Table 4.4 shows the results for definitions. The correct answers are indicated below the terms in Table 4.3. The responses in both tables were categorised according to the part of the equation that was identified.

Table 4.3

Parts of an Equation (N = 61)

Term	Correct answer		Partly correct		Incorrect answer	
	Codes	f	Codes	f	Codes	f
Coefficient $\mathbf{3}$	T1; T2; T3; T4; T5; T6; T7; T8; T9; T10; T11; T12; T13; S1; S2; S3; S4; S5; S6; S7; S8; S9; S10; S12; S13; S14; S15; S16; S17; S18; S20; S21; S22; R1, R2, R4, R5, R6, R7, R10, R13, R14; R16; R17; R18, R19, R22, R23; R26	49			S11; S19; R3; R8; R9; R12; R16; R20; R21; R24; R25	11
Variable $\boldsymbol{x}$	T1; T3; T4, T5; T6; T8; T9; T10; T11; T12; T13; S1; S2; S3, S4; S5; S6; S7; S8; S9; S10; S11; S12; S13; S14; S15; S16; S17; S20; S21; S22; R1; R2; R4; R5; R6; R7; R8; R9; R10; R11; R12; R13; R14; R16; R17; R18; R19; R20; R22; R23; R24	52			T2; T7; S18; S19; R3; R15; R21; R25; R26	9
Constant $\mathbf{4; 19}$	T3; T4; T5;T8; S1; S3; S4; S9; S13; S15; S21; R1; R17; R24; R26	15	T1; T2; T6; T9; T12; S2, S5; S7; S10; S11; S14; S16; S17; S18; S20; S22; R2; R3; R4; R5; R6; R7, R8; R9; R10; R15; R16; R18; R19; R20; R22; R23; R25	33	T7; T10; T11; T13; S6; S12; S19; R12; R13; R14	10
Algebraic operator $+; \times$	S17	1	T1; T2; T3; T4; T5; T10; T11; T13; S1; S2; S3; S9; S10; S12; S20; S21, S22; R2; R5; R6; R9; R10; R13; R14; R16; R20; R23;	30	T6; T7; T8; T9; T12 S4; S5; S6; S11; S13; S15; S16; S18;	17

Term	Correct answer Codes	f	Partly correct Codes	f	Incorrect answer Codes	f
Algebraic operator			R24; R25; R26		R1; R3; R15; R17	
Term $\mathbf{3x}$; $\mathbf{4; 19}$	T3; T4; T7; S3; S4; S10	6	T1; T2; T5 T6; T8; T10; T13; S2; S7; S8; S9; S11; S14; S15; S16; S20; S22; R5; R6; R8; R10; R13; R17; R23; R24	25	T9; T11; T12; S1; S5; S6; S13; S17; S18; S19; S21; R1; R7; R9; R14; R15 R16; R20; R25; R26	20
Expression $\mathbf{3x+4}$; $\mathbf{3x}$	T1; T5; S2; S3; S4; S7; S8; S9; S10; S11; S14; S16; S20; S21; S22; R1; R15	17	T4; T10	2	T2; T3; T6; T7; T8; T9; T11; T12; T13; S1; S5; S6; S12; 13; S15; S17, S18; R3; R5; R6; R13; R16; R17; R20; R23; R24; R25	27

Table 4.3 shows the parts of the equation that were identified, teachers' codes and the frequencies. Also, the answers for identifying the parts of the equation are classified as correct, partly correct or incorrect in the table. Partly correct answers refer to:

- Not identifying both 4 and 19 as constants.
- Not identifying all the terms i.e., $3x, 4$ and 19.
- Including all the four fundamental operators $(+, -, \div, \times)$as part of the given equation, or including an equal $(=)$sign, or writing only $(+)$ as an operator for the given equation.
- Indicating that $3x + 4$ and 19 are expressions.

The table also shows that most teachers were able to identify a variable, and also, a coefficient. However, only one teacher was able to correctly identify all the algebraic operators in the equation. This means that almost all the teachers in this study had inadequate knowledge of operators. More importantly, the table shows that all teachers who were identified as Grade 8 teachers were able to identify a coefficient. Moreover, of the 17 teachers who were able to identify the expression of the equation, 76% of them taught in Grade 9.

It can be seen from the table that only one teacher did not identify a coefficient. The table also shows that most of the teachers were able to identify 3 as a coefficient. Only 18% of the teachers did not give the correct answer. About 10% of the teachers (S11, R8, R9, R12, R15 and R24) identified $3x$ as a coefficient. Then, S19 and R8 identified $2x + 6 = 14$, whereas R3 and R25 identified 4 as a coefficient. Taking into account those who did not answer the question, it implies that about 20% of the teachers could not identify a coefficient from the given equation.

The table shows that most of the teachers were able to identify x as a variable, but 15% of the teachers (T2, T7, S18, R3, R15, R21, R25, R26 and S19) identified $4, 19, 3x$ and $8 + 3$ as variables. This shows that not all the teachers could identify a variable from a given equation.

Table 4.3 also shows that only about 25% of the teachers were able to identify both 4 and 19 as constants, while most could identify only one of the constants. It might be that most were confused by 19 which is on the other side of the equal sign and they interpreted it as not part of the equation. About 7% of the teachers (T13, S6, S15 and R14), subtracted 4 from 19 and made the answer 15 a coefficient. Also, T10 and S12 applied the same principle but missed a sign, and got (-15) as a coefficient. This shows that some teachers could not add or subtract integers. On the other hand, about 5% of the teachers (S8, R11 and R21) did not even answer the question. This indicates that not all the teachers could identify constants from the given equation.

Only S17 was able to identify both $(+)$ and $(\times)$ as algebraic operators in the given equation. However, 49% of the teachers were able to identify $(+)$ as an algebraic operator, but could not see $(\times)$ in $3x$. Also, some wrote all the algebraic operators ($\times$, $\div$, $+$, and $-$) of which, some are not part of the given equation. Also, some teachers thought $(=)$ is an operator. Further, about 13% of the teachers identified the given equation $3x + 4 = 19$ as an algebraic operator. On the other hand, about 21% of the teachers did not attempt the question. This shows that most teachers have inadequate knowledge of algebraic operators.

The table shows that only 10% of the teachers (T3, T4, T7, S3, S4 and S10) were able to identify $3x, 7$ and 19 as the terms of the given equation. However, 41% of the teachers could identify only one or two of the terms. That aside, 33% of the teachers identified the other parts of the equation instead, where most identified $3x + 4$ as a term. On other hand, 16% of the teachers did not even attempt the question. Considering these facts, it can be concluded that many of the teachers could not differentiate a term from the other parts of an equation.

The last part to be identified was an expression. Only about 28% of the teachers were able to identify $3x + 4$ as an expression. However, T4 identified $3x + 4$ and 19 as expressions, whereas T10 identified $3x - 9$ as an expression. Furthermore, about 28% of the teachers identified the given equation as an expression. Then again, 33% of the teachers did not attempt the question. Nevertheless, this shows that more than 80% of the teachers could not correctly identify the expression from the equation.

Next is Table 4.4, which shows the results of the definitions of the parts of the given equation. The answers were classified as acceptable definitions, partly acceptable definitions and unacceptable definitions.

TABLE 4.4

Definitions of the Parts of an Equation (N = 61)

Term	Acceptable definition		Partly acceptable definition		Unacceptable definition	
	Codes	f	Codes	f	Codes	f
Coefficient	T1; T2; T3; T5; T6; T7; T10; T11; T13; S1; S2; S3; S4; S7; S8; S9; S10; S12; S13; S24; S16; S17; S18; S19, S21; S22; R1; R2; R4; R6; R7; R10; R16; R23; R26	35			T8; T9; T12; S5,S15; S20; R5; R17; R21; R24; R25	11
Variable	T1; T3; T2; T5; T6; T10; T11; T13; S1; S2; S3; S4; S6; S8; S9; S10; S12; S13; S14; S16, S17; S19; S21; S22; R1; R2; R4; R6; R7; R9; R10; R11; R12; R13; R16; R19; R20; R23; R24; R25	40			T7; T8; T9; T12; S15; S18; R5; R17; R26	9
Constant	T1; T2;T3; T5; T6; T10; T11; T12; T13; S1; S3; S6; S8; S9; S10; S12; S13; S14; S15, S16; S17, S18; S19; S21; S22; R4; R5; R6; R7; R10; R20; R23; R24; R25; R26	35			T7; T8; T9; R2; R16; R17	6
Algebraic operator	T5; T11; S1; S2; S3; S9; S10; S21; S22; R10; R16; R20; R23; R25; R26	15	T1;T2;T3; T13; T12; S17; R5; R6	8	T6; T7; T8; T12; S6; S13; S15; S16; R2; R17	10
Term	T5; S2; S3; S4; S9	5	T1; T3; S10, S16, S21; R2, R10; R16, R23	9	T2; T8; T10; T11; T12; S1; S6; S13; S14; S15; S17; S19; S21; S22; R5; R6; R7; R11; R17; R24	20

Term	Acceptable definition		Partly acceptable definition		Unacceptable definition	
	Codes	f	Codes	f	Codes	f
Expression	T3; T5; S3; S4, S8, S10; S22; R2; R7; R23	10	S1; S9, S14; S16; S21; R10	6	T1; T2; T7; T8; T10; T12; T13; S6; S12; S13; S15; R5; R13; R16; R17; R20	16

Table 4.4 shows that most teachers were able to give acceptable definitions of variable, a coefficient, and a constant. On the other hand, only 10% of the teachers could give an acceptable definition of a term, and none of those teachers, taught in Grade 7.

Further, the table shows that 57% of the teachers were able to provide acceptable definitions of a coefficient. Acceptable definitions provided by the teachers include:

- Coefficient is the value that is left when the variable is taken off. It is the no. accompanying variable (T1).
- Coefficient is the number next to a variable (S1).
- Coefficient is a number that multiply variable on equation (R1).

These definitions show that these teachers had knowledge of what a variable is. It can also be seen from the table that 18% of the teachers could not provide acceptable definitions of a coefficient. The given unacceptable definitions of coefficient include:

- 3 is the numerical coefficient (T8).
- The first term or the x term (R15).

To indicate that 3 is a numerical coefficient does not explain what a coefficient is. Also, the answer given by R15 indicates that the teacher had no idea of what a coefficient is. The table shows that about 25% of the teachers did not attempt to answer this question. Therefore it can be concluded that more than 40% of the teachers could not define what a coefficient is.

The next part of an equation that was defined was a variable. Table 4.4 shows that about 66% of the teachers gave acceptable definitions of a variable. The given acceptable definitions include the following:

- Alphabetic character representing a number (T3).
- Unknown in an equation or expression (S1).

These show that the teachers had knowledge of what is a variable. On the other hand, the table reflects that 15% of the teachers could not provide acceptable definitions. Examples of unacceptable definitions are given below.

- We want values of x (T12).
- It can be an alternater (R5).

The first definition shows that the teacher had an idea that a variable is an unknown value in the equation, but did not know how to define it. However, the second definition is hard to understand. Further, about 20% of the teachers did no answer the question. This suggests that more than 30% of the teachers were unable to define a variable.

Next on the list of definitions was definition of a constant. The table shows that 57% of the teachers were able to define a constant. The definitions that were acceptable are the following:

- Number without variables (T13).
- It is a number that can not be changed by any thing, it stays as it is. (R25).

These show an understanding of a constant. That aside, about 10% of the teachers provided answers which were not acceptable as definitions of a constant. For example,

- Equation (T7).
- A number on its own it can be an alphabet in the stand of a number (R2).
- The number that will be used to get the answer by taking it to the other side and it will change the operation side (R16).

These show that the teachers definitely did not have any idea of a constant. Confusing a constant and an equation clearly shows that T7 had no knowledge of the two. In addition, about 33% of the teachers could not even attempt to define a constant. Therefore, it can be concluded that more than 40% of the teachers did not know how to define a constant.

The table shows that only about 25% of the teachers could define an algebraic operator in an acceptable way. The following definitions were taken as acceptable:

- The sign that tells us what we are supposed to do (T11).
- The operation that we are using to calculate (R10).

However, about 13% of the teachers gave definitions which were regarded as partly acceptable. They are:

- Any operation e. g. addition, subtraction, division, multiplication (T2 and T3).
- The sign which divides the terms (T13).
- Operator between constant and variable (R5).

The first response was regarded as partly correct because the teachers did not indicate that an operator is a sign that informs us of an operation. Then the second definition is partly correct because, although reference is given to addition sign in the equation, operators like '×' and '÷' do not separate terms. While that is the case, the third definition was regarded as partly correct because sometimes the sign operates between variables. On the other hand, about 33% of these teachers gave definitions which were not acceptable. The following are examples of definitions which were not acceptable.

- Put the value of $x + 0$ the y – axis (T12).
- Is a branch of Maths that deals with operation (T6).
- Put all the number with variable on its own side and the one without at its own side (S6).
- By grouping like terms (S13).

It can be seen that these teachers did not have any idea of algebraic operators. All of the responses are irrelevant to algebraic operators. On the other hand, about 46% of the teachers did not answer the question. This implies that more than 60% of these teachers could not define an algebraic operator.

Next on the line of definitions was a term. Table 4.4 shows that only about 8% of the teachers were able to give acceptable definitions of a term. The definitions include:

- In algebra a term is either a single number or variable or numbers and variables multiplied together, terms are separated by $+$ or $-$(S3).

Fifteen percent of the teachers gave definitions which were regarded as partly acceptable. For example:

- Coefficient multiplied by a constant (S16).
- An expression consisting of a number and a variable (S21).
- Parts that an expression is made up of (S10).
- This is what can be separated with algebraic operators (T1).

The first and the second definitions were regarded as partly correct because the teachers disregarded the fact that a constant or a variable only can be a term in an expression. Also, the third definition is partly correct because the teacher did not consider that a coefficient is also part of an equation but cannot be regarded as a term. The last statement does not explain much although it is true that terms are separated by algebraic operators. In addition, not all algebraic operators separate terms. The table further shows that about 33% of the teachers gave definitions of a term that were not acceptable. Some of those unacceptable definitions are listed below.

- A number separated by addition sign and multiplication sign (T10 and S2).
- 2; 8; 32, combination/number of a number and variable in numerical pattern or mathematical expression (R7).
- Count the number of terms in an expression (S6).
- The number which follows like a pattern (S19).

- An interval or representation between operational signs in an equation or expression (S22).

The first response was regarded as unacceptable because multiplication sign does not separate terms. Then, the next three responses, the teachers confused a term in the equation and terms of a sequence. Then in the last response, the teacher regarded a term as an interval. All these answers show that the teachers had inadequate knowledge of the definition of a term in an equation. In addition, about 44% of the teachers did not respond to this question. This shows that most of the teachers could not define a term.

The last part of the equation that was defined was an expression. It is shown in Table 4.4. Only 16% of the teachers were able to give an acceptable definition of the term. Some of the acceptable definitions are listed below.

- A given problem without an equal sign (T5).
- Group of terms separated by $+$ or $-$ (R2).
- Has no equal sign/a list of terms separated by $+, -, \times$ or $\div$ (R7).

The above responses show that the teachers had knowledge of an expression. Although R7 indicated that $(\times)$ or $(\div)$ separate terms, which is incorrect, the definition was accepted as correct in this case. Then, about 10% of the teachers gave partly correct definitions of an expression. The definitions include:

- Term added to the constant (S16).
- Terms without an equal sign (S21).
- Polynomial consisting of two or more terms (S1).

The first definition shows that the teacher did not regard a constant as a term. Then, in the next response, S21 did not consider the fact that a constant or a variable are regarded as terms, but they are not expressions. Also, in the last statement, S1 did not consider the fact that there are expressions with only one term. On the other hand, about 26% of the teachers gave definitions that were regarded as unacceptable. Examples of those definitions are listed below.

- Express consist of only one term (T2).
- $3x + 4$ are value on the left hand side. 19 is a constant of the right side (T8).
- If we want to solve for $x, x = 5$ (S13).
- It defines how many terms do they have (T13).

In the first definition, T2 thought that expressions consist of only one term, whereas there are expressions with more than one term. Then the definition given by T8 shows a confusion of an expression with an equation. Also, the example given by S13 suggests that an expression is the unknown value in the equation. On the other hand, T13 confused an expression with a sequence. While this is the case, 48% of the teachers did not respond to this question. This shows that most of these teachers could not define an expression.

Table 4.3 and Table 4.4 show that not all the teachers were able to identify or to define all the parts of the given equation. Only S3 and S10 were able to do both with the exception of the definition of an operator. However T5 was also able to define all the parts of the equation, but partly identified some parts of the equation. Also, S9 was able to identify all the parts with the exception of one, but could not define them all. Nonetheless, the results show that more than 50% of the teachers could not define at least three of the parts of the equation. This suggests that most teachers had inadequate knowledge of parts of an equation. Therefore, the teachers had inadequate knowledge of what is intended for learners to learn about equations.

> Question 5 stated that in algebra, information can be represented in different ways, e.g. verbally, in flow diagrams, in tables, by formulae, by equations, by graphs, etc. The teachers had to select any three ways to represent the following information.
>
> A certain car rental company charges R300 per day plus R1, 50 per kilometre for their five-seater cars. Represent this charge of their car rental per day in three different ways.

According to the responses, the teachers selected to represent the information using an equation, flow diagram, table and graph. Therefore, their responses are classified according to these selected representations. Also, their responses were classified as correct or incorrect in Table 4.5 below.

Table 4.5

R*epresentation of a Car Rental (N = 61)*

Representation	Responses				No responses	
	Correct answer		Incorrect answer			
	Codes	f	Codes	f	Codes	f
Equation	T1; T3; T5; T13; S10; S13; R13; R20; R23	9	T2; T7; T8; T9; T12; S1; S4; S5; S6; S8; S9; S15; S16; S17; S19; S21; S22; R2; R3; R6; R8; R11; R14; R15; R16; R18; R19; R21;R24; R26	30	T4; T6; T10; T11; S2; S3; S7; S11; S12; S14; S18; S20; R1; R4; R7; R9; R10; R12; R17; R22; R25	21
Table	T5; S13; R5	3	T3; S6; S10; S15; S16; S19; S22; R5; R8; R15; R18; R20; R24	13		
Flow diagram	T1	1	T3; S5, S8, S15; S16; S17; S19; R15; R16; R23; R24	11		
Graph	T5	1	T9; S10; S22; R5; R16	5		

The table shows that 66% of the teachers represented the information in at least one way, whereas 34% did not attempt to answer the question. The table also shows that most of the teachers represented the information using an equation and very few used a graph. In all the cases, it is shown that out of all the teachers who attempted a representation, less than 30% of them were able to represent the information correctly in that chosen representation.

The table also shows that out of 39 teachers who chose to represent the information in an equation, only 23% of them were able to represent the information correctly. To represent the cost of the car rental per day using an equation, variables need to be specified.

For example, if x is the number of kilometres travelled per day, then the equation will be represented by

$Charge = 300 + 1,5x$; (where the charge per day is represented in rands).

About 10% of the teachers (T1, T3, T5, T13, S13 and R13) represented the information in a similar way although none of them specified the variables. However, S10, R20 and R23 wrote the equation of the total cost of the rental for the period that the car is rented. Only R20 was able to specify the variables. Their equation is represented this way:

$$C = 300x + 1,5y,$$

The table also shows that 49% of the teachers represented the information incorrectly using an equation. Some of the incorrect equations used are listed below.

- $R300 + R1,50 = 5$ seater (S1)
- $R300 + R1,\frac{50}{km} = 5 \text{ seater}$ cars (S4)
- $300y + 1,50 = 5$ (S6)
- $300d + 1,5\ km = 5s$ (S8)

These teachers show misunderstanding of variables. That is, they associate their chosen variables with the car in this case. This follows from the teachers calculating five-seater car instead of calculating the cost for hiring the car per day. Further, all of them with the exception of S4 did not take kilometres travelled per day into consideration. Therefore, it can be concluded that these teachers could not translate word problems into equations that they need to teach to learners. It also suggests that these teachers had inadequate knowledge of an equation or equality.

Table 4.5 also show that very few of the teachers were able to represent the information correctly in a table. For example, T5 represented the information in a table as follows:

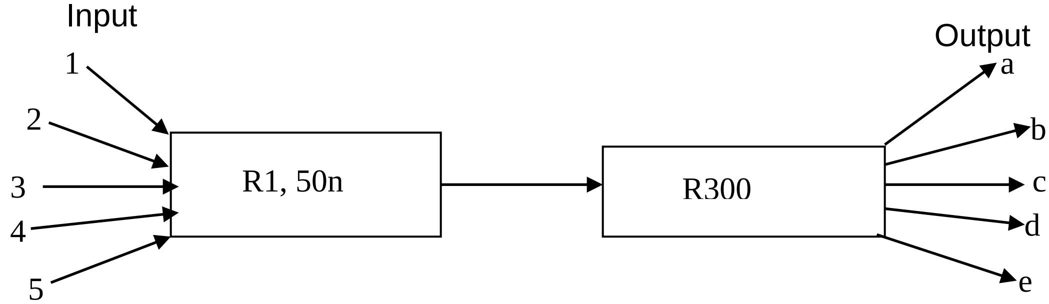
No. of kilometres	0	50	100	150	200	250	300
Total cost	300	375	450	525			

This shows that the teacher was able to interpret the information in a table. The choice of the number of kilometres also shows that the teacher understood this practical situation. However, about 81% of the teachers who selected this representation could not represent the information correctly. For instance, R15 represented the information as follows:

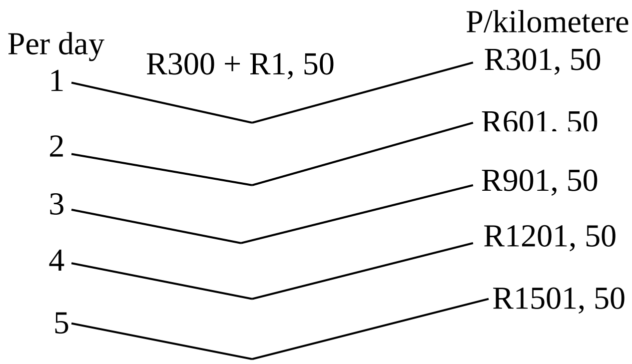
Table	Input		
	Output	5	360,3

The input values are not clearly specified, that is, it is not clear what is being calculated. Therefore, it can be concluded that this teacher did not understand the representation he is supposed to be teaching to the learners.

Table 4.5 further shows that only T1 tried to represent the information correctly in a flow chart even though the connection of R1, 50n and R300 is not clear.

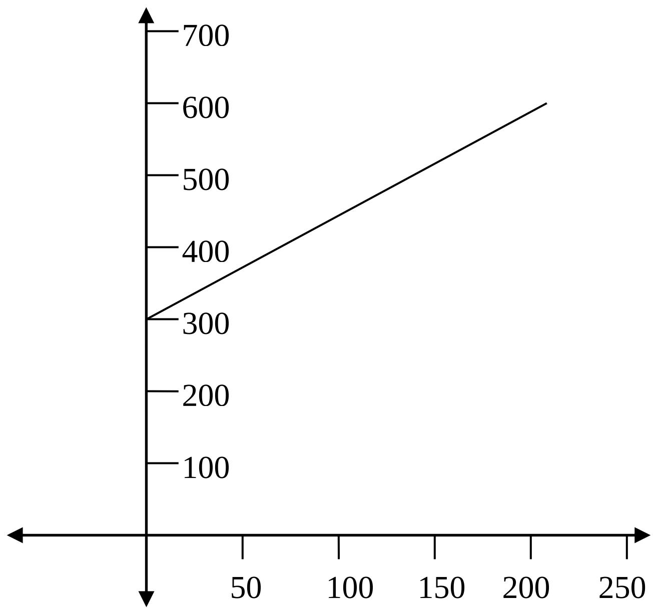


This shows some understanding of representing information using a flow diagram. The input values are clearly indicated, although practically, a hired car would not travel only one kilometre in a day. On the other hand, 18% of the teachers were unable to represent the information correctly in a flow diagram. For instance, S5 represented the information as follows:



The interpretation is that the input values are the number of days. It is not clear where or how to get the output values. This follows from the absence of variables in the given expression. Therefore, it can be concluded that the teacher had inadequate knowledge of flow diagrams that he should teach to learners.

Furthermore, Table 4.5 shows that only T5 was able to show the information correctly on a graph. Using the equation: $300 + 1{,}5k = cost/cost = 300 + 1{,}5k,$ T5 was able to represent the information as follows:



Although k was not defined in the equation, it is assumed that it represents the number of kilometres per day. Also, though the axes of the graph were not labelled, they show that they represent the cost and the number of kilometres travelled per day. This shows that the teacher had knowledge of graphs though the knowledge is not sufficient because of the unspecified variables. While this is the case, the Table 4.5 shows that 8% of the teachers represented the information incorrectly on a graph.

Most importantly, Table 4.5 shows that only one teacher (T5) was able to represent the information correctly in three different ways. Next to give the correct representations were T1 and S13 who represented the information in two different ways. This shows that more than 80% of the teachers could not represent the information correctly in any way. This suggests that most of the teachers had inadequate knowledge of different representations that they need to teach to learners. That is, most teachers could not represent the word problem algebraically. This shows inadequate knowledge of word problems.

Question 6 indicates that sometimes when you teach learners, you realise that they have difficulties in learning algebra. The teachers had to describe any five difficulties that learners have in the learning of algebra. They had to use examples in their descriptions.

The difficulties which the teachers identified were categorised according to the themes from their responses. The identified themes are: operations; expressions; equations; representations; general (algebra related) difficulties; numbers and non-algebraic difficulties in Table 4.6 below. In addition to the categories, the difficulties that the teachers identified under each category are reflected in the table. In cases where more than one teacher had identified the same difficulties and similar examples, then only one of them is reflected in the table. Similarly, if one teacher had identified similar difficulties, one is reflected. Furthermore, teachers who identified the difficulties and their frequencies are shown in Table 4.6. Also, the frequencies for each category are reflected in the table.

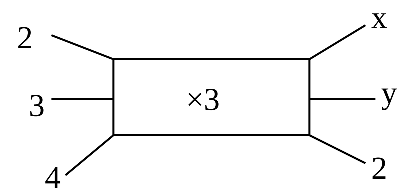
Table 4.6

Difficulties That Learners Have When Learning Algebra (N =61)

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
Operations				**3**
-Learners find it difficult/tend to forget the use of order of operation/can apply BODMAS in an incorrect way.	$2x+5=10 \rightarrow \frac{2x}{2}+5=\frac{10}{2} \rightarrow x+5=5 \rightarrow x+5-5=5-5 \rightarrow x=0$;	T1	1	
-Order of operations, multiplication, division	$a \times a = a^2; \frac{2x-4}{x+1}$	R3	1	
-Learners forget $(-)$ sign when multiplying a negative number	$\frac{2x}{2y} \cdot \frac{-2x}{2y} = \frac{4x^2}{4y^2}$	S3	1	
Expressions				**29**
-Finding the coefficient.	$3x+4$	T2	1	
-Identification of variables.	Let a particular value or number be equal to- etc	S20	1	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-On writing numbers with variables.	$x \times 2 = x2$	T5	1	
-Learners are confused by variable.	$2x + 1$	R1	1	
-Variable X numbers.	$4x = 4 \times x$	R17	1	
-Can't define term, operators, degree of a polynomial, constant, variables.	$x + 1 + 4y = 0$	S18; S22 R7	3	
-Problems understanding concepts, monomial, binomial, trinomial	failing to define the concepts	T10	1	
-To construct expressions if an expression is given by words and are supposed to write it as an expression.	There are three times as many girls as boys in school	T11	1	
-Learning with things they can touch example explaining a variable, constant as a number. -Understanding mathematical concepts in order to use them in daily basic life.	School situated around the main road watch the car that passes by eg. Isuzu, corolla etc but the school does not change, that make coefficient is multiply by the number of same model	R4	1	
-Expressions and equations learners fail to identify the two equation or Differentiating between expression and equation.	$6x + 3$ and $6x + 3 = 0$ $3x + 4;\quad 3x + 4 = 19$	T10; S1 S9; S18; S20	5	
-Deriving an equation	$300D + 150x$	R23	1	
-Finding the rule	$300 \times D + 15 \times n$	R23	1	
-Learners treat expressions as Equations.	When asked to simplify $2x^2 + 4x + 2$, they equate the expression to zero, which lead to a wrong answer	S1	1	
-Confusing addition and multiplication	$a \times a = 2a$	S16	1	
-Multiplying exponents instead of adding exponents of the same base.	$x^2 \times x^3 = x^6$	T13; S11	2	
-They can't apply laws of exponents.	$2^a \times 3^a = 6^a$, $(a \times b^2)^2$	S16; S21	2	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-Identifying terms.	$3(x-2)+4 \rightarrow 3x-6+4$	T11	1	
-Unable to identify like term.	$x^2+xy+2x^2$	S1	1	
-Identifying like terms in an expression and adding such.	$ab+bxy-2ba-2bxy$; they may add bxy and $-2bxy$ only, not realising that ab and ba are like terms	S10	1	
-Learners have problems adding or subtracting like terms.	$ab^2-ay^2+3ay^2+6ab$	T12; S22	2	
-To add and subtract expressions (taking + and – into consideration).	$2x^2-3x^3+x^3-4x^2$ $=-2x^2-2x^3$	T11	1	
-Adding unlike terms.	$4p+3q=7pq$	S16; R17	2	
-Adding like terms.	$3x^2+2x-x+x^2=0$ $\rightarrow 4x^4+x^2$	S9	1	
-Subtracting unlike terms.	$10y-5=5$	R17	1	
-They add $2+4$ or subtract $5-3$.	$2x+4$ $5p-3$	T6	1	
-Subtracting like terms.	$-2x-4x=2x$	S9	1	
-Not add like terms when they add	$2x+2x^2+x=5x^4$; they add exponent instead of adding $2x+x+2x^2 \rightarrow 3x+2x^2$	T13	1	
-Addition of terms that are not of the same degree.	$x+x^2=3x$ instead of $x+x^2=x+x^2$	S12	1	
-Unable to find the LCD. -Unable to add or subtract fractions.	$\frac{2}{4a}-\frac{4a}{2a^2}$; $\frac{a}{b}+\frac{b}{c}$	S1	1	
-Dividing algebraic expressions Incorrectly.	$\frac{\cancel{2}a-2b}{\cancel{2}}=a-2b$	S16	2	
-To simplify algebraic expressions	$6a^3b^2 \div 3ab=\frac{6.a.a.a.b.b}{3ab}$ $=2a^2b$	T11		
-How to solve problems with division equations.	$12x \div 4=$	R6	1	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-Factorise by taking out common factor.	$\frac{2x^2}{2}+\frac{4}{2}=x^2+2$ instead of $2x(x+2)$;	T10, S13	2	
-To factories. -Unable to factorise	$a^2+7a-14$; $2xa+4a^2$; $x^2+2x+1=0$	T12 S1; S6	3	
-Remove one number in brackets. -Unable to find the product.	$x(x+1)=x^2+1$ instead of x^2+x; $ax(2x+4y)$	T13 S1	2	
-They failed to substitute the value given the equation.	$2x^2-3x+4$ if $x=2 \rightarrow 2(2)^2-3(2)+4=2(4)-6+4=8-6+4=6$	S12	1	
-Substitution; use of substitution.	Calculate the value of y if $x=-2$ $3+x$	R20, R5; R26	3	
-Others fail to work with flow diagrams		R6	1	
-Using input and rules to define output values		R7	1	
Equations				**41**
-It becomes a problem to identify the constant, variable, and coefficient	$3x+4=7$	S18; S22; R15	3	
-Try to understand how a variable and coefficient can be together	$3x+4=9$	T6	1	
-Learners are confused by equal sign in the middle of a number. They are familiar to the equal sign in the beginning of a number	$2x+1=7$	R1	1	
-Can't interpret word problem into algebraic expression or equation	Three times any number minus 4 is 15	S16; S17; S21; R2; R5; R20; R24; R25	8	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-Difficulty formulating number sentences	$3 + x - 4 = 15$	R11; R24	2	
-Number sentences and verbal descriptions.	$\square - 12 = 36$	R7	1	
-How to represent the statement in the form of an equation.	$2y + 2x = 4$	S20; S22	2	
-Problem solving	A lady buys 50 oranges per week. How many orange will she buy in 3 weeks	R22	1	
-Solve for variables	They did not know how to solve and they were confused because they know in maths with deal with numbers not letters;	R16	1	
-Solving variables	$2x = 3 \rightarrow x = 2 - 3 = -1$ instead of $2x = 3 \rightarrow \frac{2x}{2} = \frac{3}{2} \rightarrow x = \frac{3}{2}$	S12; R6	2	
-Solving the unknown or variable by first grouping the like terms	$5 + x = 16 \rightarrow x = 16 - 5 \rightarrow x = 11$	S17	1	
-To add like terms together and solve the unknown	$3x + 2x + 3 + 5 = 0$	S6	1	
-Calculations/simplifying -Solving for the unknown	$p + 4 = 10$	S19; R10; R22	3	
-Fail to do or work the equations/ solving linear equations.	$2p + 5 = 18$	T6; T3	2	
-Solving number sentence with integers	$5x - 6 = 9$	R5	1	
-Do not use inverses	$x + 1 = 6 \rightarrow x + 1 - 1 = 6 \rightarrow x = 6$	S15; R2; R8	3	
-Learners fail to apply same operations on both sides of an equation	$2x + 5 = 10 \rightarrow 2x + 5 + 5 = 10 + 5$; $x + 1 = 3 \rightarrow x = 3 + 1 = 4$	T1; T3;T6; T8; T9; T13; S11;	7	
-Balancing the equation	$2x = 4 \rightarrow x = 4$	R26	1	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-using incorrect inverses when solving equations --	$3x+1=6 \rightarrow 3x=5 \rightarrow x = 5-3=2$	S16; R6	2	
-To change constant number to the left side	They didn't know we change the operation when we take the constant to the left side of equation	R16	1	
-If variable is $(-)$ negative	They didn't know we must also multiply both sides by $(-)$ by negative sign	R16	1	
-Divide by coefficient number	They didn't know we multiply both numbers the right numbers and the left number by the coefficient number	R16	1	
-when they have to solve problem they forgot to put like terms together	$3x+4=2x+14 \rightarrow x=\frac{4}{3}$ and $x=7$	T9	1	
-Solving equations, hardly equate, hardly group like terms together	$3x+4=19 \rightarrow 3x=19-4 \rightarrow \frac{3x}{3}=\frac{15}{3}$ $\rightarrow x=5$	R17	1	
-Simplifying the equation	$x^2+1+2=0 \rightarrow (x+1)(x+2) \rightarrow$ $x=-1$ or $x=-2$	S18	1	
-They forget to divide at the end	$2x+3=4 \rightarrow 2x+3-3=4-3 \rightarrow 2x=1$	S5	1	
-Ignoring the coefficient when solving equation	$3x+4=0 \rightarrow 3x=-4$	T9	1	
-They put equal sign in the wrong place	$2x+3=4 \rightarrow 2x+3-3=4-3 \rightarrow \frac{2x}{2}=x\frac{1}{2}$	S5	1	
-To find the value of x and y	$x-4=6 \rightarrow x=6+4 \rightarrow x=10$	T11; T12	2	
-Double variables in an equation -Simultaneous		R23 T12	2	
-Two equations and two variables	Work with first equation and the answer must be used on the other equation	R16	1	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-Solve equation without understanding	$2x-1=x+2 \rightarrow 2x-x=2+1 \rightarrow x=3$	T13	1	
-Some learners are likely to straggle solving equations that involve fractions	$3x^2+\frac{2x}{3}+3=0$	S1	2	
-They have some difficulties in solving algebraic equations. They have difficulties of using the L.C.M.	$\frac{x+2}{2}+\frac{x+3}{3}=4$	S7	1	
-To prove whether the statement is correct by substituting the unknown with the answer you got to make the statement True	$5+x=16 \rightarrow 11+5=16 \rightarrow 16=16$	S17; R11	2	
-How to factorise the equation by solving for	$2[x^2-3x-4]=0 \rightarrow 2[(x-4)(x+1)]=0 \rightarrow x-4=0$ or $x+1=0 \rightarrow x=4$ or $x=-1$	S13	1	
Representations				**5**
-Cannot find coordinates	$(3;-1)$ and $(5;3)$	T12	1	
-Interpreting graphs	Given a line graph which shows distance and time	R10	1	
-Drawing/plotting a graph	Draw a graph of $y=3x+1$	R20; R24	2	
-They get confused. They don't differentiate between the flow diagram and the table		R15	1	
General (algebra related)				**4**
-Learners cannot use algebra in real life	Water consumed by a certain number of people in a week is 5 kilo litres. How many people uses the water	R24	1	
-Unable to give practical examples in real life situation		R7	1	
-How to relate the problem to their everyday lives	Calculating money	R11	1	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-Algebra has many topics that makes learners to be ignorant of other topics		S3	1	
Numbers				**25**
-Use of operation	Plus, which means add, subtract which is minus, interval , different etc	S20, S22, R15	3	
-BODMAS rule	Learn the rule and how to use it in the number	R21	1	
-Order of operation	$39-(16+9)\times 2$	R22	1	
-They forget to use bodmas rule	$2-3\times 4$	T7	1	
-Number operation and Bodmus	$\frac{(2\times 6)+4}{6+4}$	T3	1	
-Not following the Bodmas Rule	$4+2-(4-3)=4+2-4-3$ instead of $4+2-4+3=5$	S11	1	
-Applying BODMAS rule, to simplify expressions		R26	2	
-Using operation signs in Bodmas rule, learners just simplify without following Bodmas rule	$\frac{2}{3}+\frac{3}{4}\times\frac{3}{2}=\frac{8+9}{12}\times\frac{3}{2}=\frac{17}{12}\times\frac{3}{2}=\frac{51}{12}$	S4	1	
-Multiplying or divide numbers with different signs	$-3\times 4=12$ it should be -12 $-3\times -4=-12$ instead of positive $12.(+12)$; $\frac{-12}{4}=3$ instead of -3	T4; T5; T8; S12; R9; R13	6	
-On multiplying numbers, this must suppose to be 12 but a learner add instead of multiplying	$4\times 3=7$	T5	1	
-Finding the lowest common denominator	$\frac{4}{6}+\frac{3}{5}$	T2; T8	2	
-Learners may forget to change the numbers when the sign in between is (÷)	$\frac{1}{2}\div\frac{1}{2}=\frac{1}{2}\times\frac{2}{1}=1$	T4; T7; T8; T2; S14	5	

Description of difficulties that the learners have in learning algebra.	Examples	Codes	f per diff	f per cat
-Learners forget to change the sign when changing from division to multiplication	$\frac{1}{2} \div \frac{4}{3} = \frac{1}{2} \times \frac{3}{4}$	T3	1	
-Adding and subtracting, learners have a problem in adding and subtracting integers	$-4+2$; $4+5-6+-3=6$; $27+34$; $4-9$; $4-(-9)$; $2+(-2)=-4$ $-2+5=7$ instead of $-2+5=3$	T2, T4; T5; T8; S3 S12, R9, R13, R20	9	
-When dealing with exponents	2^3 (learners tempted say two multiply by three which will result as six being the answer $2^3 = 2+2+2$ instead of $2 \times 2 \times 2$	S14 R26 S13	3	
-How to find the match multiples	$\frac{2}{4} + \frac{3}{5} = \frac{2}{4} = \frac{4}{8} = \frac{6}{12} = \frac{8}{16} = \frac{10}{20}$ $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20}$	R21	1	
-Working with fractions they forget to add the numerator while changing mixed number to improper fraction	$2\frac{1}{2}$	T3	1	
-Difficulties in converting mixed fraction to improper fraction	$2\frac{1}{2} = \frac{4}{2}$	S3; R13	2	
-Add/simplify the mixed fraction	$1\frac{2}{4} = \frac{6}{4}$	R21	1	
-Changing fractions to improper fractions, some learners may add those numbers and write them out	$5\frac{1}{4} = \frac{5+4+1}{4} = \frac{10}{4}$	S4	1	
-Adding and subtracting fractions (they add and subtract what they have in numerators and denominator)	$2\frac{1}{2} - \frac{3}{4} + \frac{1}{3} = \frac{5}{2} - \frac{3}{4} + \frac{1}{3} = \frac{3}{1}$	T4; R13	2	
-Working with challenging fractions that involve multiply type of then in one problem/expression	$1\frac{1}{2} + 0{,}2 \times 3\frac{9}{3}$	R26	1	
-Comparing fractions	$\frac{7}{8} > \frac{5}{6}$	S4	1	

Description of difficulties that the learners have in learning algebra	Examples	Codes	f per diff	f per cat
-Counting in fractions	$0;\ \frac{1}{2};\ 1;\ 1\frac{1}{2};\ 2;\ 2\frac{1}{2}\ldots$	R22	1	
-Completing a sequence	$1;\ 3;\ 5;\ 7;\ 9;\ldots$	R22	1	
-Writing numbers in words	$47 =$ forty seven	R22	1	
-How to convert from fraction to a decimal fraction	$\frac{10}{4} = 0{,}25$	R21	1	
-How to convert to percentages	$0{,}25 = \frac{25}{1} \times \frac{100}{1}$	R21	1	
-Finding prime factors of 3 different number and identifying the HCF	$436\ 375$ and 492	R19	1	
-Difficult to identify irrational numbers and rational numbers. -They will classify $\sqrt{25}$ as irrational, even after explaining to them that that $\sqrt{25}$ is a perfect square, also showing them without using a calculator	e.g. $\sqrt{25}$ They fail to simplify first, then thereafter classify	S8; S15	2	
-They use calculators without writing steps	$\frac{1}{3} \times \frac{1}{4}$	T7	1	
Non algebraic				**6**
-Difficult using a calculator		S3	1	
-Profit and loss	The price of a shirt was $R18$ and now is $R10$	S21	1	
-Measuring	Distance, time and weight	R11	1	
-Copying wrong work from their peers		R7	1	
-Drawing of bar and histogram	Histogram has no gaps, bar has gaps learners could confuse the two graphs	R17	1	
-When doing constructions of angles	Construct an angle of 90^0without using a protractor	S14	1	

Description of difficulties that the learners have in learning algebra	Examples	Codes	f per diff	f per cat
-Transformation geometry	Rotate 180^0clockwise and indicate the co-ordinates of the image	S14	1	

Table 4.6 shows that many difficulties were identified in numbers, expressions and equations. Also, 8% of the teachers identified seven difficulties about numbers, 11% of the teachers identified ten difficulties dealing with expressions and 13% of the teachers identified 8 difficulties in equations.

In the first category, operations, though not the focus in algebra but in arithmetic, two main difficulties were identified. The first difficulty indicates that "learners find it difficult/tend to forget the use of order of operation; "$2x + 5 = 10 \rightarrow \frac{2x}{2} + 5 = \frac{10}{2} \rightarrow x + 5 = 5 \rightarrow x + 5 - 5 = 5 - 5 \rightarrow x = 0$" (T1). Also, the example $a \times a = a^2$, is regarded as "order of operation, multiplication" (R3). These examples do not tally with the difficulties mentioned. The first example shows incorrect application of multiplicative inverse while the second one does not show any difficulty. This suggests that these teachers did not have knowledge of order of operations. That is, these teachers had inadequate knowledge of mathematical concepts. It also shows that the teachers had inadequate knowledge of analysis of learners' difficulties.

The second difficulty is reflected as "Learners forget $(-)$ sign when multiplying a negative number; $\frac{2x}{2y} . \frac{-2x}{2y} = \frac{4x^2}{4y^2}$" (S3). In this case, the example might mean that learners have difficulties dividing or multiplying expressions with different signs. If that is the case, it means the teacher will not provide the learners with the necessary help they need because the diagnosis of the problem is incorrect. However, when the learners forget, it does not imply that they do not know; they need to be shown the importance of those signs. However, if learners show difficulties of how to do it, then the concept needs to be clarified further for them to understand. In this case, the teachers showed inadequate knowledge of learners' difficulties.

In the second category, expressions, several groups of difficulties were identified. The first group in this category is identification of parts of an equation and definition of terms. Some of the identified difficulties do not really qualify to be regarded as difficulties. For example, "finding the coefficient; $3x + 4$" (T2); and "problems understanding concepts monomial, binomial and trinomial" (T10). The problem with these identified difficulties is that teachers did not understand the definitions of mathematical terms themselves. This follows from the fact that most of the teachers could not define or identify parts of an equation as it has been seen in Table 4.3 and Table 4.4. Also, the teachers confuse expressions with equations. This is seen in the statement, "learners fail to identify the two equations; $6x + 3$ and $6x + 3 = 0$" (T10), where both an expression and an equation are referred to as equations. This is also shown in the statement: "Deriving an equation; $300D + 150x$" (R23). Learners will not differentiate an expression from an equation if the teachers themselves do not know the difference.

T11 has indicated that learners are unable to change word sentences into number sentences, giving: "there are three times as many girls as boys in school", as an example. However, the same teacher could not detect the learner's difficulty when the same sentence was represented wrongly in an equation. This shows that some of the difficulties that learners have, are caused by the fact that the teachers themselves have the same difficulties as shown by their performance in Table 4.3 and Table 4.4. Also, in this section, S1 has indicated that "Learners treat expressions as equations; when asked to simplify $2x^2 + 4x + 2$, they equate the expression to zero, which leads to a wrong answer". It is not clear whether the teacher realised the difficulty when he wanted to test the learners' knowledge of like and unlike terms or the teacher made a mistake on the instruction. This choice of an expression to simplify might be the one that is confusing to the learners. They might see that the expression cannot be simplified, and then decide to factorise it. After factorising they will be tempted to find the roots because the instruction itself does not correspond with the given expression. This implies that unclear instructions or incorrect choice of expressions might result in learners' misunderstanding of concepts. However, unclear instructions might be the result of the teachers' inadequate knowledge of concepts. Therefore, it can be concluded that the teachers reflected inadequate knowledge of the learners' difficulties in this section.

The second identified group of difficulties in this category is that learners cannot write terms of an expression in a conventional way. Instead, they write variables before they write coefficients. For example, the teacher identified it as "on writing numbers with variables; $x \times 2 = x2$" (T5). Although the example shows what the difficulty is, the teacher could not explain it. Also, the teacher shows inadequate knowledge of mathematical concepts. This follows from referring to 'terms with variables and numbers' as just 'numbers and variables'. Hence the teachers had inadequate knowledge of what he intends learners to learn.

The third identified group of difficulties that learners face in this category is when simplifying expressions in the form of exponents. For instance, "they [learners] can't apply laws of exponents; $2^a \times 3^a = 6^a$" (S16). However, the explanation is inadequate because it does not explain what the learners do wrongly to show that they cannot apply the exponential laws. On the other hand, the provided example shows the opposite of the identified difficulty. This shows inadequate knowledge of error analysis, which is the result of inadequate knowledge of learners' difficulties.

There are teachers who were able to identify difficulties which correspond with their given examples. For instance, "multiplying exponents instead of adding exponents of the same base; $x^2 \times x^3 = x^6$" (T13). This shows the ability to analyse learners' errors, which then suggests the knowledge of learners' difficulties.

The fourth group of difficulties were identified when adding and subtracting like terms. However, the given examples do not reflect the difficulties identified. For instance, "Put or adding like terms together; $ab^2 - ay^2 + ab + 2ab^2 + 3ay^2 + 6ab$" (T12) and "To add and subtract expressions (taking $+$ or $-$ to consideration); $2x^2 - 3x^3 + x^3 - 4x^2 = -2x^2 - 2x^3$" (T11). These examples do not show what learners do wrongly to show their misunderstanding. In some instances, teachers were able to explain the difficulties that learners have although not fully. For example, "not to add the like terms when they add; $2x + 2x^2 + x = 5x^4$; they add exponent…" (T13). Also, "adding unlike terms; $4p + 3q = 7pq$" (S16). In the last example, although learners add unlike terms, it shows that the focus of the learners is only on the addition of the numbers, but not on the variables. In the identified difficulty, "subtracting like terms; $10y - 5 = 5$" (R17), the teacher did not explain much. However, the example shows that learners just add or subtract the numbers and disregard the variables. Again, it shows that learners think when simplifying expressions, the answer must have only one term (Booth, 1988; Booth et al., 2014; Kuchemann, 1978, Tirosh, Even & Robinson, 1998). While still on addition of like terms, a teacher indicated that when "adding like terms (they [learners] add the terms with power 2 together and get the answer to the power of 4); $3x^2 + 2x - x + x^2 = 0 \to 4x^4 + x^2$" (S9). This shows that the learners group the like terms together, but instead of considering the coefficients only when adding, they also add the exponents. The teachers did not fully explain what learners do to show that they do not understand. Therefore, this poor explanation of learners' difficulties can be attributed to inadequate knowledge of the learners' difficulties.

The fifth group of difficulties identified shows that learners do not have knowledge of commutative property. Hence they see like terms as unlike terms when their variables are arranged differently. This is shown in this example: "Identifying like terms in an expression; $ab + bxy - 2ba - 2bxy$ they may add bxy and $-2bxy$ only, not realising that ab and ba are like terms" (S10). The teacher did not explain why ab and ba are like terms. By not mentioning the commutative property in this instance suggests that the teacher had no knowledge of the concept. Inadequate knowledge of mathematical concepts implies that the teacher had inadequate knowledge of what he intends the learners to learn.

The sixth group of difficulties was identified when simplifying algebraic fractions. Some of the identified difficulties are listed below.

- Unable to add or subtract fractions; $\frac{a}{b} + \frac{b}{c}$ (S1).
- Dividing expressions incorrectly; $\frac{\cancel{2}a-2b}{\cancel{2}} = a - 2b$ (S16).
- To simplify algebraic expressions; $6a^3b^2 \div 3ab = \frac{6.a.a.a.b.b}{3ab} = 2a^2b$" (T11).

The first difficulty does not explain what learners do to show that they have difficulties. The example provided does not show the difficulty either. In the second difficulty, the example shows what learners do, but the difficulty was not fully explained. The difficulty as shown in the example shows that learners are unable to apply the distributive property. However, the teacher did not explain the difficulty fully. Also, the third explanation does not indicate any difficulty, and the example shows what learners should do instead of identifying their difficulties. Therefore, this inadequate explanation of learners' difficulties, suggests that the teachers have no knowledge of them.

In the seventh group of difficulties, teachers noted: "To factorise; $a^2 + 7a - 14$" (T12). Neither the expression was factorised nor the difficulty explained. Similarly, the difficulty "unable to find the product; $ax(2x + 4y)$" (S1), was not clarified fully to show the learners' misunderstanding. The example also does not illustrate the difficulty. In the identified difficulty, "Factorise by taking out common factor; $\frac{2x^2}{2} + \frac{4}{2} = x^2 + 2$ instead of $2x(x + 2)$" (T10), the explanation given, contradicts the example given. The impression is that learners confuse simplification and factorisation. That is, instead of simplifying the expression, they factorise it. The explanation by the teacher does not indicate where the problem lies. While in the difficulty "Remove one number in brackets; $x(x+1) = x^2 + 1$ instead of $x^2 + x$" (T13), shows that learners have no knowledge of distributive property. On the other hand, the teacher also shows lack of mathematical vocabulary. This follows from referring to distributing over a binomial as 'removing one number in brackets'. This clearly shows that the teachers had inadequate conceptual knowledge. It can be concluded that the inability to explain learners' difficulties suggests inadequate knowledge of learners' difficulties.

It is also indicated that substitution is one of the difficulties that learners have. For example, one of the teachers wrote "substitution; calculate the value of y if $x = -2$" (R20). This choice of an example will be a challenge to learners. It is because there is no expression to substitute into. The problem is incomplete, showing guess work on the part of the teacher. Therefore, these problems suggest inadequate knowledge of the learners' difficulties.

In the third category, equations, some teachers have identified that learners have difficulties in identifying parts of an equation. As it has been indicated earlier, this will not be a difficulty if learners are taught the definitions of the concepts so that they can use those to identify. However, it has been shown in Table 4.3 and Table 4.4 that most teachers were not able to identify or define the parts of the given equation. For example, R15 who indicated that learners have this difficulty could not identify any of the parts that he indicated as difficulties to learners. This only shows that learners cannot master what their teachers do not understand. Therefore, it can be concluded that these teachers had inadequate knowledge of what they intend the learners to learn.

The highest number of difficulties identified in this category is about solution of equations. For example:

- They have some difficulties in solving algebraic equations. They have difficulties of using the L.C.M; $\frac{x+2}{2}+\frac{x+3}{3}=4$ (S7).

The statement and the example do not explain how the learners solve problems of this nature to show their misunderstanding. Even in the next identified difficulty, although it does not form part of the senior curriculum, the learners' difficulties are not indicated when dealing with problems of this nature. The following difficulty was identified:

- Two equations and two variables; work with first equation and the answer must be used on the other equation (R16).

Simultaneous equations are part of algebra curriculum in the FET band, but not in senior phase curriculum. This lack of clear explanation of the learners' difficulties, suggests that these teachers had no recall of the learners' difficulties in these grades. As Shulman (1987) indicates, PCK requires comprehension and reflection amongst others. These teachers' responses suggest that they did not reflect on the lessons they taught, and also, they did not analyse their learners' difficulties. Hence, they reflect inadequate knowledge of learners' difficulties.

A number of the difficulties listed in this category do not explain exactly what the learners do to show that they have difficulties. Also, the accompanied examples were not worked out to illustrate the problems that the learners have. For example, it is indicated that "sometimes they [learners] fail to do or work the equations; $2p+5=18$" (T6). What or how learners fail, is not specified or shown in the example. Another example states: "[Learners] try to understand how a variable and coefficient can be together; $3x+4=9$" (T6). This implies that when letters are introduced as variables to the learners, they are not linked to a placeholder, which the learners are familiar with from the lower grades. This is supported by R16 who stated, “They [learners] did not know how to solve and they were confused because they know in maths we deal with numbers not letters”. Deducing from the teachers’ statements, the learners’ confusion were caused by the teachers’ failure to build on what the learners already knew. Since the learners had already used a placeholder before, understanding why a letter was used in the place of a placeholder, would not be difficult. On the other hand, the teachers’ answers suggest that the teachers did not have any recall of the learners’ difficulties, hence, they only thought of what might confuse the learners without thinking it through. It also suggests that these teachers do not always reflect on their learners’ answers, which would help in identifying where the difficulties lie.

Most importantly, the learners’ misunderstanding of variables shows that they do not know that these variables are substitutes of numbers which they must determine to make the equation balanced. Actually, these learners do not know what the equations mean. This is shown in the statement, “Learners are confused by equal sign in the middle of a number. They are familiar to the equal sign in the beginning of a number; $2x + 1 = 7$” (R1). This implies, either that these teachers were guessing these difficulties, or if the difficulties were true, the learners were given equations to solve without being introduced to the concept first. If the teachers guessed the answers, this reflects a lack of recall as a result of a lack of reflection on the teachers’ part. This implies that when they assessed learners, they did not analyse the learners’ answers to detect where the root of the problem lay. Hence, they did not recall the learners’ misunderstanding in this section.

In some cases, the given examples have been worked out, but they do not show the problems the learners have. For instance, “solving the unknown or variable by first grouping the like terms; $5 + x = 16 \to x = 16 - 5 \to x = 11$” (S17). Also, “How to factorise the equation by solving for x; $2[x^2 - 3x - 4] = 0 \to 2[(x - 4)(x + 1)] = 0 \to x - 4 = 0$ or $x + 1 = 0 \to x = 4$ or $x = -1$” (S13). The statements explain what is being done in the examples, but these do not show any difficulty. In other words, the teachers should have stated that learners do not group the like terms first when solving equations or the learners should solve the equation by first factorising the expression, if those are the difficulties experienced by the learners. The learners' difficulties were not explained or illustrated in these examples.

In other cases, the explanation given shows the opposite of the example provided. For example, "Solving equations, hardly equate, hardly group like terms together; "$3x + 4 = 19 \rightarrow 3x = 19 - 4 \rightarrow \frac{3x}{3} = \frac{15}{3} \rightarrow x = 5$" (R17). This does not show the errors that the learners do instead of grouping like terms together. Similarly, in the given statement, "when they have to solve problem they forgot to put like terms together; $3x + 4 = 2x + 14 \rightarrow x = \frac{4}{3}$ and $x = 7$" (T9). This shows that learners, instead of starting by adding inverses both sides to group the like terms when solving equations, start by equating each expression to zero. Thereafter, they simply move the constants to the other side of the equations without considering the additive inverses. They use multiplicative inverses correctly. The teacher did not explain all these. This shows that the teacher had inadequate knowledge of error analysis. In other cases, the teachers' analysis of the learners' misunderstanding is partly correct. For example, "Using incorrect inverses when solving equations; $3x + 1 = 6 \rightarrow 3x = 5 \rightarrow x = 5 - 3 = 2$" (S16). In this case, the learner correctly added an additive inverse both sides, but failed to multiply both sides of the equation by the multiplicative inverse. This shows that the learners confuse the inverses. Therefore, it can be concluded that the teachers' knowledge of learners' errors was inadequate.

In a few cases, the identified difficulties correspond with the given examples. For instance, the statement, "when they have to apply what is done on the right side must also be done on the left side they do it on one side; $2x + 3 = 4 \rightarrow 2x + 3 - 3 = 4 \rightarrow 2x = 4 \rightarrow x = 2$" (S5), corresponds with the example. This statement however, does not fully explain the difficulty mathematically. That is, the teacher did not mention any use of inverses. This shows lack of mathematical vocabulary. Again, another difficulty which corresponds with the example is that learners "Can't interpret word problem into algebraic expression or equation: Three times any number minus 4 is 15; $3 + x - 4 = 15$" (S16). The teacher could not explain exactly what the learners do in this case to show the difficulty. This difficulty was also identified by most teachers who could not identify learners' difficulties when interpreting word problems in Table 4.7. This implies that the teachers' difficulties are the learners' difficulties.

In the fourth category on representations, only four difficulties were identified. The first difficulty identified in this category indicates that learners "cannot find coordinates" (T12). This does not explain much about the difficulty. It might mean that learners do not know how to draw a Cartesian plane correctly, hence they cannot find the coordinates, or the learners might be interchanging the coordinates. This shows inadequate knowledge of learners' difficulties. The second difficulty states "Interpreting graphs; given a line graph which shows distance and time" R10. This does not explain the difficulties that learners have. Similarly, the third identified difficulty, "drawing/plotting a graph; draw a graph of $y = 3x + 1$" (R20), does not explain or illustrate the learners' difficulty. All these show that the teachers had inadequate knowledge of the learners' difficulties. Again, teachers have identified that "[Learners] get confused. They don't differentiate between the flow diagram and the table" (R15). In this case, no example was provided to illustrate this difficulty. To sum the provided answers in this category, inadequate information was provided about the learners' difficulties. This suggests, therefore, that the teachers, who provided answers in this category, had inadequate knowledge of the learners' difficulties.

In the fifth category, general (algebra related difficulties), two main difficulties were identified. The first difficulty is that learners cannot apply algebra in their lives. For example, the identified difficulty, “Learners cannot use algebra in real life; water consumed by a certain number of people in a week is 5 kilo litres. How many people use the water?” (R24). This identified difficulty, does not explain what the learners do to show that they cannot apply algebra. Further, the difficulty suggests that the learners understand algebra, but they cannot apply it. It is not clear how the teacher assessed the application in daily life, or how that affects the learning. Looking at the given example and the difficulty differently, it is possible that the teacher wanted to indicate that the learners cannot solve word problems algebraically. In that case, the word problem will be difficult to solve because it does not indicate that the people consume the same amount of water. Subsequently, incomplete information in word problems will always be difficult to interpret algebraically. For that reason, the teacher shows inadequate knowledge of word problems. Also the teacher had insufficient knowledge of the learners' difficulties.

The second difficulty identified in this category indicates: “Algebra has many topics that makes learners to be ignorant of other topics” (S3). This answer does not explain how these many topics make it difficult for learners to learn algebra. This sounds like a concern than a difficulty. Therefore, the teacher who identified this difficulty in this category shows no knowledge of the learners' difficulties in algebra.

In the sixth category, numbers, a number of teachers identified different difficulties. However, the difficulties they listed are purely arithmetic. For example, “Adding and subtracting fractions (they add and subtract what they have in numerators and denominator); $2\frac{1}{2}-\frac{3}{4}+\frac{1}{3}=\frac{5}{2}-\frac{3}{4}+\frac{1}{3}=\frac{3}{1}$”; (T4), and “Completing a sequence; $1;\ 3;\ 5;\ 7;\ 9;\ldots$” (R22). This shows that some teachers think that arithmetic is the same as algebra. Hence many difficulties were identified in this category. This suggests that a number of teachers do not have adequate knowledge of algebra curriculum in the senior phase.

In the last category, non-algebraic difficulties, general difficulties which are not specific to algebra were identified. These difficulties include ‘profit and loss’, ‘measuring’ and ‘transformation geometry’. This implies that these teachers think that everything that is done in mathematics is part of algebra. This shows the teachers’ inadequate knowledge of the algebra curriculum.

In summary, some of the difficulties identified in this question suggest that some teachers did not recall any difficulties that learners encounter when learning algebra. In some cases, teachers could not explain the difficulties that the learners have. In other cases, some of the given examples contradict the explanation given. Also, most of the explanations and the given examples do not show any difficulty. This implies that the teachers had inadequate knowledge of the learners’ difficulties. This could be the result of lack of error analysis when assessing learners and also, lack of reflection in learners’ work or lessons taught.

> In Question 7, the teachers had to study the statements that were taken from some of the learners at a school in Seshego. They had to identify the statements that are true and the ones that are false.

The teachers’ responses were classified into six categories. Each of the learners’ difficulties represents a category in Table 4.7 below.

Table 4.7

Responses of Teachers to Learners’ Difficulties When Learning Algebra (No = 61)

Statement	Teachers who indicated True		Teachers who indicated False	
	Codes	f	Codes	f
You can’t add $5c, 5b$ and $5t$ because they are like 5 cabbages, 5 beetroots and 5 tomatoes	T1; T4; T5; T6; T7; T9; T10; T12; T13; S1; S2; S4; S6; S7; S8; S9; S10; S12; S13; S14; S15; S16; S17; S18; S20; S21; S22; R2; R5; R8; R9; R10; R11; R12; R13; R14; R15; R16; R18; R20; R22; R24; R26	43	T2; T3; T8; T11; S3; S11; S19; R1; R3; R6; R17; R19; R21; R23	14

Statement	Teachers who indicated True		Teachers who indicated False	
	Codes	f	Codes	f
$3x + 4 = 19$ and $3y + 4 = 19$ are not the same because they have different letters	T1; T2; T6; T12; S1; S6; S11; S17; S20; R26	10	T3; T4; T5; T7; T8; T9; T10; T11; T13; S2; S3; S4; S7; S8; S9; S10; S12; S13; S14; S15; S16; S18; S19; S21; S22; R1; R2; R3; R4; R5; R6; R7; R8; R9; R10; R11; R12; R14; R15; R16; R17; R18; R19; R20; R21; R22; R23; R24; R25	49
If you add three onto $4p$ you get $7p$	T3; S18; R3; R6; R11	5	T1; T2; T4;T5; T6; T7; T8; T9; T10; T11; T12; T13; S1; S2; S3; S4; S6; S7; S8; S9; S10; S11; S12; S13; S14; S15; S16; S17; S19; S20; S21; S22; R1; R2; R4, R5; R7, R8; R9; R10; R12; R13; R14; R15; R16; R17; R18; R19; R20; R21; R22; R23; R24; R25; R26	55
You can't do $P + Q = 10$ because there isn't an answer	T1; T2; T3; T4;T5; T7; T9; T10; T12; S1; S4; S6; S7; S9; S11; S14; S16; S17; S19; S20; R1; R5; R9; R12; R13; R15; R16; R17; R18; R21; R22; R26	32	T6; T8; T11; T13; S2; S3; S10; S12; S13; S15; S18; S21; S22; R2; R3; R4; R6; R7; R8; R10; R11; R14; R19; R23; R24; R25	26
In this school there are three times as many girls as boys, so if b stands for the number of boys and g stands for number of girls, then in this school, $b = 3g$	T1; T2;T4; T5; T6; T7; T9; T12; T13; S1; S2; S3; S6; S7; S9; S11; S12; S14; S16; S18; S19; S20; S21; S22; R1; R2; R3; R5; R6; R7; R8; R10; R12; R14; R16; R17; R20; R24; R25; R26	40	T3; T8; T10; T11; S4; S10; S13; S15; S17; R4; R11; R13; R15; R18; R19; R21; R22; R23	18
$-x$ is a negative number	T1; T2; T3; T7; T9; T12 S2; S3; S4; S8; S9; S13; S14; S17; S18; R1; R2; R4; R5; R6; R7; R8; R10; R11; R14; R15; R16; R17; R18; R19; R20; R21; R24; R25; R26	35	T4;T5; T6; T8; T10; T11; T13; S1; S6, S7, S10; S12; S15; S16; S19; S20; S21; S22; R3; R9; R12; R13; R22; R23	24

Further, the teachers responded by indicating either true or false to the statements made by the learners. Then, their codes were classified accordingly in each category. The frequencies of the responses are also indicated in the table. All these statements given by the learners are false, indicating that the learners have misunderstanding. The table shows that out of the six difficulties that the learners have, most of the teachers could identify only two. These are: adding unlike terms and thinking that different variables always represent different values. The table also shows that most of the teachers could not identify the learner's difficulty when a learner associated variables with objects. Also, a high number of the teachers could not identify any errors when a learner misrepresented a word problem in an equation. The teachers' responses are discussed in full in the next paragraphs.

> The first difficulty states: You can't add $\mathbf{5}\boldsymbol{c}, \mathbf{5}\boldsymbol{b}$ and $\mathbf{5}\boldsymbol{t}$ because they are like $\mathbf{5}$ cabbages, $\mathbf{5}$ beetroots and $\mathbf{5}$ tomatoes.

The table shows that about 70% of the teachers indicated that the statement is true. However, the statement given by the learner is partially correct. Meaning, adding the three unlike terms will give $\mathbf{5}\boldsymbol{c} + \mathbf{5}\boldsymbol{b} + \mathbf{5}\boldsymbol{c}$, but not one term. Although R4 and S5 did not indicate that the statement is true, their answers, given below, do not indicate that they disagree with the learner.

- You can add $5 + 5 + 5 = \frac{15}{3} = 5cbt$ (S5).
- No you can't because variables are not the same (R4).

The answer as given by S5 shows that the teacher also had difficulties when adding like and unlike terms. In addition, the learner associated the variables with objects, but not with numbers (Booth, 1988; Kuchemann, 1978; Tirosh, Even & Robinson 1998). This shows the learner's difficulty. However, only about 23% of the teachers were able to detect the learner's problem. Further, R7 and R25 did not respond to this question. Taking this into consideration, it can be concluded that most of the teachers also had this difficulty of associating variables with objects. Hence, they could not detect the learner's difficulty.

> The second statement states that $\mathbf{3}\boldsymbol{x} + \mathbf{4} = \mathbf{19}$ and $\mathbf{3}\boldsymbol{y} + \mathbf{4} = \mathbf{19}$ are not the same because they have different letters.

The statement suggests that the learner thought that different letters always represent different values (Booth, 1988; Kuchemann, 1978; Tirosh, Even & Robinson 1998). Table 4.7 shows that about 16% of the teachers could not detect the learner's difficulty. Although S5 and R13 did not indicate whether true or false to the statement, their answers, which are given below, show that they disagree with the learner's statement.

- Are the same because x and y are variables, so you can use any letter of alphabet as a variable (R13).
- Are the same because $x + 4 = 19$ & $3y + 4 = 19$
  $x = 15$ & $3y = 15$
  $x = 5$ & $y = 5$ (S5).

However, most of the teachers could detect the learner's error. This suggests that the teachers also had the same difficulty.

The third difficulty indicates: If you add three onto $\mathbf{4p}$ you get $\mathbf{7p}$.

The statement shows that the learner ignored the variable and only concentrated on adding the numbers. It is said that some learners do not accept expressions with addition or subtraction sign e.g. $3 + 4p$ as complete answers (Booth, 1988; Booth et al., 2014; Kuchemann, 1978; Tirosh, Even & Robinson, 1998).They tend to add or subtract the terms to make the answer one term. However, about 90% of the teachers were able to detect the learner's misunderstanding. This suggests that almost all the teachers had knowledge of addition of like and unlike terms. This also suggests that the remaining 10% had the same problem as the learner. These include S5 who did not indicate that the statement was true but implied it in the answer he gave. Therefore, it can be concluded that only a few teachers could not identify this difficulty.

> The fourth difficulty indicates: You can't do $\boldsymbol{P}+\boldsymbol{Q}=\mathbf{10}$ because there isn't an answer.

The statement shows that the learner did not understand that each of two variables has its value, and that the sum of the values is 10. Also, the value of P is dependent on the value of Q or vice versa so that their values will sum up to 10.The table shows that more than 50% of the teachers also could not identify this difficulty. Hence they indicated the statement as true. Their statements show that these teachers have inadequate knowledge of addition of variables, i.e. inadequate knowledge of like and the unlike terms. This is shown in some of their comments. For example,

- Yes $P+Q=PQ$ (S5).
- False, $P+Q=10$ is the same as $PQ=10$ (S18).

This shows that most of the teachers lacked a deeper knowledge of variables and equations. Hence, they could not identify this difficulty.

> The fifth difficulty states: in this school there are three times as many girls as boys, so if $\boldsymbol{b}$ stands for the number of boys and $\boldsymbol{g}$ stands for number of girls, then in this school, $\boldsymbol{b}=\mathbf{3}\boldsymbol{g}$.

The statement shows that the learner did not understand that for every one boy at the school, there are three girls. This implies that for the number of boys to be the same as the number of girls, $g=3b$. This shows that the learner had misunderstanding of the role of an equal sign in an equation. That is, the learner did not consider the fact that in an equation, the left hand side must always equal the right hand side. While this is the case, the table shows that more than 60% of the teachers could not identify the learner's difficulty. This suggests that these teachers also had the same difficulty.

The sixth difficulty states that $-\boldsymbol{x}$ is a negative number.

The statement shows that the learner did not understand that the variable x represents any number, i.e., either a positive or a negative number. If the number it represents is positive, then the value of $-x$ is a negative number, and if the number it represents is negative, then the value of $-x$ is a positive number. Therefore, the negative coefficient does not automatically make the value of a term to be negative. Nonetheless, more than 50% of the teachers could not identify the difficulty. It suggests that these teachers had inadequate knowledge of signed numbers. Hence, the teachers could not detect the learner's difficulty.

In view of the above, it shows that none of the teachers was able to detect the difficulties in all the statements made by the learners. This suggests that the teachers also, had the same difficulties as the learners; hence they could not detect the difficulties. Therefore, it can be concluded that these teachers had inadequate knowledge of the learners' difficulties in algebra.

> Question 8 indicates that sometimes when teaching mathematics, you realise that you have difficulties with teaching algebra. What difficulties have you encountered when teaching algebra?

Teachers' responses were classified into ten categories in Table 4.8. The categories identified are operations, equations, expressions, patterns, graphs, numbers, resources, teaching strategies, general difficulties and irrelevant difficulties. Those who did not respond are also reflected in the table.

Table 4.8

Difficulties That Teachers Encounter When Teaching Algebra (N = 61)

	Resposes			No responses		
Difficulties	Codes	f	%	Codes	f	%
Operations	T5; T8; T11; S1; S4; S17 R3; R15; R22	9	15	T7; S7; S12; S16; S22; R1; R9; R10; R12; R13; R14; R19; R20	13	21
Equations	T3; T4; T9; T11; T12; T6; S9; S14; S20; R2; R4; R5; R16; R17; R22; R23; R25	17	28			
Expressions	T2; T3; T10; S1; S2; S6; S9; S10; S13; S20; R17; R23	12	20			
Patterns	S11; S21	2	3			
Graphs	T4; T12; S11; R23	4	7			
Numbers	T4; T8; S10; S11; R11; R22; R23	7	11			
Resources	T2; T13	2	3			
Teaching strategies	T13; S1; S5; S8; S15; S19; R6; R15; R16; R18; R21; R24; R25	13	21			
General difficulties	T1; S11; S18; R7; R8; R21; R24; R26	8	13			
Irrelevant difficulties	S3; S13	2	3			

Table 4.8 shows that more than 21% of the teachers did not respond to this question. S7 clearly indicated "no difficulty", and hence classified under no response. Also, the table shows that most difficulties were experienced when teaching equations, followed by teaching strategies and teaching expressions. Only 3% of the teachers experienced difficulties when teaching patterns, while 4% experienced difficulties when teaching graphs. The table further shows that about 13% indicated general difficulties in teaching while two of the teachers gave answers that are irrelevant to the teaching of algebra.

In the first category, operations, most teachers indicated that learners had difficulties in using the operations: addition, multiplication, subtraction and division.

Also some indicated that learners confused some of the operations, but did not explain how. The difficulties as identified by the teachers are presented below.

- The difficulties with operations $(\times; \div; +)$ and $(-)$. When dealing with operation you must use (S4, S17 and R3).
- Learners they don't understand or differentiate between multiplication sign and addition sign (T5).
- Most learners get confused when coming to division and multiplication (T8 and T11).
- Most learners do not know how or when the negative sign is used (T11).
- Learners don't know the sums, difference, quotient and product. The know but when it is in a sentence they get confused (R15).

These difficulties, do not explain what learners do to show that they have difficulties using operations. To say learners are confused by the signs does not explain their confusion. Some indicated that they had difficulties when applying order of operations. They indicated the difficulties they as follows:

- Bodmas rule. Sometimes we use operation wrongly (S17).
- Order of operations (R22).
- Applying rules in products and simplification (S1).

These statements indicate that these teachers had inadequate knowledge of the application of the order of operations. However, they did not explain how they apply it wrongly. Also S1 did not indicate the difficulty they face when simplifying or using rules of products. Therefore, it can be concluded that these teachers did not recall how they applied the order of operations.

In the second category on equations, the teachers noted several difficulties. The first group of difficulties in this category shows that learners cannot identify parts of an equation. However, the first difficulty identified by T6 states that "Learners cannot understand e.g. $3x + 4 = 19$ because it is a variable, constant, and coefficient". It is not clear whether the learners cannot identify or define the parts of the equation or whether the learners do not understand what an equation is. The second difficulty indicates "learners have problems in identifying the terms in part of an equation even though they can define the terms" (T3). This implies that learners can define the parts but cannot identify them. This is hard to believe because the learners would use the definitions to identify the parts, unless they do not understand the definitions. Again, the teachers' responses do not indicate any difficulties that the teachers themselves faced when teaching equations. Also, most teachers could not identify most parts of an equation. In addition, they could not interpret a given word problem. Further, they did not indicate any difficulties that they had when teaching equations. This implies that the teachers did not have knowledge of their own teaching difficulties when teaching equations.

The second group of difficulties under this category show difficulties when solving equations. The difficulties are listed below:

- To find values of x and y when you are given 2 equations (T12).
- Forget writing equal sign when solving problems (T2).
- Solving equations (T10).
- Problem solving (R22).
- Learners are not able to find the values of x (T11).
- Learners in most cases leave the problem unsolved or simplified (R17).
- They [leaners] do not easily understand how to solve the equation (T6, R4, and R17).
- Learners do not understand why do we divide when there is coefficient (T9).
- Learners have problems solving linear equations,
  e. g., $3x + 4 = 19 \rightarrow 7x = 19 = x = \frac{19}{7}$ (T4).
- Cross cancelling number with operation in between and numbers without operation i.e. terms (R23).
- Tables need pattern and an equation to created (R23).
- Learners forget to change the sign that goes with the number (T2 and R2).
- Learners do not understand why we change the sign (T9).

All these statements are about the solution of equations. None of the statements explains the difficulties in detail. The main problem lies with the understanding the concept of equations. Almost all the statements indicate problems with the use of inverses when solving equations. The example given by T4 shows the problem. In addition, lack of knowledge of working with like and unlike terms. Hence, they disregard the variables and just add the numbers. This is also implied by the last two statements. However, the teachers did not explain much on what the learners do. This also implies that the teachers did not analyse the learners' errors when assessing the learners' work. Hence, they could not recall the errors. The teachers did not mention the difficulties that they, as teachers, experience.

Some teachers indicated that they encountered difficulties when solving simultaneous equations. According to CAPS, simultaneous equations are not part of senior phase syllabus. This shows inadequate knowledge of senior phase algebra curriculum. Further, the teachers show inadequate knowledge of mathematics vocabulary. This follows from indicating that learners 'do not change the sign' and also by talking about 'cross cancelling'. Again, it clearly shows that the teachers do not teach the meaning of the equality sign when teaching equations. Further, it shows that these teachers do not stress the balancing of the equations. Hence they do not mention any use of inverses. This implies that the teachers have inadequate knowledge of equations. Therefore, their knowledge of subject matter is not sufficient. In addition, they did not reflect on their teaching difficulties.

The last difficulties identified in this category are:

- Changing word problems to mathematical problems (S9, S14, S20 and R5).
- To interpret the problem (R25).

These difficulties were shown in question 5 where most teachers were unable to represent a word problem in different ways. It was shown that most teachers could not interpret word problems. Therefore, learners will also have the same problem because teachers cannot properly teach what they do not understand. While that is the case, the aim of algebra is to interpret practical problems algebraically. This however cannot be achieved if the teachers themselves cannot solve word problems algebraically. This suggests that the teachers had inadequate knowledge of interpretation of word problems. Yet, none of the teachers indicated this difficulty. This shows lack of knowledge of teaching difficulties that were experienced when teaching word problems.

In the third category, expressions, several difficulties were identified. The first answer noted is

- Careless mistakes by the teacher (omitting brackets or equal sign when solving problems) (T2).

This however, cannot be classified as a difficulty. It is as it is stated, 'careless mistake'. Then, in the next group of difficulties under this category, although the teachers did not explain the root of the learners' difficulties, it is clear that some of the difficulties show that learners lack understanding of the definitions of terms. As a result, it is difficult for the learners to differentiate between concepts. This is shown in the following statements

- Differentiating variables (S20).
- Learners confuse expressions with equations (S2).

The two difficulties have already been noted earlier. It has been shown that some teachers could not define parts of an equation; hence the learners cannot identify or define them. For that reason, it can be concluded that the teacher had inadequate knowledge of the definition of terms in algebra. This follows from the teachers not being able to define the identified terms.

The second group of difficulties in this category is about simplification of expressions, and also substitution into an expression. The teachers identified the difficulties as follows:

- Expressions (R22).
- Putting the like terms together before we simplify (T9, T10 and R23).
- They always want to add or subtract not to simplify (T6).
- They sometimes have a problem because they sometimes want to add $3x + 4$ and they did not get the answer (T6).
- They sometimes have a problem because they sometimes want to subtract $5b - 2$ and they did not get the answer (T6).

- Mix addition and multiplication of variables $x \times x = 2x$ (S10).
- Learners are unable to use exponential laws (S11).
- Learners fail to replace the alphabet by the number (T2).
- They forget immediately to replace the number they should multiply to get the answer (T2).
- When you give learners an example, they memorise the first one. In the expression you have take out the common factor, each an every equation they are going to remove a common factor even if there is no common factor (S13).

Although the first answer does not explain the difficulties that are experienced when dealing with expressions, the assumption is that most difficulties are experienced when simplifying. For example, T6 indicated that “they always want to add or subtract not to simplify”. If an example was given in this case, the statement would be clear. This failure to explain the difficulties experienced suggests that the teachers had insufficient knowledge of them. Further, in some cases, the explanations were not such that the difficulty explained would be easily understood. Even in cases where an expression is given as an example, the expression was not simplified to illustrate the difficulties that learners have. Further, all the difficulties identified in this category, are the same as the difficulties identified in Question 6. Furthermore, the teachers show inadequate knowledge of algebraic vocabulary. For example, variables were referred to as alphabets in some cases. In addition, the last answer did not indicate whether the learners could not substitute when asked to evaluate, or whether they could not substitute to prove that the unknown values were correct. Furthermore, the answers in this section do not indicate the difficulties that the teachers themselves encounter when they teach. This suggests that these teachers did not reflect back on the lessons they already taught or they did not analyse the learners’ work. Therefore, it can be concluded that these teachers had inadequate knowledge of difficulties they experienced when teaching simplification of expressions.

The last group of difficulties identified in this category are about factorising expressions. The teachers indicated the following difficulties:

- Learners cannot factorise (S2).
- Factorise expressions with big numbers (S6).
- Factorising expressions with coefficient of x^2 being greater than one e.g $5x^2 + 18x - 8 = 0$ (S6).
- Factorisation in a large expression of fractions dividing each other (S9)

All these answers do not explain what learners do wrongly when factorising. In addition the expressions mentioned in the statements are dealt with in the grades beyond the senior phase. Grade 9 curriculum requires learners to factorise trinomials with 1 as a coefficient of x^2 or when the coefficient of x^2 is a factor of the other terms for them to start by taking out the common factor when factorising. This suggests that these teachers had inadequate knowledge of senior phase curriculum. Also in this section, the teachers did not indicate the difficulties that they have as teachers when they taught factorisation. This also suggests that these teachers did not reflect on the lessons that they taught. Therefore it can be concluded that these teachers had inadequate knowledge of difficulties that they experienced when teaching factorisation.

In the patterns category, only two of the teachers identified difficulties as follows:

- Learners sometimes fail to describe the rule used in writing a particular pattern (S11).
- Learners cannot extent number patterns with negative numbers or fractions (S21).

These difficulties do not explain what learners do wrong when they deal with patterns. On the other hand, the teachers did not indicate what made it difficult for them as teachers to explain patterns to learners such that the learners could understand. Again, it suggests that the teachers did not study the learners' answers to discover their misunderstanding. Therefore, it implies that the teachers did not have knowledge of the difficulties that they encountered when they taught patterns.

In the category on graphs, the following difficulties were identified.

- Learners have a problem of drawing linear graphs. In this case they do not know which one is x-intercept or y-intercept and also to write the function in standard form. E. g. $3x - 4 = 2y \rightarrow 2y = -3x + 4 \rightarrow y = -\frac{3}{2}x + 2$ (T4).
- How to draw the graphs if you do not have coordinates (T12).
- Learners fail to plot points on grid and numbering the axis correctly e. g. x-axis and y-axis (S11).

These difficulties indicate that the learners cannot draw graphs when they are given equations. They also indicate that the learners confuse coordinates. On the other hand, the teachers did not indicate what difficulties they have as teachers when they teach graphs. This might also be the result of lack of reflection of the lessons taught. Therefore, it can be concluded that these teachers had no knowledge of the difficulties they had as teachers when they taught graphs.

In the category, numbers, the difficulties as identified by the teachers are indicated below.

- Working with fractions (T8).
- Learners have problems in addition and subtraction of integers, e. g. $10 - 20 = 10$ (T4).
- Cannot subtract a big number from a small one (S10).
- Fractions, decimals, ratios, percentages, integers, expanded notations, division of whole numbers, place values, (R22).
- Group numbers (R23).
- Convetion of units (R11).

All these difficulties listed are arithmetic difficulties. This suggests that these teachers did not know the difference between arithmetic curriculum and algebra curriculum as numbers are taught in arithmetic. However, the teachers did not explain what the learners do wrong to show their misunderstanding when working with numbers. For example, they should explain what learners do wrong in grouping numbers.

In category on resources, the teachers indicated the following:

- Adding and subtracting without the aid of a calculator (T2).
- There are no more resources or materials that can be used which is touchable (T13).

However, in the senior phase, learners use a lot of mental calculations. Most of the problems that learners do in this phase do not need calculator use. Therefore, if the learners cannot add correctly, it implies that they have not mastered how to add when in lower grades. Then teachers should develop other strategies to help learners overcome their problem. Also, the teacher did not explain how lack of resources disadvantages the teaching of algebra. Further, there is no mention of examples of touchable resources and how their unavailability impacted negatively on their teaching of algebra.

In the category on teaching strategies, different groups of difficulties were identified. The first group of difficulties about teaching methodology is listed below.

- Some need special attention (R26).
- Not all learners understand the topic immediately (R26).
- They are not able to understand it easier (S5).
- Learners don't grasp the terms which are used and calculations sometimes are difficult (S19).
- To teach learner steps to be followed (R25).
- How to present it (R18).
- Teaching methodology to introduce a lesson to the learners in such a way that all learners will understand (T13).
- The teaching approach to the learners is also difficult (T13).
- I am not emphasising most of the things in words so that learners should understand. (S8).
- Failing to find out problems that lead to learners' misunderstanding and not knowing how to help them understand what I am teaching (S15).
- To make learners understand that some of the unknown numbers can become known (R24).
- When I have two equations to solve and teach learners how to solve it (R16).
- Explaining to learners about the variable standing for a particular number or value (S1).
- I did not do lesson preparations correctly and in time (R21).

This shows that 20% of the teachers in this category had methodological problems. Almost all the statements show the difficulties that the teachers face to make learners understand what is being taught. They indicate lack of effective ways of teaching algebra. These statements clearly show the teachers' inability to develop different strategies to help learners understand the algebraic concepts. As a result, these teachers felt there was poor learning in their classes.

The next group of teaching strategies reflect the teachers' inadequate subject matter knowledge. The responses of the teachers are listed below.

- Sometimes it is difficult for me to understand the problem, how can I teach learners things which I do not understand (R25).
- I did not have knowledge of mathematics as it is the first time I teach maths in GR 7 (R21).
- I am struggling in working with algebraic equations and expressions. How to relate the equations and expressions on daily life experiences (R6).
- The book I use do not have the algebraic term like coefficient, I knew that there is a variable and constant and expression but coefficient is for the first time (R15).
- I did not take mathematical rules into consideration (R21).
- Some of the equations are difficult. You don't find the values of x and y (T12).
- I had poor usage of mathematical teaching AIDS e.g. fractions, charts and number chords were not used (R21).

All these statements show that these teachers had inadequate knowledge of algebra that should be taught in the senior phase. Most of the difficulties were raised by R21. This teacher's main difficulty was teaching mathematics for the first time. This shows that the teachers were not competent in teaching algebra. This is clearly stressed by R25 who indicates that he/she cannot teach something he/she does not understand. This suggests that the teacher skips some of the topics that he/she does not understand. The statement given by R15 also reveals that some teachers only teach what is in the textbooks that they use. They do not use different books to supplement the books they use. Also, they do not follow the curriculum stated. As a result, it can be concluded that these teachers had inadequate PCK of algebra curriculum in the senior phase.

In this general category, the teachers' difficulties are listed below.

- Many learners have difficulties in applying it to daily life situations (T1).
- Learners fail to integrate most of the things (T13).
- Learners are very slow in thinking and they tend to guess the answer (R24).
- Learners did not have knowledge of mathematics. I will find when I ask questions during the lesson (R21).
- Running after the syllabi/curriculum and letting learners behind not recapping/remedial work (R7).
- Overcrowding in the class makes it hard to teach algebra, lack of teaching aids, time duration (S18).
- LOLT is a barrier, this is caused by that not always mathematics concepts can be explained in mother tongue (R8).
- The language barrier is a problem. I spend more time in the language of mathematics and still learners cannot make sense of it (R24).

Considering the first two difficulties, the teachers did not explain how they assessed that learners could not apply algebra in their lives or how learners could not integrate algebra. Besides, it is not clear how the difficulties made it hard for teachers to teach algebra. This suggests that the teachers might have guessed the answers. This is due to the lack of connection between these learners' difficulties and the teaching of algebra. Also, how the learners' guessing of answers affects the teaching is not indicated. The last statements show some of the contextual factors that influence the teaching of algebra. The scope of work that needs to be covered at a specific time makes teachers to focus on work coverage and not on teaching for understanding. Also, overcrowding in classes in most cases denies learners an opportunity for special attention. Further, algebra is about solving practical problems. As a result, language of instruction sometimes is a barrier in interpreting the word problems as it has already been proven in some of the word problems given earlier.

In the last category, irrelevant responses, teachers indicated the following difficulties:

- When coming to probability I understand the concept by just don't know how to impact the knowledge to my learners that topic is more practical and I fill I will fail to make them understand (S3).

- Learners are sometimes unable to find mean, median, mode and range. Instead of rearranging the terms they will just give the answers (S11).

It can be seen from these difficulties that these teachers did not have knowledge of algebra curriculum that is taught in the senior phase. It shows that they think everything that is taught in mathematics is part of algebra. As a result, it can be concluded that these teachers had inadequate knowledge of algebra curriculum in the senior phase.

In summarising this section, it is shown that almost all the difficulties identified were about learners, with the exception of the difficulties mentioned in teaching strategies. Further, the identified difficulties do not specify or describe the learners' difficulties. In addition, most of the difficulties identified here are the same as the difficulties identified in Table 4.6. This suggests that these teachers had no knowledge of analysis of learners' answers to detect the realised difficulties. Most importantly, the teachers could not identify the difficulties that they had as teachers when they taught algebra. Their responses suggest that these teachers did not plan for their lessons, and as such, they could not reflect on the difficulties that they had as teachers when they taught algebra. This shows that the teachers had inadequate knowledge of teaching difficulties.

In Question 9, teachers were asked whether it was important to teach algebra, and to give reasons for their answers.

The reasons given for the importance of algebra are summarised in Table 4.9 below. The reasons were categorised according to the themes given. Then the reasons were categorised into application, thinking and reasoning skills, content-based reasons and experience.

Table 4.9

Reasons Why Algebra Needs to Be Taught (N = 61)

	Responses			No responses		
Reasons for teaching algebra	Codes	f	%	Codes	f	%
Application	T1; T2; T3; T4; T5; T8; T13; S3, S4; S5; S6; S8; S9, S10; S11; S12; S13; S14; S15; S16; S17; S19; S20; S21; R2; R4; R5; R13; R14; R20; R25	31	51	S2; S22; R1; R9; R17; R18; R19	7	11
Thinking and reasoning skills	T10; T11; S1; S18; R10; R15; R20; R23; R24; R26	10	16			
Content-based reasons	T6; T9; T12; T13; S1; S7; S14; R3; R6; R8; R16; R21; R22; R23; R24	15	25			
Experience	T7; T11	2	3			

The table shows that 11% of the teachers did not respond to this question, and the majority of those who did not respond to this question taught in Grade 7. However, all the teachers who answered this question said 'yes' to the first part of the question. Then almost all of them gave reasons which were summarised as indicated in Table 4.9, with the exception of R7, R11 and R12 who did not give reasons to the question. The table also shows that most teachers gave reasons which were classified in the application category. It also shows that the most number of teachers who gave reasons in the application category, taught in Grade 9. Further, the reasons given by teachers who taught in Grade 7 were spread almost equally in the first three categories. Further, the reasons in the last category, experience, were identified by only two teachers who both taught in Grade 8. Also, only 10% (T11, T13, S14, R20, R23, and R24) of the teachers gave more than one reason which fell in different categories.

It is shown in the table that 51% of the teachers gave reasons in the first category, application. The reasons given are stated below.

- Yes. It makes mathematics to be interesting as it can be applied in a daily life situation; Makes maths meaningful (T1 and S5).

- Yes, in every day for real life situation they live mathematics (T13, S19, R2, R4 and R5).
- Yes, because it helps us to solve the problems in our everyday life because in algebra we solve the unknown (R13).
- It is also important because almost everything you do in daily life involves algebra (T4, S8, S9, S14 and S22).
- Yes, people or learners should familiarise themselves with operation because they (learners) use them daily. It also helps them to count (S17).
- Yes, we use algebra in everyday life like when shopping, when you want to calculate the distance from Tzaneen to Nkowankowa (S11).
- It helps you to solve lots of problems in a real life situation, e.g. adding and subtracting. In real life you can use it in finance etc, when you are doing models you can use algebra (T2).
- Yes, learners or everybody can know how to solve problems in real life, e.g. when you want to divide/share something to people. You have to use division sign (T5 and S12).
- Yes, it is because it assists the learner to solve problems (S6, S13, S14, S15, S16, R14, R20 and R25).
- Yes, I think it is important so that learners know how to simplify things, how to share, how to calculate money and money more (S3 and S4).
- Yes, in everyday life it assists in financial e.g. To check profit, loss & break even points: to interpret graphs & understand what they mean (T3).
- Yes, because it is a basic to financial (T8).
- Yes, when tiling, painting or doing some paving, you need to calculate areas, perimeter, find cost/area etc., divide some articles using ratio (S10).

These reasons indicate that algebra can be applied in daily lives. Some of the given examples for application in daily life situations under this category are: “adding and subtracting”, (T2), and “checking profit and loss”, (T3). These examples do not say much because one can add or check profit without the knowledge of algebra. However, application of algebra is visible when formulae are used for calculation. The use of formulae makes calculations easier than having to do a lot of calculations and making errors in the process. As indicated earlier on, a lot of formulae are used in finance, when finding the area, perimeter, etc. Also, interpretation of graphs is another example given in this category. This skill is learnt in algebra. However, more than 50% of these teachers in this category did not indicate where or how algebra is applicable in real life. For example, T13 indicated that in everyday life situation “they live mathematics”, but that does not explain how. This suggests that most of the teachers only knew that algebra is applicable in daily lives but did not know where or how it is applicable.

In the second category, which is about thinking skills, the following reasons were identified:

- Yes, it helps in reasoning skills (T10, S1 and R17).
- It helps learners to think critically when coming to sharing (T11, S1 and R17).
- It brightens the mind of learners at the real life (S18).
- Yes, it makes learners to be constructive. They become critical thinkers (R10).
- Yes, it promotes reasoning capacity (R15).
- Challenges brainstorming (R17).
- The use of variable is exciting and challenging and stimulate critical thinking (R23).
- Yes, thinking and reasoning power can be developed, (R24).
- Yes, it is another part discipline of mathematics that contribute to develop the cognitive thinking of individual child/learner (R26).

All of these reasons in this category indicate that algebra helps in thinking and reasoning skills. However, how the skills are developed is not indicated. This suggests that these teachers did not know how the skills are developed in algebra.

The third category was mainly about content knowledge. The reasons classified in this category are indicated below.

- I think skills learned can be implore in other branches of maths (R23).
- Yes, because it is the section where learners should score high marks if they understand the section (T4).
- Yes, because learner must know different topics including algebra, they must get enough knowledge from the teacher about algebra and practice very well; they must be able to solve different equations (T6).
- Yes. It teaches children about like terms and unlike terms (S7).
- Yes, for the learner to understand how to get the unknown number from the equation (T9).
- Yes, because we find many resolutions on that algebra; How to solve equations and to factorize thats how algebra is all about (T12 and R16).
- To equalize things they need to know how to equate the equation (T13)
- Allow following rules and principles e.g. BODMAS, ASSOCIATIVE, COMMUTATIVE, etc (S1 and S14).
- Yes, learner are able to solve problems using different methods (R3).
- Learners learn how to substitute numbers with letters. Learners should know how to do expressions and equations. They are able to represent equations on flow diagrams and they must refer them to everyday life (R6).
- Yes, so that learners can be in a position to use operational signs (R8).
- So that learners can use letters instead of place holders (R8).
- Algebra is important because it gives one background of numbers and how to use numbers in fractions intiger etc. (R21).

- Algebra is a key to other mathematical concepts as it will enable learners to be able to solve problems in maths using the knowledge or numbers gained in algebra (R21).
- Yes, it is important to teach algebra because learners to learn more on calculations, finance, counting. Learners learn how to use or operate a calculator (R22).
- Yes, because solving the unknown is useful (R23).
- Learners can learn to justify the solutions after making some conjectures (R24).
- To be able to analyse a given data/or be able to represent it (R20).

The first reason shows that R23 is aware of the importance of algebra in some other parts of mathematics. On the other hand, all the other reasons are about the ideas that are taught in algebra or in mathematics. These reasons are irrelevant to the question because they do not explain why algebra should be taught. For instance, “it teaches children about like terms and unlike terms” (S7). It was not clarified how this knowledge would help learners in the long run. This suggests that these teachers did not have knowledge about how important algebra is in their lives.

In the fourth category, experience, the teachers stated the reasons why algebra should be taught as follows:

- Yes, while teaching am gaining more experience (T7).
- Yes. Because it makes learners to practice more often (T11).

These two reasons do not show why algebra should be taught. Besides, teachers gain experience from teaching any subject. Also, practising more often does not imply that the subject should be taught. Therefore, the two reasons provided in this section suggest that these teachers did not know the importance of algebra.

To summarise, algebra is the most important branch of mathematics. Algebra is used in some branches of mathematics like calculus, geometry, etc. Also, there are subjects like physics and accounting, which use algebra in their calculations. Furthermore, all formulas used daily in finance, when calculating area and perimeter, etc., are the result of application of algebra. However, most teachers could not give the reasons why algebra should be taught. This suggests that most of these teachers did not have knowledge of why algebra is important in their lives.

4.2.2 Analysis of data from CoRe

As indicated earlier, teachers completed CoRe matrices in groups according to the grades that they taught. Therefore, there are different CoRes for Grade 7, for Grade 8, as well as for Grade 9. However, the Grade 9 teachers were divided into two groups, and therefore completed two different matrices.

Table 4.10 below shows the CoRe which was completed by the Grade 7 teachers. Then, CoRes for grades 8 and the two Grade 9 groups are presented in Appendix H, Appendix I and Appendix J respectively. Each table categorises the teachers' responses according to the eight questions of the CoRe and also according to the big ideas that were identified.

Table 4.10

Group 1 CoRe on Senior Phase Algebra

Grade 7	**IMPORTANT ALGEBRAIC IDEAS/CONCEPTS** Big Idea 1 Algebric expressions	Big Idea 2 Algebric equations	Big Idea 3 Graphs
What you intend the learners to learn about this idea.	-Recognise and interprete rules and relation represented in symbolic form. -Identify variables and constant. -Translation of symbol statements into algebric expression. -Usage of algebraic terminology.	-To analyse and interprete number sentences. -To unable to solve and complete number sentences.	-Interpretation of graphs. -Analysing, interpreting and drawing of graphs on linear or non-linear and on constant, increasing and decreasing.
Why it is important for learners to learn this.	-Interprete rules or relation represented in symbolic form. -Identify variables and constants in formulae. -Formulate algebraic expression. -Mathematical language saves time its economical. -Solving mathematical problems.	-Balance of things in life. -They can simplify expression easily. -Budget. -Solving of situation fairly.	-Ability to inteprete and analyse information on a graph. -Ability to information by drawing graphs. -Saves time and space.
What else you know about the idea (that you do not intend learners to know yet.	-Identifying and classify conventions for writing algebraic expressions. -Expand and simplify algebraic expressions e.g. grouping of like and unlike terms. -To recognise and identify coefficients and exponents in algebraic expressions.	-Extend solving equations to include: using additive and multiplicative inverses and using laws of exponents. -Simultaneous equations. -Use substitution in equations to generate tables of ordered pairs.	-Use tables or ordered pairs to plot points and draw graphs on the Cartesian plane.
Difficulties connected with teaching this idea.	-Learners are unable to translate word sentences into number sentences. -Misunderstanding the terms	-Application of additive and multiplicative inverses in solving problems, -inability to switch form one	-Lack of drawing skills.

	Big Idea 1 Algebric expressions	Big Idea 2 Algebric equations	Big Idea 3 Graphs
	(terminology) e.g. adding and increasing.	property to the other. -Unable to analyse and Interpret number sentences that describe the situation	
Knowledge about learners' thinking which influences your teaching of this idea	-Transition from the usage of □ to variables. -They cannot relate notations with real life.	-Balancing of objects on a scale. -Solving for x/unknown is complicated.	-They think of bar graphs and pie charts.
Other factors which influence your teaching this idea	-LOLT -Overcrowded classes -More work in a short space of time. -Arrangement of topics	APPLY TO ALL	APPLY TO ALL
Teaching procedures (and particular reasons for using these to engage with this idea)	-The display of chart with symbolic information. -Introduce variables, constant and coefficient so that learners will be able to use mathematical language in writing word problems as number sentences. -Learners will identify those terms in an expression.	-Start by the prior knowledge of □ before introducing variables. -Teach learners how to solve for the unknown using additive and multiplicative inverses of numbers.	-Introduce lines of the graphs to learners i.e. horizontal lines $-x$-axis vertical lines $-y-$ axis. -Introduce dependent and independent variables showing them that independent variables are written on the x $-$axis and dependent on the y $-$axis. -Show them again the different types of graphs: how to identify whether the graph is linear, non- linear or constant. -Draw graph for the learners to interpret, teach them how to interprete and how to draw graphs.
Specific ways of ascertaining students' understanding or confusion around this idea	-Continuous assessment (classwork, homework, tests, assignment, classroom activities orally) and give feedback. -Sharing of different techniques of solving problems by learners.	APPLY TO ALL	APPLY TO ALL

The responses of the teachers are analysed per question according to the big ideas/concepts that the teachers identified in the grade that they taught. For that reason, each group of teachers had to start by identifying the main ideas/concepts that are taught in algebra in the particular grade that they taught.

With regard to the *big or main ideas/concepts* that the teachers had to identify, the Grade 7 and Grade 8 groups identified the same three main concepts, which are algebraic expressions, algebraic equations and graphs. Also, both groups of Grade 9 identified algebraic expressions and algebraic equations as big ideas, but none of these groups regarded graphs as part of algebra. In addition, the first group of Grade 9 teachers also identified exponents and functions and the second group identified factorisation. It implies the first group of Grade 9 did not regard exponents as expressions. Also, the second group of Grade 9 regarded factorisation as concept separate from expressions. Further, the second group of Grade 9, Grade 7 and Grade 8 teachers did not see functions as part of algebra. Furthermore, none of the groups identified patterns as the main concept that is taught in senior phase algebra. This suggests that all these teachers in their different grade groups had inadequate knowledge of the main concepts that are taught in senior phase algebra. This follows from none of the groups identifying all the main concepts that are taught in the grades that they taught.

> Then, the first question of the CoRe that was answered was: *What you intend learners to learn about this idea.*

The Grade 7 teachers were able to specify all what the learners should learn about algebraic expressions and graphs in Grade 7 according to CAPS document. For example, they indicated that the learners should learn to recognise and interpret rules and relation represented in symbolic form. However, they did not indicate that the learners should solve equations, should determine numerical values of expressions, or identify parts of the given equation.

The Grade 8 teachers were able to specify some of the concepts that they wanted learners to learn about the main concepts. For example, they indicated that learners should learn to solve equations, analyse and interpret equations, analyse and interpret graphs, add and subtract like and unlike terms. However, ideas like expansion and simplification of expressions, evaluation of expressions, determining square and cube roots of algebraic terms were not identified.

The first group of Grade 9 teachers only indicated that they wanted learners to simplify and expand expressions, but said nothing about factorisation. Also, the group did not indicate that learners should learn to interpret graphs. On the other hand, the second group indicated that they wanted learners to simplify expressions by factorisation. This indicates that these teachers did not understand the difference between factorisation and simplification.

The results show that in all these grades, the teachers did not identify all the ideas falling under the main ideas that they identified. That is, they did not specify all what the learners need to learn about the main concepts in their particular grades. It implies that these teachers did not have adequate knowledge of the curriculum coverage of the concepts that they identified in the grades that they taught.

> The second question answered in the CoRe was: Why it is important for the learners to learn this?

Most of the reasons given by the Grade 7 teachers do not show that the teachers had knowledge about the importance of the concepts they identified. They indicated what the learners needed to learn about the concepts instead of showing where or how the learners can apply the ideas. For instance, they indicated the following,

- Identify variables and constants in formulae, formulate algebraic expression.

- They can simplify expressions quickly.
- Graphs saves time and space.

Almost all the above reasons given are content based and they did not mention how the ideas are applicable. However, some of the reasons show that the ideas can be applied but the teachers did not explain how they are applicable in the reasons given. For example, they indicated:

- Budget.
- Solving of situation fairly.
- Ability to interprete and analyse information on a graph.

It is not indicated how equations are applicable in budget. Also, it is not clear how algebra is used to solve situation fairly. While it is a known knowledge that algebra can be used to solve practical situations, the impression given here is that the situation can be solved, but if algebra is not used, the situation cannot be solved fairly. How this fairness comes to play is not explained. On the other hand, the teachers indicated that learners would be able to interpret and analyse information on graphs.

Some of the reasons supplied by Grade 8 teachers show the importance of algebra while others do not. For example, they were able to indicate that algebra develops "learners' thinking and reasoning skills", but they did not clarify how. On the other hand, they indicated that: "Learners must be able to label the heading of the graph", which does not indicate how this knowledge will help the learners in the long run.

The reasons given by the Grade 9 teachers show the importance of algebra while others do not. For instance, they indicated that the learners would be able to simplify expressions, to group like and unlike terms, to solve the unknown to be known or plotting points on the Cartesian plane. It was not explained how and where this knowledge is applicable. However, they indicated that knowledge of equation is applicable in real life, but did not explain further.

As mentioned earlier, algebra is the most important branch of mathematics. As a result, teachers of algebra are expected to know where algebra is applied in real life situations. For example, they should know that algebra is used in some branches of mathematics like calculus, geometry, etc. Also, they should have identified of other subjects like physics and accounting that use algebra. Furthermore, they should know that all formulas that are used daily in finance, when calculating area and perimeter, etc., are the result of the application of algebra.

Most of these reasons show that these teachers did not have sufficient knowledge of how or where the identified concepts can be applied. Also, even in cases where they indicated that algebra is applicable in real life, they did not explain how or where it can be applied. Therefore, it can be concluded that these teachers had insufficient knowledge of the importance of the ideas that they identified.

> Then, the third question stated, what else you might know about this idea (that you don't intend learners to know yet)?

When answering this question, the Grade 7 teachers were able to identify some of the topics that the learners were not taught in Grade 7, but were to be dealt with in the next grades. For example, they indicated that learners would be taught to identify and classify conventions for writing algebraic expressions, simultaneous equations, use tables or ordered pairs to plot points and draw graphs on the Cartesian plane.

The Grade 8 teachers also identified some of the concepts about the main concepts that learners would do in the next grades. For example, they indicated that learners would factorise, draw linear graphs given equations and would determine the equations from the given linear graphs.

The Grade 9 teachers were able to identify some of the ideas about the big ideas that the learners were not supposed to do in that particular grade, but were to do in the next grades. For example, they indicated that learners were not supposed to do simultaneous equations, multiply binomials and polynomials or solve third degree equations.

The evidence shows that all the teachers in the different grades were able to identify the topics that the learners were not supposed to do in the grades they were in. This suggests that the teachers were able to identify the ideas about the main ideas that learners would do in the next grades.

> The fourth question was to identify difficulties/ limitations connected with teaching this idea.

The Grade 7 teachers indicated that "learners are unable to translate word sentences into number sentences, misunderstanding the terms (terminology) e.g. adding and increasing, application of additive, multiplicative inverses in solving problems and lack of drawing skills", but the teachers did not clarify what learners do to show that they have difficulties. Further, examples were not given to clarify the situation. While this is the case, the teachers did not explain the difficulties that they encountered when they taught the identified ideas.

Difficulties that were identified by Grade 8 teachers were mostly contextual. For instance, "Lack of resources, using free hand when drawing graphs, instead of using T- square". These teachers did not indicate the difficulties that learners encountered when they learnt the identified ideas. Also they did not mention the difficulties that they encountered connected to the teaching of the identified ideas.

The Grade 9 teachers indicated the difficulties that learners have when learning algebra. For example, they indicated that "learners sometimes forget to apply laws and rules used in exponents, learners tends to consider or write expression as equations, or mix simplifying expression and solving equations”. In these cases the teachers did not explain or give example of what learners did to show these difficulties. On the other hand, the teachers identified “overcrowding, not having resources to explain the terminology, strategies of making teaching equations lasts”. The teachers did not explain how these factors interfere with the teaching of algebra.

In summary, the teachers did not explain what the learners did to show that they had difficulties. Also examples were not supplied to illustrate these difficulties. This suggests that the teachers had insufficient knowledge of the difficulties that are connected with the teaching of the identified concepts.

> The fifth question that was answered stated: Knowledge about learners’ thinking that influences your teaching this idea.

The Grade 7 teachers indicated “the transitions from using placeholders to using variables, learners have problems relating notations with real life and learners have difficulties solving for unknown”. However, the teachers did not explain how these factors influenced the teaching of the identified concepts. Also, they indicated “balancing of objects on a scale” and that “the learners confuse bar graphs with pie charts”. These ideas however, are not taught in senior phase algebra. These responses show the teachers’ inadequate knowledge of learners’ thinking.

The Grade 8 teachers indicated only the concepts that are difficult for learners to understand as the thinking that influences the participants’ teaching. For example, they mentioned “Lack of understanding on mathematical concepts, e.g. root, exponent” and that “learners cannot differentiate dependent and independent variables”. No clarity was provided on how the given information influenced their teaching.

The Grade 9 teachers also identified some of the learners' difficult sections that they teach as the thinking that influenced their teaching. For example, difficulties in factorising or grouping like terms, but they did not indicate how their teaching changed because of these difficulties. They also indicated that some learners who have knowledge of some concepts may be bored. Again, how these learners changed their teaching was not indicated.

These responses suggest that the teachers did not have any recall of how their teaching was influenced when they taught the identified concepts. This follows from the teachers' inability to explain how their teaching changed as a result of the learners' thinking.

> In the sixth question, the teachers had to state other factors that influence their teaching of the identified ideas.

The Grade 7 teachers indicated contextual factors as the other factors that influenced their teaching. For example, they mentioned overcrowded classes, lack of resources and the schedule. However, they did not explain how their teaching was influenced by these factors.

Similarly, the Grade 8 teachers indicated that lack of resources is one of the factors that influenced their teaching. Other factors that influenced their teaching included the topics that learners found hard to understand. For instance, the application of additive inverses when solving equations, learners found it difficult. However, there is no clarity on how the given factors influenced their teaching.

Also, the Grade 9 teachers indicated that the contextual factors influenced their teaching. One group specified lack of resources and overcrowding, but the other group did not specify. The group that did not specify included the behaviour of learners and, gifted learners and time takers as the factors that influenced their teaching. However, they did not specify how these influenced their teaching.

Considering these other factors that the teachers indicated as those that influenced their teaching, it can be concluded that the teachers were able to identify them. However, failure to explain how those factors influenced their teaching, suggest that the teachers did not know how their teaching was influenced when they taught the identified ideas.

> The seventh CoRe question was to determine the teaching procedures (and particular reasons for using these to engage with this idea)

The Grade 7 teachers indicated the steps they followed when they taught expressions, equations and graphs. For instance, they indicated that they displayed chart with symbolic information when they introduced expressions. However, they did not explain what information is symbolic. Then they introduced variables, constants and coefficients. Also, they indicated that when they introduced variables, they first link them with the placeholders. Further, when they introduced graphs, they started with the axes. Regarding all the steps they followed when teaching the identified ideas, they did not give the reasons why they followed those steps.

Also, the Grade 8 teachers indicated the steps that they followed when they taught the identified ideas. However, the steps they followed were not specific. For example, they only indicated introduction of the topic, without specifying how they introduced the topic. As a result, they were unable to give reasons why they followed the steps they specified.

For Grade 9 teachers, one group indicated the methods they used during the lesson. For example, they indicated class discussions, demonstrations group discussions, etc. They did not specify any of the procedures they followed when teaching the identified ideas. Another group just indicated that they linked the new topic with their previous knowledge. They also indicated that they gave examples then they let the learners do their own problems following the examples. They also, did not specify a particular procedure.

The Grade 7 teachers specified the steps they followed when teaching the identified topics although they did not give the reasons for following those procedures. On the other hand, the Grade 8 and Grade 9 teachers were not specific about their procedure. This suggests that these did not follow any particular procedures when teaching the identified concepts, hence they could not give reasons why they taught the way they did.

> In the eighth question, teachers had to identify specific ways of ascertaining learners' understanding or confusion around the ideas.

The Grade 7 teachers indicated that they gave learners continuous assessment to ascertain that they understand or misunderstand the concept. They mentioned classwork, homework, tests, assignment and classroom activities as the form of assessment they used. They also indicated that learners share different techniques of solving problems. In this case, it is not clear how the teachers assessed learners' when they discussed. In case of Grade 8, the teachers, mentioned "the correct use of mathematical language when teaching", "contextualising problems" and "encouraging logical and critical thinking", and "being realistic" as the ways they used to check learners' understanding. However, they did not explain how they assess whether the learners have understood what they were taught or not. Then, in case of Grade 9 teachers, they indicated homework, classwork, tests, assignments, projects, investigations, group activities as ways of ascertaining that the learners understood or not.

It can be seen that the Grade 7 and the Grade 9 were able to specify the assessment methods that they used when teaching. On the other hand, the Grade 8 teacher mentioned assessment as their teaching procedure.

4.3 SUMMARY

The findings revealed that most teachers could not identify the main concepts that are taught in algebra in the grades that they taught when they wrote the test as individuals. Then, out of those who identified variables, expression, equations, graphs and functions as the main concepts, most were Grade 7 teachers. Only 8% could identify expressions, 7% identified equations, only one identified patterns, 5% identified graphs and also 5% identified functions. Only two teachers could identify at least three of the main concepts. The rest identified sub- concepts as the main concepts. In addition, most of these teachers identified only one sub-concepts as the main concepts that is taught in algebra. Also, when identifying the concepts as grade groups, none of the groups could identify all the main ideas that they taught even when they referred from CAPS documents. However, they were able to indicate some of the concepts about the identified main ideas that they wanted learners to learn. While that is the case, the results of the test also revealed that most of the teachers could not identify or define all the parts of an equation. Only two of the teachers were able to identify and define the parts of the given equation. Also, only three of the teachers could represent the given word problem in at least two different ways. This shows that most teachers had inadequate content knowledge of algebra in the grades that they taught. That is, they lacked the relevant knowledge of the ideas and what needs to be taught about the ideas that are taught in their grades.

The results also revealed that most participants did not know the importance of algebra. However, a few of them were able to indicate that algebra is applicable in everyday life but failed to explain how or where it is applicable.

The knowledge of learners' difficulties is a very important factor in the development of PCK. Some of the teachers were able to identify learners' difficulties in the test. However, most of the difficulties identified did not correspond with the examples given. Most of the answers given as difficulties did not explain what those difficulties were. In addition, in most cases, the examples given showed the correct way of solving the problems given, but not the learners' errors. Also, in the compiled CoRes, the answers given as difficulties did not explain the difficulties. Again, when teachers were given statements made by the learners to identify their difficulties, most of the teachers could not identify those difficulties in the given statements. Further, they could not specify the difficulties that they, as teachers, experienced when teaching the ideas they identified. Consequently, the teachers displayed inadequate knowledge of the learner' difficulties.

Teachers' observation and analysis of learners' responses when teaching certain topics usually make teachers change their ways of teaching those topics to meet the learners' needs. However, the teachers in this study were unable to specify any of the learners' thinking that influenced their teaching. They only indicated topics that were difficult for learners to understand as factors that influenced their teaching, but did not explain how their teaching was influenced by those difficulties. They also indicated that lack of resources influenced their teaching, although they did not explain how.

It is indicated that teachers with developed PCK follow certain procedures when teaching certain topics and they have reasons for following those procedures. On the contrary, the results revealed that the Grade 8 and Grade 9 teachers in the study did not show any specific ways they followed when teaching the algebra ideas that they identified. However, the Grade 7 group indicated their procedure, but could not give the reasons why they followed those procedures. Hence, it can be concluded that the teachers in this study showed inadequate PCK of algebra.

4.4 CONCLUSION

The chapter described findings from the test that the teachers wrote as individuals and the CoRes that the teachers compiled in groups. The findings were based on the test that teachers wrote as individuals and the CoRe matrices that they compiled in grade groups. The findings were based on identifying the main concepts that are taught in algebra in the grades that the teachers taught, what they wanted learners to learn about the identified concepts, the importance of learning algebra, the knowledge of learners' difficulties, teaching procedures, the factors that influenced their teaching and the ways of ensuring that the learners understood what was taught. The next chapter presents the summary, recommendations and conclusion drawn from this study.

CHAPTER 5: SUMMARY, RECOMMENDATION AND CONCLUSION

5.1 INTRODUCTION

This chapter aims to summarise, recommend and make a conclusion about the findings of the study that explores the algebra pedagogical content knowledge (PCK) of senior phase teachers in Limpopo. The findings on the teachers' PCK are summarised and interpreted using the CoRe, that is, the main concepts that are taught in senior phase algebra, what the teachers intend learners to learn about the concepts, why it is important to learn algebra, difficulties connected with the teaching of algebra, factors that influence the teaching of algebra, teaching procedures and specific ways of ascertaining learners' understanding of the concepts.

5.2 SUMMARY AND INTERPRETATION OF THE RESEARCH FINDINGS

- According to the CoRe, the first thing the teachers needed to do was to identify the big ideas (main concepts) that are taught in their respective grades.

The reported study revealed that most of the teachers could not identify variables, expression, patterns, equations, graphs and functions as the main ideas that are taught in algebra when they were writing a test as individuals. They could only identify concepts that could be learnt about the main concepts as the main ideas/concepts. Only three of the teachers were able to identify at least two main concepts. Also, the teachers could not identify all the concepts that are taught in the grades that they taught when they completed the CoRe matrices as grade groups. It was also found that none of the grade groups identified patterns as one of the main concepts taught in algebra. Furthermore, the results revealed that some teachers could not differentiate between algebra and arithmetic. This follows from the teachers identifying operations and numbers as the main concepts that are taught in algebra. This indicates that the teachers had insufficient knowledge of senior phase algebra curriculum.

- The next question of the CoRe was for the teachers to specify what they intended learners to learn about the ideas.

When the participants were compiling the CoRes, they could not identify all what learners needed to learn about the main concepts in the grades that they taught. For example, *evaluation of expressions, determining square and cube roots of algebraic terms* were not mentioned in Grade 8. Contrary to the expectations, the results revealed that the participants had a limited knowledge about variables. This follows from failure to link placeholders and letters used as variables, and also the inability to explain why different variables can be used in the same equation. In addition, most of the participants were unable to identify and to define all the parts of an equation. Furthermore, almost all the participants could not express a given word problem using different representations. This indicates that the participants had inadequate knowledge of algebra subject matter. As a result, the knowledge that they needed to impart to learners was inadequate.

- The next CoRe question was to explain why it is important to learn algebra/the identified main ideas.

Most participants indicated that algebra is applicable in our daily lives but did not explain further. Although some indicated that it is used when doing budge, but how exactly, it is not indicated. Also a few indicated that algebra helps in critical thinking, but did not explain how. Most of the reasons given were based on what needs to be taught in algebra. For example, some teachers indicated that algebra is important because learners are able to use variables. That is, the teachers could not link algebra to some other parts of mathematics like calculus, trigonometry, and geometry. Also, they could not link it to subjects like physics and accounting where formulae are used when solving problems. Furthermore, the participants did not realise that formulas that are used in all areas of life are the result of application algebra. These formulas include compound interest formula in finance, formula to calculate force in science, formulas for calculating area and perimeter. The important issue that is revealed by these findings is that most of these teachers could not identify areas where algebra is applicable. Hence, they could not specify the importance of algebra.

- The teachers had to specify what they knew about the identified ideas, that they did not intend the learners to know in that particular grade.

All the grade groups were able to indicate some of the topics about the identified ideas that learners would learn in the next grades.

- The teachers had to identify the difficulties associated with the teaching of the ideas/concepts of algebra.

When identifying difficulties/ limitations connected with the teaching of the algebraic concepts, most were not specific, i.e. most could not describe the problems that learners had in the examples that they gave. In some cases, the explanation of the problems did not tally with the examples given. In most cases the examples given did not show any difficulties. The problems were solved correctly. Furthermore, when asked to identify the learners' difficulties in the statements given by the learners, none of the teachers was able to detect all the difficulties. When compiling the CoRe, they could only identify contextual factors as difficulties connected with the teaching of the identified ideas, but could not identify the difficulties that they, as teachers had.

There are several possible explanations to these results. First, inadequate knowledge of subject matter is a contributory factor to the inability to detect the learners' difficulties. Second, the results suggest that the teachers did not give themselves time to analyse learners' errors, hence they were unable to describe them. The results imply that the teachers did not prepare for the lessons that they taught, as a result, they could not reflect on them. As it has been revealed in the literature, reflection plays a major role in the planning of lessons (Shulman, 1987). Therefore, failure to plan will result in failure to develop new comprehension and reflection which will help in reminding about successes or limitation of the previous lessons (Shulman, 1987). Therefore, it can be concluded that these teachers had inadequate knowledge of the learners' difficulties.

- The next question was the knowledge of learners' thinking that influenced their teaching.

It is possible that the reasons that made it not easy for teachers to identify learners' difficulties, might be the same reasons that made it difficult for them to identify knowledge about the learners' thinking that influences their teaching the idea. This follows from the teachers indicating only topics that are difficult for learners to understand as the thinking that influenced their teaching. For example, the Grade 8 teachers indicated that learners could not differentiate dependent and independent variables. One cannot answer this question easily if one does not plan one's lessons before teaching. In addition, if one has never reflected on the lesson taught and also has never analysed learners' errors, then recalling what one has never comprehended will not be easy, if not impossible. Hence, the answers given by the teachers suggest that there was no comprehension of learners' thinking. They could not indicate the pre-knowledge of the main ideas identified that the learners bring to class, which influenced their teaching. On the other hand, they were able to point lack of resources as the other factor that influenced their teaching of algebra. But clarity was not given on how that influenced their teaching. These findings suggest that most participants had inadequate knowledge of learners' thinking that influenced their teaching.

- The teachers had to specify the teaching procedures they used when teaching the identified concepts, and also give reasons for using those particular procedures.

The Grade 7 group indicated certain steps they used when teaching certain ideas, but they could not give the reasons why they followed those steps. The teaching procedures as indicated by some groups were not specific to the teaching of algebra or the teaching of concepts identified. For example, in case of Grade 8 group, they indicated *introduction of the topic* without specifying how the topic is introduced. Also, the grade methods indicated by the Grade 9 group, for example, discussions, telling methods, etc., do not indicate any specific procedure. This suggests that these teachers did not have any specific ways that they followed when presenting the ideas to learners. Hence they could not supply any reasons for the general ways that they had identified. This implies that planning was not done for the lessons they taught on the same topics. It is indicated that good planning is a prescription of effective teaching, hence a way towards developing PCK (Shulman, 1987). In this case, the teachers reflected they had insufficient knowledge of teaching procedures.

- The teachers had to give the specific ways of ascertaining the learners' understanding or confusion around the ideas.

The Grade 7 and Grade 9 groups were able to specify the assessment methods they used to ascertain learning of the concepts. However, none of these groups indicated how it gave feedback to the learners to ensure the improvement on the errors done. The Grade 8 group did not indicate any form of assessment, though they indicated some form of assessment under the procedures they followed when teaching. Then it can be concluded that the teachers ascertain learners' understanding.

5.3 IMPLICATIONS FROM THE STUDY

The study has explored senior phase teachers' PCK of algebra. The results revealed that the teachers were not able to indicate all the main ideas/concepts of algebra that are taught in their specific grades. They were also unable to specify all the content that they wanted the learners to learn in algebra. The study revealed that the teachers had inadequate algebra content knowledge. Inadequate subject matter knowledge limits PCK development (Karaman, 2012; Kind, 2009; Plotz, et al. 2012; Shulman, 1986; Van Driel & Berry, 2010). Hence, they could not adequately identify or describe learners' difficulties. They could not indicate their own difficulties that they realised when they taught algebra. They could not also specify the reasons for using the procedures they followed when teaching the ideas they identified. Besides the inadequate knowledge of algebra subject matter, they did not know the algebra curriculum in the grades that they taught. Furthermore, they could not give the importance of teaching algebra. However, they indicated that they did assess their learners. Also they were able to indicate some of the concepts about the main ideas that learners would do in the next grades. In view of the above reasons, it can be concluded that the participants had inadequate PCK of algebra.

5.4 RECOMMENDATIONS

According to CAPS document, algebra content knowledge that is required to be taught in the senior phase (Grades 7-9) covers these main concepts: variables, expressions, patterns, equations, graphs (limited to the drawing of linear graphs), functions and relationships. Then, teachers should know the definition of algebraic terms; be able to identify parts of an equation, determine input values, output values or rules for patterns and relationships; determine a relationship in different ways; be able to simplify algebraic expressions; factorise algebraic expressions; analyse situations in a variety of contexts in order to make sense of them, and represent and describe situations in algebraic language, formulae, expressions, equations and graphs. Therefore, all mathematics teachers in senior phase should have a good knowledge of these topics as prescribed. This implies that continuous professional development (CPD) programmes of algebra should be arranged for practising teachers to intensify their subject matter knowledge. This is due to the fact that good subject matter knowledge is the foundation of the development of PCK (Karaman, 2012; Kind, 2009; Plotz, et al., 2012; Shulman, 1986; Van Driel & Berry, 2010). Then a developing PCK [since PCK develops with experience of teaching] will result in effective teaching (Hurrell, 2013; Ijeh & Nkopodi, 2013; Kanyongo & Brown, 2013; Loughran, et al., 2006; Plotz et al., 2012; Shulman, 1986; Van Driel et al., 1998). Consequently, effective teaching will yield better results.

Development of PCK should also be part of CPD programmes for the development of algebra. Therefore, lesson preparations using Shulman's (1987) cycle of activities and CoRes constructions should be a priority. The use of CoRes has proven to yield good results in the development of science teachers' PCK. Most importantly, teachers who were involved in the development of CoRe in science, viewed it as an important tool that improved their way of thinking and teaching (Bertram & Loughran, 2012). There is also a need of further studies in the development of PCK in other topics of mathematics.

The textbooks that are used in South African schools, should, in addition to the subject matter that they offer, include a short history of algebra so that teachers and learners could realise its importance. Also, the textbooks should contextualise the algebra content as much as possible for users to see its practical application.

5.5 CONTRIBUTIONS OF THE STUDY

In the literature that I reviewed, studies explored mathematics PCK of teachers in some other parts of the world, I never came across one that was done in Limpopo Province, South Africa. In addition, the use of CoRe to document or to develop the PCK of teachers was done in some subjects, mostly science, other than mathematics. Therefore, this is the first study in Limpopo Province to investigate the PCK of algebra teachers using CoRe.

5.6 LIMITATIONS OF THE STUDY

To gather data, the study used a test and a CoRe matrix. As a result, the opportunity of observing teachers in practice was not fulfilled. Also, the views of teachers about their teaching that they could not be put in writing would not be known. Further, some of the responses given by the participants could be clarified in discussions which needed interviews.

5.7 CONCLUSION

This chapter summarised and interpreted the results of this study. The implications, limitations and recommendations of this study were also specified in this chapter. The chapter also indicated the contribution of this study to literature.

5.8 CONCLUDING REMARKS

Teacher knowledge plays an important role in the effectiveness teaching. Consequently, the performance of learners is largely based on the effectiveness of teachers (South African Education, 2007; Desimone, et al., 2002; Hill, et al., 2008). However, pedagogical content knowledge is proven to be the answer to effective teaching (Shulman, 1986). This implies that the development of PCK of teachers should be a priority in teacher development programmes to improve effective teaching. It has been proven in the literature that teachers with adequate PCK have the ability to address learners' difficulties when they build on the knowledge that learners already have (Shulman, 1986). Furthermore, studies show that learners learn effectively when teaching focuses on them (Johnson & Larsen, 2012; Shulman, 1986). Most importantly, the knowledge of subject matter plays an important role in the development of PCK. Although PCK is said to develop with time, inadequate subject matter retards its development as it has been shown in this study. That is, experience did not show any significance in algebra PCK development of the teachers because of the knowledge gap that the teachers had. Hence, development of both algebra subject matter and algebra PCK of teachers need immediate attention, so that teachers and students benefit from such intervention.

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Appendix A: Test

<table>
<tr><td>PART 1</td><td>ALGEBRA</td></tr>
<tr><td>What is algebra

Glossary of algebraic terms</td><td>Algebra is a branch of mathematics that deals with properties of operations and the structures these operations are defined on. Elementary Algebra that follows the study of arithmetic is mostly occupied with operations on sets of whole and rational numbers and solving first and second order equations. What puts elementary algebra a step ahead of elementary arithmetic is a systematic use of letters to denote generic numbers.</td></tr>
<tr><td></td><td>Well, you have been teaching algebra for some time now. What are the main ideas that are taught in algebra?</td></tr>
<tr><td></td><td>One of the ideas we teach in algebra is to solve for ‘what we do not know yet’. Like, for instance, what is the missing number in the equation $\square +7 = 13$? But we all know that we no longer write the equation in that way. It is written as $x + 7 = 13$, where x is called an unknown or a variable.

Why do we write it in that way, i.e. with a variable?</td></tr>
<tr><td></td><td>Can we also write it as $y + 7 = 13$? Also give a reason for your answer.</td></tr>
<tr><td></td><td>In the equation $3x + 4 = 19$ which part is the following term?

<table>
<tr><th>Term</th><th>Part of equation</th><th>Define the term</th></tr>
<tr><td>Coefficient</td><td></td><td></td></tr>
<tr><td>Variable</td><td></td><td></td></tr>
<tr><td>Constant</td><td></td><td></td></tr>
<tr><td>Algebraic Operator</td><td></td><td></td></tr>
<tr><td>Term</td><td></td><td></td></tr>
<tr><td>Expression</td><td></td><td></td></tr>
</table>
</td></tr>
</table>

	In algebra, information can be represented in different ways, i.e. verbally, in flow diagrams, in tables, by formulae, by equations, by graphs, etc. Select any three ways to represent the following information: A certain car rental company charges R 300 per day plus R 1, 50 per kilometre for their five-seater cars.

PART 2	DIFFICULTIES THAT PUPILS HAVE WHEN LEARNING ALGEBRA	
	Sometimes when you teach learners, you realise that they have difficulties in learning algebra. Now describe any five difficulties that learners have in the learning of algebra. Use examples in your descriptions.	
	Difficulties	Examples
	Study the following statements that were taken from some learners at a school in Seshego. Identify the statements that are true and the ones that are false. a) You can't add $5c, 5b$ and $5t$ because they are like 5 cabbages, 5 beetroots and 5 tomatoes. b) $3x + 4 = 19$ and $3y + 4 = 19$ are not the same equations because they have different letters. c) If you add three onto $4p$ you get $7p$. d) You can't do $P + Q = 10$ because there isn't an answer. e) In this school there are three times as many girls as boys, so if b stands for the number boys and g for the number of girls, then in this school, $b = 3g$. f) $-x$ is a negative number.	

PART 3	TEACHING ALGEBRA
	Sometimes when teaching mathematics, you realise that you have difficulties with teaching algebra. What difficulties have you encountered when teaching algebra?
	Do you think is important to teach algebra? Give reasons for your answer.

Appendix B: Content Representation (CoRe) matrix

CoRe on Senior Phase Algebra

GRADE	**IMPORTANT ALGEBRAIC IDEAS/CONCEPTS**			
	Big Idea 1	Big Idea 2	Big Idea 3	Other
What you intend the learners to learn about this idea.				
Why it is important for learners to learn this.				
What else you know about the idea (that you do not intend learners to know yet.				
Difficulties/limitations connected with teaching this idea.				
Knowledge about learners' thinking which influences your teaching of this idea.				
Other factors which influence this idea.				
Teaching procedures (and particular reasons for using these to engage with this idea).				
Specific ways of ascertaining learners' understanding or confusion around this idea,				

Note. From "In search of pedagogical content knowledge in science: developing ways of articulating and documenting professional practice," by J. Loughran, A. Berry and P. Mulhall, 2004, Journal of Research in Science Teaching, 41(4), p. 376.

Appendix C: A letter of request to conduct a research

Enq: Bopape, M. E.

Cell: 072 282 8908

Email: BopapeM@edu.limpopo.gov.za

P. O. Box 4429

Polokwane

0700

21 January 2015

The Head

Department of Education

LIMPOPO Province

Dear Sir

REQUEST TO CONDUCT RESEARCH DURING IN-SERVICE TRAINING OF TEACHERS AND AT SCHOOLS

My name is Mamogobo Bopape, a student at the University of Limpopo. I am studying towards a Master’s degree in Mathematics Education. I am also a Deputy Chief Education Specialist in Mathematics and I am attached to MASTEC Institute.

The purpose of my study is to **explore the pedagogical content knowledge (integration of pedagogical knowledge and subject matter knowledge) of algebra that senior phase mathematics teachers have using content representations**. This is in relation to what has been stated in the Integrated Strategic Planning Framework for Teacher Education and Development in South Africa 2011- 2025 which requires teachers to strengthen their subject matter knowledge and to develop their pedagogical content knowledge for effective teaching and learning.

I therefore request permission to conduct research during the training of GET Mathematics teachers at MASTEC and also at two circuits in Capricorn District. I also request to do lesson observations of the teachers concerned at their respective schools. I also request to use the name MASTEC Institute in my work.

Teachers will be required to write a pre-test at the beginning of the training and to construct content representation (CoRes) to show their knowledge of the teaching of the topic. Teachers will be informed about the study. No teacher will be obliged to participate in the study and they will be free to terminate any time they wish to do so. Teachers who agree to be part of the study will be observed while teaching to further explore their pedagogical content knowledge. Only the information collected from teachers who sign the consent forms to be part of the study will be used. The information collected will be for research purposes only. Teachers' identities will not be revealed. The participants will not be exposed to any physical or mental harm.

I am hoping for your positive response in this regard.

Kind regards,

M. E. Bopape

Appendix D: Approval from Department of Education in Limpopo Province

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

Enquiries: MC Makola PhD, *Tel No: 015 290 9448 .E-mail:* *MakolaMC@edu.limpopo.gov.za*

PO BOX 4429

POLOKWANE

0700

BOPAPE M.E

RE: Request for permission to Conduct Research

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"EXPLORING LIMPOPO PROVINCE`S SENIOR PHASE MATHEMATICS TEACHERS PEDAGOGICAL CONTENT KNOWLEDGE OF ALGEBRA USING REPRESENTATIONS".**
3. The following conditions should be considered:

   3.1 The research should not have any financial implications for Limpopo Department of Education.

   3.2 Arrangements should be made with the Circuit Office and the schools concerned.

   3.3 The conduct of research should not anyhow disrupt the academic programs at the schools.

   3.4 The research should not be conducted during the time of Examinations especially the fourth term.

   3.5 During the study, applicable research ethics should be adhered to, in particular the principle of voluntary participation (the people involved should be respected).

Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa **- development is about people!**

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

4. Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5. The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

21/08/2015

Mashaba KM **Date**

Acting Head of Department.

Appendix E: Clearance certificate

University of Limpopo
Department of Research Administration and Development
Private Bag X1106, Sovenga, 0727, South Africa
Tel: (015) 268 2212, Fax: (015) 268 2306, Email:noko.monene@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE CLEARANCE CERTIFICATE

MEETING:	**05 July 2016**
PROJECT NUMBER:	**TREC/46/2016: PG**
<u>PROJECT:</u>	
Title:	Exploring Limpopo Provinces' Senior Phase mathematics teachers' Pedagogical content knowledge of algebra using content representations
Researcher:	Ms ME Bopape
Supervisor:	Dr KM Chuene
Co-Supervisor:	Dr RS Maoto
Department:	Mathematics, Science and Technology Education
School:	Education
Degree:	Masters in Education

PROF TAB MASHEGO
CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE

The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: **REC-0310111-031**

Note:
i) **Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee.**
ii) **The budget for the research will be considered separately from the protocol.**
PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.

Finding solutions for Africa

Appendix F: Teachers consent form

TITLE: EXPLORING A GROUP OF LIMPOPO PROVINCES' SENIOR PHASE MATHEMATICS TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF ALGEBRA USING CONTENT REPRESENTATIONS

The following information is provided to help you decide whether you wish to participate in the present study or not.

- You are not in any way obliged to participate in the study.

- You are free to withdraw at any time without any fear of affecting your relationship with the facilitator or the Institute.

- The purpose of this study is to determine pedagogical content knowledge of algebra that teachers have.

- You will be given tasks at the beginning of the programme. Your responses in the tasks will be used in the study.

- Do not hesitate to ask questions about the study before participating or during the study.

- I would be happy to share the findings with you after the research is completed.

- Your name will not be associated with the research findings in any way, and only I will know your identity.

- There are no known risks and/or discomforts associated with this study.

If you agree that I use your responses for the study, please attach your name and signature on the consent form.

Bopape, M. E.   Student (9548548)   University of Limpopo   (072 282 8908)

	SURNAME & INITIALS	SIGNATURE
1		
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Appendix G: Participant profile form

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

MASTEC

MASTEC EDUCATOR’ PROFILE REGISTRATION			
1. PERSONAL DETAILS			
1.1. SURNAME & INITIALS:			
1.2. ID:			
1.3. GENDER:			
1.4. RACE:		NATIONALITY:	
1.5. CELL NUMBER:		E-MAIL ADDRESS:	
2. WORK DETAILS			
2.1. DISTRICT:		CIRCUIT:	
2.2. SCHOOL:		PRIMARY/SECONDARY:	
2.3. SCHOOL PHYSICAL ADDRESS:			
2.4. SCHOOL TEL NO.:			
3. QUALIFICATIONS			
3.1. PROFESSIONAL QUALIFICATIONS IN MATHEMATICS			
NAME OF QUALIFICATION	INSTITUTION	NO. OF YEARS TRAINING	YEAR OBTAINED
3.2. HIGHEST ACADEMIC QUALIFICATION IN MATHEMATICS			
NAME OF QUALIFICATION	INSTITUTION	NO. OF YEARS TRAINING	YEAR OBTAINED
4. EXPERINCE IN TEACHING MATHEMATICS			
GRADES TAUGHT		NUMBER OF YEARS IN THAT GRADE	
EDUCATOR’S SIGNATURE:			

Appendix H: Group 2 CoRe on senior phase algebra

Grade 8	**IMPORTANT ALGEBRAIC IDEAS/CONCEPTS** Big Idea 1 Equations	Big Idea 2 Graphs	Big Idea 3 Algebraic expressions
What you intend the learners to learn about this idea.	-Solve equations by inspection e.g. $2x = 100$ $\therefore x = 50$ -Solve equations by substitution e.g. $x^2 = 4$ substitute 5 in for x -Use laws of exponents -Use additive and multiplicative inverses e.g. $2x - 6 = 10$ -Analyse and interpret equations.	-Analyse and interpret graphs. -Recognise dependent and independent variables. -Understand linear and non-linear relationships. -Interpret features of graphs such as maximum and minimum values. -Understand the difference between discrete and continuous graph. -Draw graphs using ordered pairs to plot points on a Cartesian plane.	-Interpret rules. -Identify variables and constants in formulae. -Add and subtract like terms -Identify and classify like and unlike terms. -Recognise and identify coefficients and exponents.
Why it is important for students to learn this.	-To be able to solve problems based on the given information. -To develop learners' thinking or reasoning skills. -To be able to translate algebraic symbols in words.	-Learners will be able to communicate information by looking at the graph. -Learners will be able to compare different things using graphs. -Learners must be able to label $x - axis$ and $y - axis$ and the heading of the graph.	-Mathematics is a language to be good at mathematics at any language. -To learn new words and understand what those words mean. -Learners should be able to identify variable and constant in the expression, e.g. $6m + 23; 23$ is the constant and 6 is the coefficient. -Learners should be able to add/subtract like terms. -Learners can be able to translate word problems into mathematical language using symbols.

	Big Idea 1 Equations	Big Idea 2 Graphs	Big Idea 3 Algebraic expressions
What else you know about the idea (that you do not intend learners to know yet).	-Using factorisation -Equations of the form: product of factors is equal to zero.	-Linear graphs do have $x\text{-}intercept$ and $y - intercept$, they also have gradient. -Drawing linear graphs from given equations. -Determine equations from given linear graph.	-Using factorisation. -Different of two squares -Trinomials of the form, e.g. $ax^2 + bx + c$ where a is a common factor
Difficulties/limitations connected with teaching this idea.	-Lack of resources, e.g. textbooks. -Teaching approach. -Language barrier.	-Lack of resources, using free hand when drawing graphs, instead of using T-square.	-Lack of resources. -Overcrowded classes, larger group comprises face-face interaction. -Communicating variety of strategies at different levels, e.g. group work. -Time constraints.
Knowledge about learners' thinking which influences your teaching of this idea.	-Lack of understanding on mathematical concepts, e.g. root, exponent. -Translation of algebraic symbol into English translation.	-Learners cannot differentiate dependent and independent variables. -Learners intend to confuse discrete and continuous information. -Learners are unable to plot points on a Cartesian plane.	-Learners cannot identify and classify the difference between the like and unlike terms. -Cannot recognise and interpret rules in symbolic form. -Learners cannot work on addition and subtraction of like terms. -Cannot identify variables and constants in a given expression.
Other factors which influence your teaching this idea.	-The issue of knowing that when we solve equations we use additive and multiplicative inverses. -Learners intend to add unlike terms.	-Lack of resources.	-They cannot differentiate algebraic expression like monomial, binomial, trinomial and polynomial.

	Big Idea 1 Equations	Big Idea 2 Graphs	Big Idea 3 Algebraic expressions
Teaching procedures (and particular reasons for using these to engage with this idea).	-Prior knowledge from learners by asking questions. -Introduction of lesson/topic. -Interacting with learners by question and answer method and solving problems. -Presentation -Class activity -Problem solving approach/problem centred learning.	-Prior knowledge from learners by asking questions. -Introduction of lesson/topic. -Presentation of the lesson -Interacting with learners by giving them Problem to draw. -Class activity/homework -Project-based learning Cognitively-guided instruction	-Prior knowledge from learners by asking questions. -Introduction of lesson/topic. -Presentation of the lesson -Interacting with learners by giving them problems to solve. -Class activity/homework -Problem-solving/ problem c centred learning.
Specific ways of ascertaining students' understanding or confusion around this idea.	-Use of correct mathematical language to clarify mathematical concepts.	-Promote correct use of mathematical language. -Encourage logical and critical thinking. -Be realistic. -Enhance problem solving skills.	-Teach in context bringing real life situation into the classroom to enhance problem solving skill.

Appendix I: Group 3 CoRe on senior phase algebra

Grade 9	**IMPORTANT ALGEBRAIC IDEAS/CONCEPTS** Big Idea 1 Exponents	Big Idea 2 Expressions	Big Idea 3 Equations	Big Idea 4 Functions
What you intend the learners to learn about this idea.	-Laws and rules of exponents e.g. $a^n + a^m = a^{m+n}$) -Learners must be able to simplify using the rules and laws in solving problem. e.g. $2a^2 \times a = 2a^3$	-To simplify and expand expressions e.g. (i) $2(x+3)$ (ii) $\frac{2x^2+4x^2y+2x^2y}{8x^2y}$	-Able to solve equations e.g. $x + 7 = 15$	-Completing tables and drawing of graphs.
Why it is important for students to learn this.	-Assist the learners to solve equations and simplify expression.	-The ability to group like terms and simplify expressions.	-To learn about sharing things or to apply in real life situation.	-To plot the points on the Cartesian plane
What else do you know about the idea (that you do not intend learners to know yet?	-Consider negative exponents, that is application of law no 4 and no5. -Growth and decay .	-Factorisation of polynomials, e.g. $2a^2 + ab + 4b^2$	-Simultaneous equations.	-Plotting of high order function e.g. quadratic equations.
Difficulties/limitations connected with teaching this idea.	-Learners sometimes forget to apply laws and rules used in exponents.	-Learners tends to consider or write expression as equations. -Mix simplifying expression and solving equations.	-Application of inverse method	-Not able to determine the equation from the given graph.
Knowledge about learners' thinking which influences your	-Previous knowledge can influence teaching positivity and negativity	-They experience difficulty in factorising expression.	-Those thinking negativity and positivity.	-Introduction of prior knowledge. -Solving the equation

	Big Idea 1 Exponents	Big Idea 2 Expressions	Big Idea 3 Equations	Big Idea 4 Functions
teaching of this idea.	e.g. Those having knowledge about exponents may be bored.		-Learners have problem of grouping of like terms e.g. $4x + 2 = 2x$	will give learners knowledge of answering.
Other factors which influence your teaching this idea.	-Type of learners you are teaching (that is depending on IQ). -Overall behaviour of the learners. .	-Gifted learners and Time takers.	-Contexual factors	-Contexual factors
Teaching procedures (and particular reasons for using these to engage with this idea).	-Group discussion, telling method, demonstration, and class discussions.	-Class discussion, group discussion, telling method, demonstration.	-Demonstration, class discussion, telling method, group discussion.	-Group discussion, class discussion, telling method, demonstration.
Specific ways of ascertaining students' understanding or confusion around this idea	-Remedial work, classwork, homework tests, assignments, investigation, projects.	-Classwork, homework, tests, assignments, investigation, projects	-Tests, assignments, investigation, projects, classworks, homeworks.	-Classwork, homework, tests, assignments, investigation, projects

Appendix J: Group 4 CoRe on senior phase algebra

Grade 9	**IMPORTANT ALGEBRAIC IDEAS/CONCEPTS** Big Idea 1 Algebraic expressions	Big Idea 2 Algebraic equations	Big Idea 3 Factorisation
What you intend the learners to learn about this idea.	-Simplifying algebraic expressions by means of:factorisation -Using Operations -Expansion of algebraic expressions	-Know the difference between expression and equation. -Know the different types of equations e.g. linear, quadratic. -Write equations to describe problems situation (word problems). -Solving equations using factorisation. -Simplify equation.	
Why it is important for learners to learn this.	-Application of that knowledge in real life situation. -Recognise and interpret lives or relationships, -To improve their mathematical expressions.	-Solve unknown to be known, -Solve real life problems. -Change word problems into equations.	
What else you know about the idea (that you do not intend learners to know yet.	-Division of polynomials -Multiplication of binomials by polynomials -Working with third degree expressions -Factorisation of nominal where the coefficient of x^2 (term 1) is greater than one.	-Solve equations including surds. -Factorise sum and difference between two cubes. -Solving third degree equations and simultaneous equations.	
Difficulties/limitations connected with teaching this idea?	-Overcrowding i.e. no learner must be left behind, so the educator is unable to move to each learner. -Not having enough resources to explain the terminologies e.g. variable is	-Strategies of making teaching equations lasts.	

	Big Idea 1 Algebraic expressions	Big Idea 2 Algebraic equations	Big Idea 3 Factorisation
	represented by any letter of the alphabet and in quadratic expressions c ← is called a constant.		
Knowledge about learners' thinking which influences your teaching of this idea.	-Are able to apply generations. -Prior knowledge, are able to identify variables, exponents. -Can not differentiate between terms in an expression eg unlike & unlike terms.	-Learners forget to put a zero after zero. -Learners do not know how to expand. -Difficulty with removing a coefficient of a variable to make the equation linear. $\frac{2y}{2}+\frac{2}{2}=\frac{4}{2}$; $y+1=2$ e.g. They must be able to solve problem situation by inspection.	
Other factors which influence your teaching this idea	-Overcrowding -Lack of resources	-Overcrowding -Lack of resources	-Overcrowding -Lack of resources
Teaching procedures (and particular reasons for using these to engage with this idea)	-Definition of expressions -Linking of new topic with their prior knowledge. -Give examples and give a similar example to learners.	-Give examples and then give a similar problem to learners to solve.	-Give examples and then give a similar problem to learners to solve.
Specific ways of ascertaining students' understanding or confusion around this idea	-Group activities -Homeworks & classworks -Tests -Assignments -Projects -Exam	-Group activities -Homeworks & classworks -Tests -Assignments -Exam	-Group activities -Homeworks & classworks -Tests -Assignments -Exam