

**EMPIRICAL ANALYSIS OF INFLATION DYNAMICS: EVIDENCE FROM
GHANA AND SOUTH AFRICA**

by

ALEXANDER BOATENG

THESIS

Submitted in fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

STATISTICS

in the

**FACULTY OF SCIENCE AND AGRICULTURE
(School of Mathematical and Computer Sciences)**

at the

UNIVERSITY OF LIMPOPO

PROMOTER: Prof. M. Lesaoana

CO-PROMOTERS: Prof. H.J. Siweya

Prof. A. Belete

2017

Declaration

I declare that the thesis hereby submitted to the University of Limpopo, for the degree of Doctor of Philosophy in Statistics has not previously been submitted by me for a degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.

BOATENG, A

Date

Abstract

Using the ARFIMA (autoregressive and fractionally integrated moving average) model extended with sGARCH (standard generalised autoregressive conditional heteroscedasticity) and 'gjrGARCH (Glosten-Jagannathan-Runkle generalised autoregressive conditional heteroscedasticity) innovations, fractional integration approach and state space model, this study has empirically examined persistency of inflation dynamics of Ghana and South Africa, the only two countries in Sub-Saharan Africa with Inflation Targeting (IT) monetary policy. The first part of the analysis employed monthly CPI (Consumer Price Index) inflation series for the period January 1971 to October 2014 obtained from the Bank of Ghana (BoG), and for the period January 1995 to December 2014 obtained from Statistics South Africa. The second part involves the estimation of threshold effect of inflation on economic growth using annual data obtained from the IMF (International Monetary Fund) database for the period 1981 to 2014, for both countries.

Results from the study showed that structural breaks, long memory and nonlinearities (or regime shifts) are largely responsible for inflation persistence, hence the ever-changing nature of inflation rates of Ghana and South Africa. ARFIMA(3,0.35,1)-'gjrGARCH(1,1) under Generalised Error Distribution (GED) and ARFIMA(3,0.50,1)-'gjrGARCH(1,1) under Student- t Distribution (STD) models provided the best fit for persistence in the conditional mean (or level) of CPI for Ghana and South Africa, respectively. The results from these models provided evidence of time-varying conditional mean and volatility in CPI inflation

rates of both countries. The two models also revealed an asymmetric effect of inflationary shocks, where negative shocks appear to have greater impact than positive shocks, in terms of persistence on the conditional mean with time-varying volatility.

This thesis proposes a model that combines fractional integration with non-linear deterministic terms based on the Chebyshev polynomials in time for the analysis of CPI inflation rates of Ghana and South Africa. We tested for non-linear deterministic terms in the context of fractional integration and estimated the fractional differencing parameters, d to be 1.11 and 1.32 respectively, for the Ghanaian and the South African inflation rates, but the non-linear trends were found to be statistically insignificant in the two series. New evidence from this thesis depicts that inflation rate of Ghana is highly persistent and non-mean reverting, with an estimated fractional differencing parameter, $d > 1.0$, and will therefore require some policy action to steer inflation back to stability. However, the South African inflation series was found to be a cyclical process with an order of integration estimated to be $d = 0.7$, depicting mean reversion, with the length of the cycles approximated to last for 80 months.

Finally, the thesis incorporated structural breaks, long memory, non-linearity, and some explanatory variables into a state space model and estimated the threshold effect of inflation on economic growth. The empirical results suggest that inflation below the estimated levels of 9% and 6% for Ghana and South Africa respectively, will be conducive for economic growth.

The policy implications of these results for both countries are as follows. First, both series had similar properties responsible for inducing inflation persistence such as structural breaks, non-linearities, long memory and asymmetric response to negatives shocks - but with varied degrees of magnitude. For both

countries, the conditional mean and unobserved components such as volatility for both countries were found to be time-varying. This thesis, therefore, recommends to the BoG and the South African Reserve Bank (SARB) - responsible for monetary policies, and the Finance Ministers of both governments - responsible for fiscal policies, to take the above-mentioned properties into account in the formulation of their monetary policies.

Second, the thesis recommends that the BoG and the SARB consolidate the IT policy, since keeping inflation below the targets set of 9% and 6%, respectively for Ghana and South Africa, will boost economic growth. Third, policymakers could also design measures (monetary and fiscal policies) such as increase in interest rates, credit control, and reduction of unnecessary expenditure, among others, to control inflation due to its adverse effects on market volatility. Even though an increase in interest rates could assist in curtailing the recent and anticipated increase in inflation rates in both countries, where targets have been missed by Ghana and South Africa, it will also be prudent to legislate monetary policies around demand-supply side since the problem of both countries appears to be more of a structuralist than a monetarist. It is, therefore, recommended that both countries tighten the IT monetary policy in order to reduce inflation persistence. This will eventually impact on poverty and income distribution with ramifications for economic growth and/or development.

The fourth implication of these results is that governments and central banks should be mindful of the actions and decisions they take, in the sense that unguarded decisions and unnecessary alarms could raise uncertainties in the economy, which could, in turn, affect the future trajectory of inflation. Finally, the thesis recommends that governments of both countries strengthen the private sector, which is the engine of growth. For small and open economies such as Ghana and South Africa, this will grow the economy through job creation

and restore investor confidence.

Keywords: CPI inflation, Persistence, Structural breaks, Fractional integration, Long memory, ARFIMA model, sGARCH model, 'gjrGARCH model, Asymmetry, Non-linearity, GDP per capita, Threshold effect, State space model, GHCPI, SACPI.

Dedication

To God and Prof. L.A. Gil-Alana (University of Navarra, Spain), my wife, Mrs. Belinda Boateng and my daughter, Adelaide Boateng

Acknowledgements

I would like to express my gratitude to God Almighty. I am also deeply indebted to my supervisors Prof. M. Lesaoana, Prof. H.J. Siweya and Prof. A. Belete, for their thoughtful guidance, intellectual support and encouragement throughout the period of my study, which led to the production of this thesis.

I would like to express my sincere appreciation to Bishop Dad-Heward Mills, Prof. K. Ayisi, Prof. A. Bediako, Prof. P. Masoko, Rev. and Mrs. Mokganya, Rev. E. Ofori, and my mother's Madam F. Donkor and Madam G. Tsukudo for their support throughout the entire study. Without their help, this study would not have been completed. I am also grateful to my friends Dr. E.N. Aidoo, Dr. S. Oduro, Mr. K.A. Ababio, Mr M. Omane-Adjepong and Mr. R. Oppong, for their support. To my brothers Mr. B. Opoku, Mr. H. Aggyapong, Mr. M. Owusu and my sister Mrs. Boateng-Sarpong, thank you for being there for me.

Lastly, I would like to thank the University of Limpopo, the National Research Foundation (NRF), the Department of Science and Technology (DST), Telkom's Tertiary Education Support Programme (TESP) and the NRF-DST Centre of Excellence for Mathematical and Statistical Sciences (CoE-MaSS) for their financial support.

Contents

Declaration	i
Abstract	ii
Dedication	vi
Acknowledgments	vii
Table of Contents	viii
List of Figures	xiii
List of Tables	xv
List of Special Symbols	xvii
Distributions	xix
List of Abbreviations and Acronyms	xxi
Research Outputs	xxiv
1 Introduction and Background	1
1.1 Introduction	1
1.2 Structural Breaks, Long Memory and Non-Linearity	4
1.3 Statement of the Problem	7

1.4	Research Questions	11
1.5	Aim of the Study	12
1.5.1	Objectives of the study	12
1.6	Scope of the Study	13
1.7	Structure of the Study	15
2	Literature Review	18
2.1	Introduction	18
2.2	Theoretical Framework of Inflation Modelling in Relation to Monetary Policies	19
2.3	Inflation Targeting and Inflation Persistence	23
2.4	Unit Root Tests without Structural Breaks	28
2.4.1	Augmented Dickey-Fuller (1979) test	28
2.4.2	Phillips-Perron (1988) test	29
2.4.3	Kwiatkowski-Phillips-Schmidt-Shin (1992) test	31
2.5	Unit Root Tests with one Structural Break	32
2.5.1	Zivot and Andrews (1992) test	32
2.6	Unit Root Tests with Multiple Structural Breaks	34
2.6.1	Bai and Perron (1998) test	34
2.7	Fractional Integration Method by Gil-Alana (2008)	38
2.8	Long Memory in Time Series	38
2.8.1	Fractional Gaussian noise	40
2.8.2	The ARFIMA model	41
2.9	Sources of Long Memory	43
2.9.1	Contemporaneous aggregation	43
2.9.2	Temporal aggregation	44
2.10	Outline of Long Memory Models	46
2.10.1	Rescaled range (R/S) analysis	46
2.10.2	Modified R/S analysis	50
2.10.3	Empirical evidence of the Hurst phenomenon	51

2.11 Estimation Methods for Fractionally Integrated Parameter and ARFIMA Model	53
2.11.1 Parametric estimators	53
2.11.2 Semi-parametric estimators	55
2.12 Long Memory and Fractionally Integrated ARMA with Conditional Heteroscedastic Innovations	63
2.12.1 Autoregressive conditional heteroscedasticity model	64
2.12.2 The standard autoregressive conditional heteroscedasticity model	66
2.12.3 Glosten-Jagannathan-Runkle generalised autoregressive conditionally heteroscedasticity model	67
2.12.4 ARFIMA(p,d,q)-sGARCH(P,Q) model	68
2.12.5 ARFIMA(p,d,q)-'gjrGARCH(P,Q) model	69
2.13 The Framework of Cuestas and Gil-Alana (2015) based on Chebyshev Polynomial	70
2.14 State Space Modelling and Threshold effect of Inflation-Growth Nexus	72
2.14.1 State space model of unobserved components with Kalman filter	75
2.14.2 Estimation of state space model	75
2.15 Empirical Evidence of Inflation and Economic Growth Nexus	79
2.16 Summary	84

3 Structural Breaks and Testing for Random Walk Hypothesis in Inflation Series 87

3.1 Introduction	87
3.2 Synopsis of Inflation Dynamics of Ghana and South Africa	88
3.3 Data and Methodology	90
3.3.1 Descriptive statistics	90
3.3.2 Methodology	92

3.4	Empirical Results	92
3.5	Summary	100
4	Analysis of Long Memory and ARFIMA Modelling of Inflation	
	Series	102
4.1	Introduction	102
4.2	Empirical Evidence of Long Memory and ARFIMA Modelling . .	104
4.3	Data and Methodology	107
	4.3.1 Descriptive statistics	107
	4.3.2 Methodology	110
4.4	Empirical Results	112
4.5	Summary	116
5	Modelling Persistence in the Conditional Mean of Inflation using	
	ARFIMA-sGARCH and ARFIMA-‘gjrGARCH Models	119
5.1	Introduction	119
5.2	Data and Methodology	121
	5.2.1 Descriptive statistics	121
	5.2.2 Methodology	123
5.3	Empirical Results	123
5.4	Summary	131
6	Non-Linearity, Structural Breaks and Fractional Integration in	
	Inflation Series	134
6.1	Introduction	134
6.2	Empirical Evidence	137
6.3	Data and Methodology	138
	6.3.1 Descriptive statistics	138
	6.3.2 Methodology	139
6.4	Empirical Results	139

6.5	Summary	144
7	Threshold Effect of Inflation-Growth Nexus using State Space	
	Model	146
7.1	Introduction	146
7.2	Empirical Evidence of Thresholds effect of Inflation-growth Nexus from Ghana and South Africa	150
7.3	Data and Methodology	153
	7.3.1 Descriptive statistics	153
	7.3.2 Methodology	155
7.4	Empirical Results	158
7.5	Summary	168
8	Summary, Conclusion and Recommendations	170
8.1	Introduction	170
8.2	Summary	171
	8.2.1 Major findings	171
	8.2.2 Contributions of the study	176
8.3	Conclusion	177
	8.3.1 Policy implications	177
8.4	Recommendations and Suggestions for Future Research	180
	Appendices	210

List of Figures

3.1	Monthly CPI Inflation series of Ghana (1971:M01-2014:M10) . . .	90
3.2	Monthly Inflation series of South Africa (1995:M01-2014:M12) . . .	91
3.3	Recursive estimates of d for SACPI inflation series	99
4.1	Density graph for normality for GHCPI inflation series	108
4.2	The spectral density, periodogram, ACF and PACF of GHCPI in- flation series	109
4.3	Density graph for normality for SACPI inflation series	110
4.4	The spectral density, periodogram, ACF and PACF of SACPI in- flation series	111
5.1	Residual plots for GHCPI inflation for ARFIMA(3,0.38,1)	122
5.2	Residual plots for SACPI inflation for ARFIMA(3,0.43,2)	123
5.3	ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.37,1)-sGARCH(1,1) of GHCPI inflation series . . .	125
5.4	ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.35,1)-'gjrGARCH(1,1) of GHCPI inflation	128
5.5	ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.50,2)-sGARCH(1,1) of SACPI inflation series . . .	131
5.6	ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.50,2)-'gjrGARCH(1,1) of SACPI inflation series . .	132
6.1	The behaviour of GHCPI inflation series	139
6.2	The behaviour of SACPI inflation series	140

6.3	Periodogram of GHCPI inflation series for all frequencies	142
6.4	Periodogram of SACPI inflation series for all frequencies	142
6.5	Periodogram of GHCPI inflation series for the first 50 frequencies	143
6.6	Periodogram of SACPI inflation series for the first 50 frequencies	144
7.1	Time series plots for all the variables: Ghana (1981-2014)	154
7.2	Time series plots for all the variables: South Africa (1981-2014) .	155
7.3	Residual analysis for GHCPI inflation with state space-BSM . . .	161
7.4	Residual analysis for SACPI inflation with state space-BSM . . .	162
7.5	The graph of Growth rate by GDP per capita and CPI inflation series of Ghana (1981-2014)	162
7.6	The graph of Growth rate by GDP per capita and CPI inflation series of South Africa (1981-2014)	163
7.7	Residual analysis of the selected state space model for thresholds effect of inflation-growth nexus of Ghana	166
7.8	Residual analysis of the selected state space model for thresholds effect of inflation-growth nexus of South Africa	167

List of Tables

3.1	Descriptive and Summary Statistics of CPI Inflation of Ghana and South Africa	92
3.2	Unit Root Test for GHCPI inflation series	93
3.3	Unit Root Test for SACPI inflation series	93
3.4	Multiple breakpoint tests of $L + 1$ vs L sequentially determined breaks for GHCPI inflation series	94
3.5	Multiple breakpoint tests of $L + 1$ vs L sequentially determined breaks for SACPI inflation series	95
3.6	Estimates of the fractional differencing, d parameter for GHCPI inflation series	96
3.7	Estimates of d in the context of multiple breaks for GHCPI inflation series	97
3.8	Estimates of the fractional differencing, d parameter for SACPI inflation series	98
3.9	Estimates of the fractional differencing, d parameter for SACPI inflation series	98
4.1	Descriptive and Summary Statistics of CPI Inflation of Ghana and South Africa	107
4.2	Hurst estimates of modified R/S analysis for GHCPI and SACPI inflation series	112
4.3	Parameters estimates of long memory for GHCPI and SACPI inflation series	113

4.4	Parameter estimates of ARFIMA models for GHCPI inflation series	114
4.5	Parameter estimates for ARFIMA models for SACPI inflation series	115
4.6	Residual diagnostics test for ARFIMA (p,d,q) models selected for GHCPI and SACPI inflation series	116
5.1	Criteria for ARMA(p,q) selection	124
5.2	Fitted models for GHCPI and SACPI inflation series	125
5.3	ARFIMA(3,d,1)-sGARCH(1,1) under GED for GHCPI inflation series	126
5.4	ARFIMA(3,d,1)-'gjrGARCH(1,1) under GED for GHCPI inflation series	127
5.5	Assessment of goodness of fit under GED for GHCPI inflation series	128
5.6	ARFIMA(3,d,2)-sGARCH(1,1) under STD for SACPI inflation series	129
5.7	ARFIMA(3,d,2)-'gjrGARCH(1,1) under STD distribution for SACPI inflation series	130
5.8	Assessment of goodness of fit under STD distribution for SACPI inflation series	132
6.1	Estimates of d in the context of non-linear trends for GHCPI and SACPI inflation series	140
6.2	Estimates of d with a linear model for GHCPI and SACPI inflation series	141
6.3	Estimate of d in the context of cyclical fractional integration for SACPI	144
7.1	Descriptive and Summary Statistics	156
7.2	State space model of unobserved components of GHCPI with Kalman filter	159

7.3	State space model of unobserved components of SACPI with Kalman filter	160
7.4	Pairwise Granger causality test for CPI inflation and growth rate by GDP per capita for Ghana and South Africa	161
7.5	State space model of unobserved components with Kalman filter in estimating the threshold effect of inflation-growth nexus of Ghana	164
7.6	State space model of unobserved components with Kalman filter in estimating the threshold effect of inflation-growth nexus of South Africa	165
7.7	Serial correlation statistics of residuals on the response variable: Growth rate by GDP per capita for Ghana and South Africa . . .	168

List of Special Symbols and Abbreviations

TB	break date
δ^2	volatility
DT_t	dummy variable corresponding trend shift
DU_t	dummy variable corresponding to mean shift
Δ	first difference
k	lag periods
y_t	time series data (CPI inflation series)
ε_t	white noise residuals
μ	mean
Z_{it}	vector of exogeneous variables
m	number of breaks
$T_1 \dots T_m$	break dates
$\rho(\tau)$	autocovariance function
d	fractional differencing parameter
ρ_j	autocorrelation function
$B_H(t)$	Fractional Brownian motion
$\Gamma(\cdot)$	Gamma function
H	Hurst exponent
$\theta(L)$	autoregressive component
$\phi(L)$	moving average component

L	backshift operator
$f_u(\cdot)$	spectral density function
p, q	integer orders
R_N	rescaled range sample mean
Q_n	rescaled range test statistic
y_{tGDP}^*	growth rate per capita (Gross Domestic Product)
y_t^T	trend component
y_t^C	cyclical component
T	sample size
$L_w(\theta)$	local whittle estimator
$P_{i,T}$	Chebyshev polynomial
L_E	exact maximum likelihood estimator
δ	threshold value for inflation-growth nexus
F	transition function
$P_{0/0}$	covariance matrix
$\beta_{0/0}$	initial states
σ_t^2	conditional variance
γ_j	leverage effect
$I(0)$	stationary process
$I(1)$	unit root process
$I(d)$	fractionally integrated process
<i>iid</i>	Independently and Identically Distributed

List of Distributions

Normal Distribution

The probability function of the normal distribution is given by:

$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

where μ and σ denotes the location and scale parameters respectively. The case of $\mu = 0$ and $\sigma = 1$ will reduce the normal distribution to standard normal distribution:

$$f(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

Student- t Distribution

If $z \sim N(0, 1)$ and $\mu \sim \chi^2(r)$ are independent, then the random variable $T = \frac{z}{\sqrt{\frac{\mu}{r}}}$ follows a student- t distribution with r degrees of freedom.

The probability density function of T is given by:

$$f(t) = \frac{\Gamma(\frac{r+1}{2})}{\sqrt{\pi r} \Gamma(\frac{r}{2})} \times \frac{1}{(1 + \frac{t^2}{r})^{\frac{r+1}{2}}}$$

for $-\infty < t < \infty$.

Generalised Error Distribution

The generalised error distribution is a three parameter distribution belonging to the exponential family with the conditional density function given by:

$$f(x) = \frac{\kappa e^{-0.5|\frac{x-\alpha}{\beta}|^\kappa}}{2^{1+\kappa^{-1}}\beta\Gamma(\kappa^{-1})}$$

with α , β and κ denote the location, scale and shape parameters, respectively.

List of Abbreviations and Acronyms

ADF	Augmented Dickey-Fuller test (1979)
AIC	Akaike Information Criterion
AR	Autoregressive model
ARCH	Autoregressive Conditional Heteroscedasticity
ARFIMA	Autoregressive and Fractionally Integrated Moving Average
ARMA	Autoregressive and Moving Average
BIC	Bayesian Information Criterion
BoG	Bank of Ghana
BP	Bai and Perron test (1998)
CPI	Consumer Price Index
CPIX	Consumer Price Index Excluding Mortgage Prices
DAC	Directional Accuracy test
EMH	Efficient Market Hypothesis
EML	Exact Maximum Likelihood
ERP	Economic Recovery Programme
FBM	Fractional Brownian Motion
FGN	Fractional Gaussian Noise
FSA-PGD	Faculty of Science and Agriculture - Post Graduate research Day
GDP	Gross Domestic Product
GED	Generalised Error Distribution

‘gjrGARCH	Glosten-Jagannathan-Runkle Generalised Autoregressive Conditional Heteroscedasticity
GHCPI	Ghana Consumer Price Index
GLS	Generalised Least Squares
GMM	Generalised Method of Moments
GPH	Geweke and Porter-Hudak (1983)
IMF	International Monetary Fund
IT	Inflation Targeting
ITMPF	Inflation Targeting Monetary Policy Framework
J-B	Jarque-Bera test
KPSS	Kwiatkowski–Phillips–Schmidt–Shin test (1992)
LB	Level Beak
LL	Log Likelihood
LM	Lagrange Multiplier test
LSTAR	Logistic Smooth Transition Autoregressive model
LW	Local Whittle
MAE	Mean Absolute Error
M1	Money supply aggregate that includes physical money-both paper and coin
M2	Money aggregate that includes assets that are highly liquid but are not cash
M3	Aggregate money supply that includes M2 as well as large time deposits
MPC	Monetary Policy Committee
MSE	Mean Square Error
NIV	Non-parametric Instrumental Variable
NLLS	Non-Linear Least Squares

Norm	Normal Distribution
OECD	Organisation for Economic Co-operation and Development
OL	Outlier
OLS	Ordinary Least Squares
PP	Phillips-Perron test (1988)
PSTR	Panel Smooth Transition Regression model
RMSE	Root Mean Square Error
R/S	Rescaled Range
SACPI	South Africa Consumer Price Index
SADC	Southern African Development Community
SAP	Structural Adjustment Programme
SARB	South African Reserve Bank
SASA	South African Statistical Association
SBM	Standard Brownian Motion
SETAR	Self Exciting Threshold Autoregressive model
SIV	Semi-parametric Instrumental Variable
STAMP	Structural Time series Analyser Modeller and Predictor
STAR	Smooth Transition Autoregressive model
STD	Student-t Distribution
Stats SA	Statistics South Africa
sGARCH	Standard Generalised Autoregressive Conditional Heteroscedasticity
USD	United States Dollar
VAR	Vector Autoregressive
VECM	Vector Error Correction Model
VR	Variance Ratio
ZA	Zivot and Andrews test (1992)

Research Outputs

The following sections give a list of research outputs from this thesis.

Peer Reviewed Journal Publications

1. Boateng, A., Lesaoana, M., Siweya, H.J., Belete, A. and Gil-Alana, L.A. (2016). Long Memory and ARFIMA Modelling: The case of CPI Inflation Rates for Ghana, *Journal of Developing Areas*, 50(3): 288-304.
2. Boateng, A., Lesaoana, M., Siweya, H.J., Belete, A. and Gil-Alana, L.A. (2017). Non-Linearities, Structural Breaks and Fractional Integration in the Analysis of the Ghana and South African CPI Inflation Rates, *Journal of Developing Areas* (Accepted).
3. Boateng, A., Lesaoana, M., Siweya, H.J., Belete, A. and Gil-Alana, L.A. (2016). Unit Root tests and Structural Breaks: An Application to South African CPI Headline inflation rates, *Economic Change and Restructuring*. (Under Review).
4. Boateng, A., Lesaoana, M., Siweya, H.J., Belete, A. and Gil-Alana, L.A. (2017). Modelling persistence in the conditional mean of inflation using the ARFIMA process with sGARCH and 'girGARCH innovations: The case of Ghana and South Africa. *African Review of Economics and Finance* (Accepted).

Conferences and Related Events

1. World Social Science Forum. DST Funded event to interact and collaborate with international experts. 13-17 September 2015.
2. Boateng, A., Lesaoana, M., Siweya, H.J. Belete, A. and Gil-Alana, L.A. (2015). Long Memory and ARFIMA Modelling: The case of CPI Inflation rates of Ghana and South Africa. South African Statistical Association (SASA 2015), 29 November-3 December 2015, University of Pretoria, South Africa.
3. Boateng, A., Lesaoana, M., Siweya, H.J. Belete, A. and Gil-Alana, L.A. (2015). Long Memory and ARFIMA Modelling: The case of CPI Inflation rates of Ghana and South Africa. Faculty of Science and Agriculture Postgraduate Research Conference (FSA-PGD 2015), 1-2 October 2015, Bolivia Lodge, Polokwane, South Africa.
4. Boateng, A., Lesaoana, M., Siweya, H.J. and Belete, A. Modelling persistence in the conditional mean of inflation using the ARFIMA process with sGARCH and 'girGARCH innovations: The case of Ghana and South Africa. African Review of Economics and Finance Conference, 11-12 August 2016, Kwame Nkrumah and University of Science and Technology (KNUST), Ghana.
5. Boateng, A., Lesaoana, M., Siweya, H.J. and Belete, A. Modelling persistence in the conditional mean of inflation using the ARFIMA process with sGARCH and 'girGARCH innovations: The case of Ghana and South Africa. Faculty of Science and Agriculture Postgraduate Research Conference (FSA-PGD 2015), 24-25 October 2016, Bolivia Lodge, Polokwane, South Africa.
6. Boateng, A., Lesaoana, M., Siweya, H.J. and Belete, A. Modelling persistence in the conditional mean of inflation using the ARFIMA process

with sGARCH and 'girGARCH innovations: The case of Ghana and South Africa. Annual Research Meeting by Center of Excellence for Mathematical and Statistical Sciences (CoE-MaSS), 18-21 October 2016, Johannesburg, South Africa.

Chapter 1

Introduction and Background



1.1 Introduction

In recent years there has been an increase in both applied and theoretical research in time series modelling and forecasting. The research in this area has contributed immensely to the success of several economies in the world. One of the economic variables that have received much attention in time series modelling is inflation. This is because inflation is a major focus for economic policy worldwide (Hendry, 2001).

Inflation is also regarded as a crucial indicator of economic growth, the cost of living and the general well-being of a population. As a result, it has been one of the most examined topics in macroeconomics, especially from theoretical and empirical perspectives (Belkhouja and Mootamri, 2016). The principal aim of monetary policy is to stabilise prices and improve economic growth. Therefore, in order to optimise and test the efficacy of monetary policy, monetary authorities monitor inflation persistence or long memory (Belkhouja and Mootamri,

2016). The propensity of inflation rates to, in some instances, revert slowly to recover equilibrium level following a shock or, in extreme cases, be volatile and non-mean reverting, often referred to as inflation persistence, has long been an issue of concern for both policymakers and academics (Alagidede et al., 2014). For policymakers, deviations of the inflation rate from a specific target, the speed of responding to correcting measures, and the output cost of implementing, say, a disinflation policy are critical; but for academics, the underlying dynamics of inflation and how the theory fits the facts are crucial (Alagidede et al., 2014).

Persistency of inflation dynamics has also become a major concern receiving a lot of attention, from professionals and economists, as well as by the media and the general public (van Ruth, 2014). For the reason being that inflation is one of the few economic indicators which impact the whole of the economy, from the purchasing power of households to financial markets (van Ruth, 2014). It is also used as a significant benchmark for the state of the economy, with increasing inflation construed as potentially signifying a upsurge in economic activity (van Ruth, 2014). According to van Ruth (2014), the rate of inflation is defined as the year-on-year growth rate of the consumer price index. This implies that inflation does not only consider the price level itself, but also the fluctuations within and the response to inflationary shocks.

Indeed, the importance of inflation elucidates why there is keen interest in the dynamics of the inflation rate, i.e. the increasing or decreasing rate of change of the price level (van Ruth, 2014). This leads to an essential concept of inflationary pressure or shocks, which is defined casually as developments which could bring about higher rates. In order to study inflationary pressure or shocks, or directly predict inflation, it is first necessary to arrive at the basic understanding of inflation dynamics by considering the issue of persistence

emanating form or induced by structural breaks, volatility and non-linearity¹.

Basically, there are two approaches to consider, when modelling inflation: one that takes a theoretical model of how inflation works, and uses this to estimate parameters and construct forecasts (van Ruth, 2014). For example, the New Keynesian Philips curve and monetary theories belong to this category. The other elementary method is more empirical, assuming a rudimentary, general mechanism of inflation drivers and using time series procedures and methods of selecting variables to build models and construct forecasts (van Ruth, 2014). This is thesis takes an even more rudimentary approach, applying a whole range of time series models to the inflation series itself. An indication of how best to predict inflation will largely depend on the performance of these different types models.

In time series modelling, one needs to be acquainted with the properties of the series before selecting a model. Regarding inflation, the well-known properties that inform inflation persistence are structural breaks, long memory, volatility and non-linearities (or regime shifts), among others. The matter relating to time series features and inflation modelling is subject to the series from country to country. These features must be considered when selecting a time series model for a given series in order to understand and control inflation persistence².

¹Accurate characterisation of inflation dynamics is central in the choice of statistical techniques required for appropriate data analysis. Statistical inference, from time series, is typically based upon the assumption of stationarity. Nelson and Plosser (1982), among others, have demonstrated that inferences based on time series that possesses a stochastic or unit root feature are less straight forward than in the case of stationary data. This is because there is a high probability of making unreliable and misleading inferences when the variables are non-stationary.

²As stated by Harvie and Pahlavani (2006, p.181) “it is essential to correctly identify structural breaks in the data for any economy: firstly, to avoid model misspecification and coefficient bias using such data and secondly, to ensure that the test of non-stationarity of the data give correct results”.

A study conducted by Hansen (2001) revealed that a structural break may affect some or all of the model parameters. In this situation if the selected model is not capable of handling these features, then the prediction from such a model may be inaccurate. With the opinion of Morana and Beltratti (2004) the choice between structural breaks and long memory is important for a variety of financial applications, especially in the area of risk measurement, asset allocation and option pricing.

In the existing literature, few studies, especially in Sub-Saharan Africa, have empirically examined inflation persistence originating from structural breaks, long memory, volatility and non-linearity in inflation series, and those that are available are based on linear trends, intercepts or at most structural breaks at fixed points in time. This study is an attempt to bridge this gap. This thesis seeks to make a contribution to the existing literature on inflation studies by proposing a model with potential non-linear trends and structural breaks, where the errors are assumed to be fractionally integrated, $I(d)$ (Cuestas and Gil-Alana, 2016). The study will also incorporate these properties into a state space model with Kalman filter to estimate the threshold effect of inflation on economic growth. These frameworks would be applied to two Sub-Saharan African countries, i.e. Ghana and South Africa, which are countries with Inflation Targeting (IT) policy.

1.2 Structural Breaks, Long Memory and Non-Linearity

The issue of structural change is of substantial importance as far as analysis of macroeconomic time series data such as inflation is concerned. Structural change arises in many time series data for a number of reasons, such as economic crises, changes in institutional appointments, policy vicissitudes

and regime shifts, among others. An accompanying problem in relation to testing the null hypothesis of structural stability against an alternative of a one or two-time structural break. Most significantly, results may be skewed and biased towards the erroneous non-rejection of the non-stationary hypothesis if structural change are present in the data generating process, but are not permitted in the specification of an econometric model, (Perron, 1989, 1997; Leybourne and Newbold, 2003). The economic implication of such a result is to incorrectly conclude that the series under investigation has a stochastic trend process. This therefore means that any shock, whether demand, supply, or policy induced to the variable, will effects on the variable into the very long-run. It is, therefore, necessary to allow for structural breaks in the model specification process in order to obtain more reliable results in the test of non-stationarity³.

Interest in inflation persistence with a long memory process can be traced to the investigation of physical sciences data. Hurst (1951) pioneered the introduction of formal models with a long memory process and were mainly associated with hydrological studies which sort to examine how to regularise the flow of River Nile with respect to its non-periodic (flooding) cycles. This feature was described by Mandelbrot and Wallis (1968) as the “Joseph effect” referring to the biblical incident in which seven years of plenty were to be followed by seven

³Dating of potential break point is assumed to be known a priori conventionally. Test statistics are then constructed by including dummy variables representing different intercepts and slopes, thus extending the standard Dickey-Fuller process (Perron, 1989). Nevertheless, this standard has been criticised, most notably by Christiano (1992), who has contended that this approach nullifies the distribution theory underlying conventional testing. In response, a number of studies have developed dissimilar methodologies for endogenising dates, for example, Zivot and Andrews (1992); Lumsdaine and Papell (1997); Perron (1997) and Bai and Perron (2003). These studies have demonstrated that by endogenously determining the time of structural breaks, bias in the usual unit root tests can be reduced. A class of test statistics which allows for two different forms of a structural break have been proposed by Perron and Perron and Vogelsang (1992) and Perron (1997): namely, the Additive Outlier model, which is more applicable to series exhibiting an abrupt change in the mean (the crash model), and the Innovational Outlier model, which captures changes in a more gradual manner through time.

Perron (1997, p.356), for example argues that “...if one can still can reject the unit root hypothesis under such a scenario it must be the case it will be rejected under less stringent assumption”.

years of scarcity. In this sense, the long memory process refers to observations in the remote past that are highly correlated with observations in the distant future. Long memory in financial markets together with its implications was first studied by Mandelbrot (1971), who also proposed the Hurst's 'rescaled range' statistic to detect long memory behaviour in asset return data. In addition, Mandelbrot (1971) noted that if security prices exhibit long memory which induces persistence, then the influx of new market information cannot be arbitrated away, but generally stay dependently correlated in the market. Also, Lo (1991) contended that standard tests or models such as arbitrage pricing theory and capital asset pricing model are not applicable or redundant if the asset returns exhibit long memory behaviour.

The long memory also designates the high-order correlation structure of a series⁴. Such time series are characterised by separate, but non-periodic cyclical patterns. Non-linear dependence in the mean (or the first moment) of the distribution, is often caused by the presence of long memory dynamics, hence a potentially predictable component in the series dynamics. Modeling and testing non-linearity in inflation series has also attracted a significant amount of interest because non-linear models outperform their linear counterparts. Indeed, a neglected non-linearity will mean that different theories such as Quantity theory of Money will have to be re-evaluated. This means that traditional unit root tests for stationarity would be misleading, if inflation follows a non-linear process (Diebold and Inoue, 2001; Granger and Hyung, 2004)⁵.

A number of researchers have acknowledged the importance of modelling struc-

⁴If a series exhibits long memory, there is persistent dependence even between distant observations.

⁵Non-linearity in inflation may reflect different speed of adjustment toward equilibrium or the level designated by policymakers. This means that the speed of adjustment increases as the deviation of the inflation rate from the equilibrium becomes greater. For countries whose central banks tend to keep inflation within a target range, non-linearity in inflation may come from the response of monetary policy to inflation.

tural breaks, long memory and non-linearities using time series techniques, as they have attempted to develop some models (Anderson and Bollerslev, 1997). For instance, using the ARFIMA model, Gadea et al. (2004) found long memory behaviour in inflation rates of UK (United Kingdom), Italy and Spain. After the researchers had considered the issue of breaks in the series, the long memory parameter was reduced, which was a clear indication that most of the inflation series contained both features as established by other scholars such as Hyung et al. (2006) who used wavelet transformation methods to estimate long memory in Group of Seven: UK, Japan, Germany, France, USA (United States of America), Canada and Italy inflation rates. The investigators found that long memory existed in Germany and Japan series and the phenomenon may just be a consequence of structural changes. Hsu (2005) found evidence that inflation is fractionally integrated, suggesting that the differencing parameter is significantly different from zero and unity for other countries where inflation rates had both long memory and structural changes. Unlike other researchers, Lee (2005) used wavelet transformation method to estimate the long memory parameter in the monthly USA inflation rate. Lee (2005) found that inflation exhibited a degree of stochastic non-stationarity. Thus, it is essential to model and test for the existence of long memory, non-linearity and structural breaks in inflation series of Ghana and South Africa. This would allow us to investigate whether inflation series in Ghana and South Africa contains one, two, three or neither of the features, as these will be useful in the identification of proper inflation model that will produce better forecast for future planning.

1.3 Statement of the Problem

In Ghana, the real domestic product (GDP) was projected to fall from 4.0% in 2014 to 3.4% in 2015, as energy rationing, high inflation and ongoing fiscal indiscipline continued to undermine economic activity. However, as part of the

economic recovery program, Ghana is expected to receive large capital inflows, a possible Eurobond issuance up to \$1.5 billion (of which only \$1 billion would be issued with the World Bank guarantee to restructure its debt); and disbursements from development partners (World Bank Report, 2016a).

On other hand, South African growth has been stuck in low gear with real GDP growth estimated at 2.0% in both 2015 and 2016, due to a combination of domestic constraints and external headwinds emanating from the fall in commodity prices and slowdown of the Chinese economy (World Bank Report, 2016b). The inconsistent growth performance has worsened the already high unemployment, inequality, and macro weaknesses. However, a slight recovery is expected in 2017 with real GDP growth estimated at 2.4% as new electricity supply comes online. Consumer prices in South Africa increased by 4.8% year-on-year in November of 2015, up from 4.7% reported in the previous month and matching market forecasts (World Bank Report, 2016b). Inflation rate in South Africa averaged 9.31% from 1968 until 2015, reaching an all-time high of 20.90% in January of 1986 and a record low of 0.20% in January of 2004 (Trading Economics Report, 2016b).

The recent fluctuations in the economy of these two IT countries imply that inflation of Ghana and South Africa may be highly persistent. The question is whether this persistence is due to inflation series being non-stationary, i.e. having a unit root, or whether it is stationary but exhibits long memory. For this reason, policymakers have been challenged to provide continuous and sustained monetary and fiscal stimuli to support the respective countries' ambitious development plans. Another problem common to Ghana and South Africa (and many other African countries) is the lack of compatibility of sustained economic growth with price stability. The mere fact that so much theoretical and empirical attention is paid to inflation suggests that it is considered an

extremely important economic problem. Needless to say, the exercise of public policy designed to alleviate inflationary pressures requires some knowledge about properties of the series that informs the ever-changing nature of inflation persistence.

Noting the failure by economists to diagnose the root causes of inflation in many cases, Paul Samuelson and Robert Solow commented that: "...Just as generals are said to be fighting the wrong war, economists have been accused of fighting the wrong inflation ..." Samuelson and Solow (1956, p.177). This means that, neglecting the underlying properties that inform the persistency of inflation dynamics could as well lead to losing the battle against inflation. This thesis will hence delve deep into persistency of inflation dynamics emanating from structural breaks, long memory, volatility (or inflation uncertainty) and non-linearity (or regime shifts) using time series techniques in three dimensions.

First, following the research conducted by Gil-Alana and Toro (2002) on estimation and testing of an autoregressive and fractionally integrated moving average (ARFIMA) models in the real exchange rate, it was discovered that an application of parametric methods such as exact maximum likelihood in estimating ARFIMA models requires a Gaussian assumption and correct specification of the short memory parameters. Our study departs from the previous work in several aspects including the estimation of ARFIMA model in the conditional mean. A semi-parametric approach such as Local Whittle (LW) estimator by Robinson (1995) and log-periodogram regression model by Geweke and Porter-Hudak (1983) (GPH), together with a non-parametric modified rescaled range (R/S) analysis (Hurst, 1951), shall be applied to the CPI inflation series of Ghana and South Africa. The advantage of these methods over traditional methods, such as exact maximum likelihood (EML) methods, is that they do

not require the Gaussian assumption. Our study will also examine persistence induced by the presence long memory process, in the conditional mean by extending the ARFIMA model with standard generalised autoregressive conditional heteroscedasticity (sGARCH) and Glosten-Jagannathan-Runkle generalised autoregressive conditional heteroscedasticity (‘gjrGARCH) processes to describe the time-dependent heteroscedasticity under three distributional assumptions.

Second, in the existing literature, few studies have empirically examined the possibility of non-linearity in inflation persistence, especially in Sub-Saharan Africa, and those that are available are classical models based on linear trends, intercepts and/or at most structural breaks at fixed points in time. This thesis is an attempt to bridge this gap. Moreover, to analyse the persistency of inflation dynamics, empirical studies have largely used the unit root testing approach in addition to the vector autoregressive (VAR) model and vector error-correction (VECM) model. There are two underlying assumptions that characterised the use of these models. Firstly, irrespective of whether inflation is above or below its mean or level and notwithstanding of the size of the negative and positive shock that hits inflation, the speed of adjustment of inflation towards equilibrium would remain the same throughout the sample period. Secondly, this asserts that inflation series is normally distributed. However, since the first conjecture uniquely differentiates inflation between unit root and a stationary process, it inhibits a thorough investigation into inflation dynamics. Again, in as much as inflation frequently has a non-normal distribution as a result of outliers, the standard unit root testing method might yield biased estimates for the advantage of unit root instead of robust estimation results. This research challenges these two assumptions and adopts the framework of Cuestas and Cuestas and Gil-Alana (2016). The researcher has sought to make a contribution to the existing literature on inflation persistence

studies by proposing a model with potential non-linear trends and structural breaks, where the errors are assumed to be fractionally integrated $I(d)$. This framework has been applied to two Sub-Saharan African countries: Ghana and South Africa, which are countries with IT policy⁶.

Lastly, the thesis has addressed the issue of economic growth and inflation by investigating the relationship between inflation and other macroeconomic variables such as GDP by growth rate per capita, total exports of goods and services and total imports of goods and services, total investments and growth rate in population etc., by adopting a state space model of unobserved components with Kalman filter. This idea contributes to knowledge by bringing to the attention, the limitation of ordinary least squares (OLS)/generalised least squares (GLS) estimation and non-linear least squares (NLLS) techniques, among others.

1.4 Research Questions

This study has attempted to answer the following research questions:

- Do structural break(s) exist in CPI inflation series of Ghana and South Africa? What could be the consequence of a neglected structural break(s) in an econometric modelling, unit root testing and fractional integration, especially for statistical inference and forecasting?
- Does CPI inflation series of Ghana and South Africa follow a Martingale Difference Sequence?
- What is the optimal level of inflation that is consistent with economic growth in Ghana and South Africa?

⁶The proposed model with non-linear trend is based on Chebyshev polynomial in time (Hamming, 1973; Smyth, 1998).

1.5 Aim of the Study

The aim of this thesis is to empirically investigate persistency of inflation dynamics by taking cognisance of structural breaks, long memory, volatility and non-linearity in CPI inflation series of Ghana and South Africa, in relation to economic growth.

1.5.1 Objectives of the study

In furtherance of achieving this aim, the following specific objectives have been pursued:

- to determine the existence or absence of structural breaks and their significance in modelling inflation series of Ghana and South Africa;
- to analyse the long memory properties of inflation series by estimating the fractional differencing parameter using recently developed robust techniques;
- to model the persistence in the conditional mean by extending the ARFIMA process with two different conditional heteroscedastic models: sGARCH and 'girGARCH processes to describe time-dependent heteroscedascity under three different distributional assumptions;
- to model the CPI inflation series with non-linear deterministic trends and the relationship between CPI inflation and economic growth with fractionally integrated errors;
- to model the unobserved components and threshold effect of inflation-growth nexus using state space model with Kalman filter.

Each analytical chapter in this thesis has explored the above objectives with a view to providing answers to the research questions. In addition, the expectations are that answers to the objectives reflect the contribution of this study to

current literature, both on the relationship between growth rate by GDP per capita, imports and exports of goods and services, CPI inflation, finance and econometrics, in general.

1.6 Scope of the Study

The discussion of structural breaks, long memory, volatility and non-linearities in inflation series, has engaged macroeconomic researchers over a long period (Diebold and Inoue, 2001; Granger and Hyung, 2004; Lee, 2005). Advanced models based on structural breaks, long memory and non-linearities for time series modelling, especially for macroeconomic time series, have been developed over the last 5-15 years by Granger and Joyeux (1980) and Hosking (1981). However, despite the attention this topic has received, there is limited research work based on the experiences from Africa.

The empirical analyses in this study provide an extensive examination of the presence and features of structural breaks, long memory, volatility and non-linearities in explaining inflation persistence in CPI inflation series of Ghana and South Africa. As such, this thesis sits within the broader context of the empirical debate concerning whether CPI inflation is stationary or follows a unit root process. This thesis has in addition, investigated the presence or absence of long memory and non-linearities in CPI inflation of the two IT countries under study.

One of the most promising avenues of research examined in this thesis is the estimation of fractionally integrated parameter, which is the speed of adjustment of the conditional mean to an equilibrium position after a shock. This study employs a LW method developed by Robinson (1995) and log-periodogram regression model by Geweke and Porter-Hudak (1983) to examine the

presence of long memory. In the literature, semi-parametric estimators are considered to be more robust than parametric estimators such as EML by Sowell (1992) and R/S analysis (non-parametric), because semi-parametric estimator can detect long memory even in the presence of structural breaks. Another crucial point when estimating with parametric approaches is the issue of Gaussian assumption and correct model specification, otherwise the estimates are likely to be inconsistent, hence the adoption of LW estimator in this research. This thesis has also modelled the fractional integration process (or persistence) in the conditional mean by extending the ARFIMA process with two different conditional heteroscedastic models such as sGARCH and 'gjrGARCH processes to describe time-dependent heteroscedascity under three different distributional assumptions.

Another contribution of this thesis to the existing literature on inflation persistence is the proposed model with potential non-linear trends based on the Chebyshev polynomial in time, where the errors are assumed to be fractionally integrated, $I(d)$ (Cuestas and Gil-Alana, 2016). This framework is applied to two Sub-Saharan African countries: Ghana and South Africa. The study has as well addressed the issue of inflation-growth nexus by adopting a different approach involving a state space model of unobserved components with Kalman filter algorithm developed by Koopman et al. (2006). This idea contributes to knowledge by bringing to the attention the limitation of OLS and GLS estimation techniques where the errors are assumed to be following a long memory process. The researcher has been motivated to examine the persistency of inflation dynamics of CPI inflation series of Ghana and South Africa on the grounds of economic policy and statistical inference.

1.7 Structure of the Study

This thesis is subdivided into eight chapters, with Chapter 1 providing the background and rationale of the study. The chapter outlines the three main properties of financial time series: structural breaks, long memory and non-linearities, among others, which has been the subject of investigation and basis for developing new models capable of explaining persistency of inflation dynamics. The problem statement, research questions, aim and objectives, together with the scope of the study, covering the methodologies and the justification of the various time series techniques and research gaps, have also been highlighted in this chapter.

Chapter 2 deals with general literature review relevant to this thesis, covering the theories on inflation modelling, inflation targeting and persistence of inflation, structural breaks, long memory and non-linearities. Traditional techniques used in handling such properties and their shortcomings have been presented in this chapter. A justification regarding the choice of time series techniques and the new models that have been employed in this thesis have also been treated in the subsequent sections of the chapter.

Chapter 3 presents the inflation experience of Ghana and South Africa and evidence of existence and non-existence of structural breaks in relation to inflation persistence using different approaches: Dickey and Fuller (1979), Augmented Dickey Fuller test (ADF), Phillips and Perron (1988) (PP), Zivot and Andrews (1992), (ZA), Kwiatkowski et al. (1992), (KPSS), Bai and Perron (1998), (BP) and fractional integration method proposed by Gil-Alana (2008). Implications of such breaks in a model building process have also been discussed in this chapter.

Chapter 4 addresses the issue and sources of long memory and also puts for-

ward a justification for adopting semi-parametric estimators over the parametric estimators. Approaches such as R/S analysis by Hurst (1951), GPH by Geweke and Porter-Hudak (1983), EML by Sowell (1992) and LW by Robinson (1995) have been employed. This chapter has also been concerned with modelling the fractional integration process using the ARFIMA model to describe inflation persistence under three different distributional assumptions: Normal Distribution (Norm), Student- t Distribution (STD) and Generalised Error Distribution (GED).

Chapter 5 models the persistence in the conditional mean of CPI inflation series of Ghana and South Africa by extending the ARFIMA process with time-dependent heteroscedastic models - sGARCH and 'gjrGARCH under three distributional assumptions: Norm, STD and GED.

Chapter 6 proposes a new model for inflation persistence in the context of non-linear deterministic trends with fractionally integrated errors, based on Chebyshev polynomial using a framework developed by Cuestas and Gil-Alana (2016). The distinction between this new model and classical models have been covered in this chapter.

Chapter 7 explores and determines the relationship between CPI inflation rates and economic growth by adopting a state space of unobserved components with Kalman filter developed by Koopman et al. (2006) in the STAMP (Structural Time Series Analyser, Modeller and Predictor) package.

Chapter 8 provides some concluding remarks regarding the empirical research presented in the thesis. A summary of major findings, contributions of the thesis and an outline of future research directions, are also presented in this chapter.

To conclude, the prime aim of monetary policy is to keep inflation low and at stable levels as much as possible. Thus, to design an optimal monetary policy, and evaluate its efficiency, monetary authorities in developing countries monitor inflation persistence, which reflects their ability to recover stability after a shock to price levels (Belkhouja and Mootamri, 2016). The persistency of inflation dynamics has been subjected to investigation in relation to the IT policy introduced by Ghana and South Africa and its ramification on economic growth.

Chapter 2

Literature Review

2.1 Introduction

The purpose of this chapter is to review the general literature on the contentious debate on whether or not CPI inflation series follow a stationary or a unit root process, by paying special attention to the theories behind inflation modelling, inflation targeting and inflation persistence and related issues such as structural breaks, long memory, volatility and non-linearities. Traditional techniques used in handling such properties and their shortcomings have also been presented in this chapter. A justification regarding the choice of time series techniques, estimation methods and the new models that have been employed in this study have been reviewed in the subsequent sections.

2.2 Theoretical Framework of Inflation Modelling in Relation to Monetary Policies

A number of views on inflation modelling have emerged in literature, at both empirical and theoretical levels. Among such theories are the Monetarist and the Structuralists. The monetarist believes in policies that are aimed at restricting money supply. Accordingly, the monetarist view suggests that inflation is a matter of undue aggregate demand. As Milton Friedman describes it, "Inflation is always and everywhere a monetary phenomenon" (Friedman, 1970, p.11). From the perspective of the monetarist, therefore, unchecked expansion in aggregate demand, motivated by government deficits, financed in part by increases in money supply and directed credit allocation, is the main perpetrator of inflation.

Studies by Lawson (1966), Ahmed (1970), Ewusi (1977), Steel (1979), Chhibber and Shafik (1990), and Adu and Marbuah (2011), among others, asserted the monetarist hypothesis for Ghana. The monetarist dispute is particularly evident in the third phase of inflation episode in Ghana from 1972 to 1982, in accordance with a study conducted by (Alagidede et al., 2014). Expansionary monetary policy coupled with large external inflows exerted an upward pressure on domestic prices (Chhibber and Shafik, 1990). Accordingly, the period 1973-1982, for instance, culminated the years where inflation averaged 54.5% per annum, and in 1983, the Ghanaian economy recorded the highest ever inflation rate of approximately 123%. They projected a way out of demand-induced inflation courtesy monetarist theory (Alagidede et al., 2014).

Monetarists argue that the solutions are inherent in the causes, i.e. monetary and fiscal restraint. Should these fail to moderate inflation, a second best solution is to fasten restrictions and price controls that distort relative prices.

Consequently, in 1983, Ghana launched the Economic Recovery Programme (ERP) and this was followed in 1986 by the Structural Adjustment Programme (SAP). Among other goals, these two reform packages were aimed at reversing the decline in the Ghanaian economy by opening several sectors of the economy that had hitherto been rigidly controlled. Among the policies is the liberalisation of interest rates, exchange rate reform, stemming the tide of monetary growth to cure inflation, curtailing trade deficits and introducing market administered prices in the financial sector. Following the adoption of the ERP, average annual inflation plummeted from a high of 123% in 1983 to 39.7% in 1984 and a further 10.3% in 1985. According to (Alagidede et al., 2014), inflation still remains high in spite of implemented programmes such as ERP and SAP, suggesting the possibility of other existing factors other than extreme demand pressure.

In the view of the structuralists, Sowah and Kwakye (1993) argue that supply-side dynamics like food prices are necessary in price variations in Ghana. Therefore a basic weakness in the domestic production and industrial base, together with structural rigidities in the agricultural sector translate to high food prices which is translated into the general price level. Furthermore, domestic policy discrepancy and output volatility appear to be more important in explaining both the short and long-run dynamics of inflation than monetary factors (Sowah, 1994, 1996). Applying consistent fiscal policies, and attention to the supply inflexibilities, argue Sowah (1994), is the key to a successful domestic price stability structure. At one level, a ceasefire between the monetarist and structuralists views can be sought. Though structural hold-ups may form the root cause of the inflation problem, demand-supply driven factors cannot be left out (Alagidede et al., 2014). Next are studies on the two views either separately or together in trying to explain price developments in Ghana.

Thus, specific drivers of inflation such as exchange rate depreciation, wages, and exogenous shocks in the domestic food supply, petroleum prices, and government fiscal policy, among others are frequently mentioned in the literature. Dordunoo (1994) argues that the rapid exchange rate depreciation and the subsequent hikes in import prices are inflationary, and (Ocran, 2007) opines that the devaluation of the Ghana Cedi by 991% in 1983 was partly the reason for the 123% inflation recorded that year. Adu and Marbuah (2011) present an interesting amalgamation of the two views. In their investigation of the determinants of inflation for the period 1960 to 2009 in Ghana, the authors claimed that fiscal deficits, money supply and production constraints exert pressure on domestic price levels. They argue that domestic production constraints are critical while monetary growth in the long-run underlie most of the inflation experience in the recent phase. Hence they conclude that, eliminating constraints to production and controlling unrestrained monetary growth is significant to the success of the disinflation effort in Ghana (Alagidede et al., 2014).

Up until early 1970s, South Africa experienced low levels of inflation, but the oil price shock in 1973 led to a hike in inflation both globally and domestically, hence inflation became a policy priority in South Africa (Moolman and du Toit, 2004). Dr De Jongh, in his tenure as a Reserve Bank Governor, instituted a number of extra direct controls such as a ceiling on advances, deposit rate controls, exchange controls, import deposits and some direct consumer credit controls, in an effort to control the persistent increase in money supply and inflationary tendency (Kock, 1998).

The monetary controls between 1960s and 1970s gave way in the 1980s, to a general acknowledgment of the need to eliminate as many limitations as possible in a move towards market-oriented policy. The Thatcher government in the UK and the Reagan administration in the USA in the 1980s, triggered a

definite shift across the world in favour of market-oriented policy measures (Moolman and du Toit, 2004).

This modification in policy tactic was further encouraged by the liberalisation of international financial markets. South Africa followed suit by implementing a more market-oriented approach. The development gained momentum after the report of the De Kock Commission in 1984/1985. In line with the international trend at the time, the Reserve Bank, under Dr De Kock, started to line up its policies with expansions in markets, rather than to try to force markets in a pre-determined direction. More emphasis was placed on using interest rate adjustments rather than direct credit extension restrictions. Accordingly, this was a very difficult decade in the history of the Reserve Bank, operating with controlled indirect intervention in the face of prevalent international hostility and growing resistance, consequently the economic and racial policies of the government at the time (Moolman and du Toit, 2004). In the study conducted by Moolman and du Toit (2004), after the first democratic election in 1994, political hindrances were dropped.

The removal of sanctions against South Africa and the scrapping of exchange controls on foreigners, open South Africa to the world of financial market movements. During the 1990s inflation rates declined steadily from 15.3% in 1991 to 5.2% in 1999. Following countries such as New Zealand, Canada, Sweden and the UK, South Africa adopted an inflation-targeting regime in February 2000. The preliminary target for average CPIX inflation was 3% to 6% for 2002 and 2003, and between 3% and 5% for 2004 and 2005. The CPIX inflation was fixed back slowly from levels above 8% in mid-2000 and dropped to levels within the target range in September and October 2001 when it was 5.8% and 5.9%, respectively. Though, following the dramatic depreciation of the rand from R8.63/USD in September 2001 to R11.61/USD in January 2002, CPIX

intensified to levels of 12% by the end of 2002.

Notwithstanding sustained controlled monetary policy and a cumulative increase in the repo rate of 400 basis points, CPIX inflation has not been within the target range since October 2001. The Governor of the Reserve Bank and the Minister of Finance agreed that the inflation target would not be attained in 2002 or 2003, and subsequently amended the target for 2004 to 3%-6%. The fiasco to meet the target has underscored South Africa's price-vulnerability as a small open economy and has also raised questions about the efficacy of monetary policy (Moolman and du Toit, 2004).

From the synopsis given above, it appears that price increases confronted by Ghana and South Africa - and many other countries in Sub-Saharan Africa, are supply-side problems, mostly externally enforced. Hence, they cannot be held liable on excessive domestic aggregate demand (McKinley, 2008). This implies that modelling inflation series of Ghana and South Africa could be more of a structuralist problem than monetarist.

2.3 Inflation Targeting and Inflation Persistence

The principal objective of monetary policy is to stabilise prices and recover growth. Therefore, in order to optimise and assess the worth of monetary policy, monetary authorities keep an eagle eye on inflation persistence (Belkhouja and Mootamri, 2016). Indeed an estimate of persistence in inflation captures the long-run effects of shocks on the level of inflation. This means that the measure of persistence to a large extent, indicates the likely impact of shocks that may have come about as a result of structural breaks (demand/supply shocks), long memory, volatility (or inflation uncertainty) and non-linearity (or regime shifts), on the future trajectory of inflation in terms its impact on fu-

ture inflation and also its ability to recover stability. These properties have been subjected to investigation in relation the IT policy introduced by Ghana and South Africa.

New Zealand in 1990 was the first country to adopt the IT policy as an official monetary policy, and the IT policy has subsequently been implemented by a number of countries around the world, including Ghana and South Africa (Heintz and Ndikumana, 2011). IT encompasses a statement of an inflation target by the central bank, where they mostly target a narrow range of inflation rates. Central banks then employ monetary tools, such as M3, M2 and M1 in an effort to keep inflation within the target (Heintz and Ndikumana, 2011)⁷. The IT framework emphasises an augmented responsibility and obligation of the central bank, through the appraisal of its performance in achieving the target and openly disclosing the reasons for any deviation (Heintz and Ndikumana, 2011).

Recent studies have shown that there is no association between inflation targeting and growth and its volatility. For instance, Ball and Sheridan (2003) compared seven Organisation for Economic Co-operation and Development (OECD) countries with adopted IT and 13 countries without IT policy, and found that the countries with IT policy have at least the identical level of growth on an average. Also, among developing countries, IT has helped to lessen volatility in output growth (Nicoletta and Laxton, 2007).

The empirical evidence of regime fluctuations on inflation persistence has also been studied by Benati (2008). He estimates a small-scale New-Keynesian model for major industrial countries over various sub-samples by means of

⁷M1 denotes money supply aggregate of a country that includes physical money-both paper and coin; M2 denotes money aggregate that includes assets that are highly liquid but are not cash; M3 denotes aggregate money supply that includes M2 as well as large time deposits.

Bayesian methods. His main result is that the degree of inflation persistence, as captured by the coefficient of lagged inflation in a hybrid Phillips curve, drops significantly towards zero once a credible new monetary regime is in place.

For emerging market economies such as Ghana and South Africa, however, the effects of IT are mixed. Goncalves and Salles (2008) found that developing countries implementing IT experience a significant decline of inflation and in growth volatility. Using a variety of tendency score matching methods, Lin and Ye (2009) reinforced these findings. According to their results, both the level of inflation and its volatility fall after the adoption of IT.

Recently, however, Brito (2010), and Brito and Bystedt (2010) extended this line of research by taking account of common time effects. These authors found that IT has no effect on the level and the variance of inflation in emerging countries. Less supportive evidence for a reduction in inflation persistence is provided by Cecchetti and Debelle (2006), who concluded that the major change in the inflation process in the past two decades is a decline in the mean of inflation.

Levin et al. (2004) argued that the adoption of IT lowered the degree of inflation persistence in major industrial countries. The study by O'Reilly and Whelan (2005) found no change in inflation persistence over the sample period, while Tillmann (2012) provided evidence in favour of a decline in persistence since 1999. Siklos (2008) employed AR(1) model and estimated the first-order autoregressive AR(1) process for inflation for a set of emerging market countries and included a dummy variable indicating the adoption of IT. He found a reduction in inflation persistence only in a handful of emerging economies after adopting IT.

Filardo and Genberg (2010) studied the performance of an IT policy in Asia and the Pacific. They also analysed the development of inflation persistence, measured as the AR(1) coefficient for inflation and in terms of an integrated moving average representation, and found a reduction in persistence only for Korea, New Zealand, and Australia. However, in other countries, like Thailand, the Philippines, and Indonesia, persistence increased. Miles (2008) investigated the relationship between inflation volatility and inflation persistence in the Canadian pre and post inflation targeting periods, while Kontonikas (2004) focused on UK. Indeed Kontonikas (2004) and Miles (2008) examined the impact of IT on inflation uncertainty in the UK and Canada, respectively using augmented GARCH models together with a dummy variable for the post-IT period, incorporated in the conditional variance equation. Their results indicated a significant negative impact of IT on inflation uncertainty in the respective countries, which suggests that the policy has had a significant impact in decreasing inflation uncertainty.

Ghana joined the family of IT countries in March 2007 backed by the Bank of Ghana Act 2002 (Act 612) Section 3 Sub-section 2, which grants the central bank operational independence while Section 27 Sub-section 1 also disseminates the establishment of Monetary Policy Committee for the execution of IT framework. The IT framework was based on the belief that policy is designed to target inflation through an inflation forecast (Bank of Ghana, 2014). The Bank of Ghana (BoG), has since 2002 imitated the policy of price stability, with mostly low inflation and a fairly stable exchange rate (Sowa and Abradu-Otoo, 2007). But Quartey (2010) noted that although the central bank has been pursuing low inflation policies since 2002, notwithstanding, it does not follow an explicit inflation targeting framework. However, the low inflation policy framework mimics an inflation targeting regime in which a specific level of inflation is set and targeted together with the central bank and the Ministry of

Finance, but the target does not include the customary modelling and minimising of the loss functions as is typically done under inflation targeting regimes (Sowa and Abradu-Otoo, 2007).

South Africa embraced the Inflation Targeting Monetary Policy Framework (ITMPF), in February 2000, for the purposes of creating an environment favourable to low stable inflation. There is a general compromise that low, stable inflation should be the chief objective of monetary policy, especially since price stability is considered by main stream economists as a necessary precondition for faster sustainable growth. Stability is crucial for creating an environment that is conducive for the long-term financing of government and private sector debt including labour intensive industries such as construction and small to medium enterprises. Nevertheless long term planning entails agents to view central bank's forecast of inflation and policy stance in curtailing inflationary pressure as being credible (Bevilaqua et al., 2008; Miles, 2008). The IT monetary policy framework is considered by many mainstream economists as the ideal vehicle for achieving the low, stable inflation objective.

Kaseeram (2012) attempted to debate, project and assess whether inflation volatility and persistence for South Africa has improved since the adoption of inflation targeting, by accounting for structural breaks in the inflation series arising from the global fall in inflation rates via the Bai and Perron (2003), Lumsdaine and Papell (1997), and Andrews and Ploberger (1994) multiple structural breaks test. In his research, Kaseeram (2012) investigated whether inflation volatility and inflation persistence have declined since the adoption of IT in South Africa, since IT purports to anchor expectations around a target band, using GARCH, GARCH-M and AR (2). His study found no significant changes in inflation volatility and persistence over the pre and post IT periods. Analogous to the methods used by Kontonikas (2004), and Miles (2008), who

utilised GARCH and GARCH-M models together with dummy variables for the pre and post IT periods, to study volatility changes in the monthly inflation rates of UK and Canada respectively, and found a significant, negative impact of IT on inflation uncertainty in the respective countries, which suggests that the policy has had a significant impact in decreasing inflation uncertainty. Gupta and Uwilingiye (2007), and Gupta et al. (2010) investigated the shift in inflation volatility over the pre and post inflation targeting period for South Africa. Conversely, both studies used different methods to the study conducted by Kaseeram (2012), with the previous study using the factor VAR approach and the latter employing the lesser known Saphe Cracking model which is extensively used in the engineering sciences.

Overall, the results obtained from the previous studies for the two countries, might be problematic since a number of financial time series like inflation rate, is usually fractionally integrated, hence determination and incorporation of structural breaks in the context of fractional integration, might improve the results. Beyond understanding the factors responsible for inflation dynamics in the two IT countries, the thesis has gone ahead and examined the effect of IT monetary policy across two emerging countries: Ghana and South Africa, in dealing with persistence in the context of fractional integration, by paying attention to structural breaks (shocks), volatility (inflation uncertainty), nonlinearities or regime shifts among others, using Gil-Alana (2008).

2.4 Unit Root Tests without Structural Breaks

2.4.1 Augmented Dickey-Fuller (1979) test

According to empirical literature, the standard and well accepted method of detecting non-stationarity is to examine the tests for unit root using the Dickey

and Fuller (1979) (ADF). In order to investigate the stationary properties of the data based on the ADF test, an analysis of each variable is done using a unit root test based on the following equation:

$$\Delta y_t = \mu + \beta t + \alpha y_{t-1} + \sum_{i=1}^k c_i \Delta y_{t-1} + \varepsilon_t \quad (2.1)$$

where y_t is the time series being investigated, y_{t-1} is the lagged level of the series, μ is a constant, t is a time trend variable, β is the coefficient on the time trend, Δ represents the first difference operator, and k is the number of lags which are added to the model to ensure white noise residuals denoted by ε_t . The Akaike Information Criterion (AIC) and Schwartz Information Criterion (SIC) are used to determine the optimal length k .

The ADF test is mainly concerned with the estimate of α in Equation (2.1) by testing the hypothesis of $H_0 : \alpha = 0$ against $H_0 : \alpha \neq 1$. The rejection of the null hypothesis in favour of the alternative hypothesis suggests that y_t is stationary and integrated of order zero, $I(0)$. If the unit root for the first difference is rejected, then the first difference is stationary and the variable is integrated of order one. In order to ascertain the stationarity of the variables under study, the researcher began by applying the ADF test. The null hypothesis of a unit root is rejected if the absolute value of the test-statistic or t -statistic for α (in absolute value) is greater than the critical value.

2.4.2 Phillips-Perron (1988) test

Since the ADF test is known to be a low power test that is biased towards non-rejection of the unit root hypothesis, especially with short term spans of the data, the researcher then applied a Phillips and Perron (1988) (PP) unit root

test. The test regression for the PP test is given by:

$$\Delta y_t = \beta D_t + \pi y_{t-1} + \mu_t \quad (2.2)$$

where Δ is the first difference operator, D_t is the time trend, β is the coefficient on the time trend, y_{t-1} is the lagged level of the series and μ_t denotes white noise residuals. The PP test corrects for any serial correlation and heteroscedasticity in the errors $\mu_t \sim I(0)$ of the test regression by directly modifying the test statistics $t_{\pi=0}$ and $T_{\hat{\pi}}$. These modified statistics, denoted by Z_t and Z_π are respectively given by:

$$Z_t = \left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2} \right) \cdot t_{\pi=0} \frac{1}{2} \left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2} \right) \cdot \left(\frac{T - SE(\hat{\pi})}{\hat{\sigma}^2} \right) \quad (2.3)$$

$$Z_\pi = T_{\hat{\pi}} - \frac{1}{2} - \frac{T^2 - SE(\hat{\pi})}{\hat{\sigma}^2} (\hat{\lambda}^2 - \hat{\sigma}^2) \quad (2.4)$$

The terms $\hat{\sigma}^2$ and $\hat{\lambda}^2$ are consistent estimates of the variance parameters:

$$\sigma^2 = \lim_{T \rightarrow \infty} \sum_{t=1}^T E[\mu_t^2] \quad \text{and} \quad \lambda^2 = \lim_{T \rightarrow \infty} \sum_{t=1}^T E[T^{-1} S_T^2] = LRV \quad \text{with} \quad S_T = \sum_{t=1}^T \mu_t.$$

The sample variance of the least squares residual $\hat{\mu}_t$ is a consistent estimate of σ^2 and the Newey-West long-run variance estimate of μ_t using $\hat{\mu}_t$ is a consistent estimate of λ^2 . Under the null hypothesis that $\pi = 0$, the PP, Z_t and Z_π statistics have the same asymptotic distribution as the ADF t -statistics and normalised bias statistics. A point worthy of note is the fact that ADF and PP tests are asymptotically equivalent, but may differ substantially in finite samples due to the different ways in which they correct the serial correlation in the test regression.

2.4.3 Kwiatkowski-Phillips-Schmidt-Shin (1992) test

The KPSS test proposed by Kwiatkowski et al. (1992) is a Lagrange multiplier (LM) used to test the null hypothesis that a given observable series is level stationary or stationary around a deterministic trend⁸. This test takes the null hypothesis as a stationary process against the alternative hypothesis of unit root process (Pfaff, 2008), and it is given by:

$$y_t = Dt + r_t + \varepsilon_t \quad (2.5)$$

where r_t is a random walk, i.e. $r_t = r_{t-1} + \mu_t$, and the error term μ_t is assumed to be independent and identically distributed (i.i.d.): $(0, \sigma_{\mu^2})$; with D_t and ε_t denoting the deterministic trend and a stationary error, respectively. The test statistic is constructed as either the series y_t is regressed on only a constant term (level) and/or a constant with a deterministic trend (level and trend) depending on whether one wants to test level or trend stationarity. Let the partial sum series of the residuals ε_t from the regression model be:

$$S_t = \sum_{i=1}^t \hat{\varepsilon}_t, \quad t = 1, 2, 3, \dots, T \quad (2.6)$$

Then the KPSS test statistic for the null hypothesis of stationarity is given by:

$$LM = \frac{\sum_{i=1}^t S_t^2}{T^2 \hat{\sigma}_\varepsilon^2} \quad (2.7)$$

where $\hat{\sigma}_\varepsilon^2$ is an estimate of the error variance of ε^2 from the regression model.

The optimal weighting function which corresponds to the Bartlett window $w(s, l) = \frac{1-s}{l+1}$ is used as suggested by KPSS to estimate the long-run variance $\hat{\sigma}_\varepsilon^2$, defined by:

$$\hat{\sigma}_\varepsilon^2 = s^2(l) = T^{-1} \sum_{t=1}^T \hat{\varepsilon}_t^2 + 2T - 1 \sum_{s=1}^l 1 - \frac{s}{l+1} \sum_{t=s+1}^T \hat{\varepsilon}_t \hat{\varepsilon}_{t-1} \quad (2.8)$$

⁸Considering a given series, y_t , the KPSS tests the null hypothesis of stationarity, $H_0 : y_t = I(0)$ against an alternative hypothesis of unit root given by $H_1 : y_t = I(1)$.

where l is the lag truncation parameter. In this study $l = 4\lceil \frac{n}{100} \rceil^{0.25}$. The approximate upper tail critical values of the asymptotic distribution of KPSS test are taken from KPSS. Engle (2000) cautions about the use of the KPSS test because of its lack of power. Caner and Killian (2001) indicated that the KPSS test shows size distortions when the stochastic process is close to non-stationarity. Chen (2002) also examined the behaviour of the KPSS test in the presence of structural breaks and found that the test has the power to reject the null hypothesis of stationarity of the series in the presence of breaks. In another empirical study, Otero and Smith (2005) investigated the effect of the KPSS test in the presence of an outlier. They found that the power of the KPSS test to reject the null hypothesis of stationarity falls when the series has unit root with outliers. To avoid false conclusions, the Zivot and Andrews (1992) and Bai and Perron (1998) tests which are capable of handling data with breaks among others, can be employed.

2.5 Unit Root Tests with one Structural Break

2.5.1 Zivot and Andrews (1992) test

Zivot and Andrews (1992) (ZA) suggested that the break point in the series should not be determined on the basis of prior observations of the data. Based on the idea of Perron (1989) an alternative unit root test, simply ZA test, was proposed. The ZA test allows for one estimated break in the trend function under the alternative hypothesis. The null hypothesis of unit root of the ZA test is given by:

$$H_0 : y_t = \mu + y_{t-1} + \varepsilon_t \quad (2.9)$$

where y_t is the time series being tested; μ is the mean or the constant and ε_t is white noise. Under the alternative hypothesis of trend stationarity with one-time break occurring at an unknown point in time, the test considers three

different models defined as follows: H_1 :

$$\text{Model A : } y_t = \mu + \alpha y_{t-1} + \beta t + \gamma DU_t + \sum_{i=1}^k c_i \Delta y_{t-1} + \varepsilon_t \quad (2.10)$$

$$\text{Model B : } y_t = \mu + \alpha y_{t-1} + \beta t + \theta DT_t + \sum_{i=1}^k c_i \Delta y_{t-1} + \varepsilon_t \quad (2.11)$$

$$\text{Model C : } y_t = \mu + \alpha y_{t-1} + \beta t + \gamma DU_t + \theta DT_t + \sum_{i=1}^k c_i \Delta y_{t-1} + \varepsilon_t \quad (2.12)$$

where y_t is the time series being tested, μ is the mean or the constant, t is the time trend variable, Δ denotes the first difference operator, k is the number of lags which are added to the model to ensure that the residuals ε_t are white noise, DU_t represents an indicator dummy variable for the mean shift occurring at the break date TB , and DT_t is an indicator dummy variable corresponding to the trend shift. The functions DU_t and DT_t are respectively given by:

$$DU_t = \begin{cases} 1, & \text{if } t > TB, \\ 0, & \text{otherwise.} \end{cases}$$

and

$$DT_t = \begin{cases} 1 - TB, & \text{if } t > TB, \\ 0, & \text{otherwise.} \end{cases}$$

The error term ε_t is assumed to be serially uncorrelated. If the error term is autocorrelated, enough lagged difference terms are to be included, so that the error is serially independent. Model A defined by Equation (2.10) allows for a one-time change in the intercept (level). Model B defined by Equation (2.11), allows for a one-time change in the trend and Model C defined by Equation

(2.12), allows for a one-time change in both the intercept (level) and the trend. With ZA test, the break points are endogenously determined within each of the models. The test considers all points as potential candidates for a breaking point, but the final break point suggested by each model is selected recursively by choosing the value of TB with the minimum absolute value of the one-sided t -statistic for α .

2.6 Unit Root Tests with Multiple Structural Breaks

2.6.1 Bai and Perron (1998) test

Macroeconomic time series data such as inflation rates could be subject to more than one structural break. For this reason, the Bai and Perron (1998) (BP) test provides an option for testing for multiple structural breaks together with a comprehensive analysis which precludes the presence of trending regressors. In this context the CPI inflation data set fits very well since it exhibits no trending behaviour. Following the extensive work by Andrews (1993), Andrews and Ploberger (1994), Kuan and Hornik (1995), Ben and Jouini (2003), Ben et al. (2004), and Jouini and Boutahar (2005), the model and the test statistics of the BP technique are briefly discussed next.

Consider the following structural break model where all the coefficients are subject to change:

$$y_t = z_t' \delta_j + \mu_t, \quad t = T_{j-1} + 1, \dots, T_j \quad (2.13)$$

for $j = 1, \dots, m + 1$, where $T_0 = 0$, $T_{m+1} = T$, y_t is the observed dependent variable, $z \in R^q$ is a vector of covariates, δ_j is the corresponding vector of coefficients with $\delta_j \neq \delta_{j+1}$, ($1 \leq j \leq m$), μ_t is the disturbance and the parameter m denotes the number of changes or breaks (Ben and Jouini, 2003). The break

dates (T_1, \dots, T_m) are explicitly treated as unknowns.

The estimation method considered is based on the least-squares principle proposed by Bai and Perron (1998). For each m -partition (T_1, \dots, T_m) denoted by T_j , the associated least-squares estimate of δ_j is obtained by minimising the the sum of squares residuals:

$$\sum_{i=1}^{m+1} \times \sum_{t=T_{i-1}+1}^{T_j} (y_t - z_t' \delta_i)^2$$

under the constraint that $\delta_i \neq \delta_{i+1}$ for $(1 \leq i \leq m)$.

We define $\lambda_j = \frac{T_j}{T}$ with $0 \leq \lambda_1 \leq \dots \leq \lambda_m \leq 1$. According to Ben and Jouini (2003), BP imposes some constraints on the possible values of the break dates. They define break dates as a set for some arbitrary small positive number ε : $\Lambda_\varepsilon = (\lambda_1, \dots, \lambda_m); |\lambda_{i+1} - \lambda_i| \geq 1 - \varepsilon$, in order to restrict each break date to be asymptotically distinct and bounded from the boundaries of the sample, as there are enough observations to identify all subsample parameters:

$$\delta = (\delta_1', \delta_2', \dots, \delta_{m+1}')' \quad (2.14)$$

Let $\hat{\delta}(T_j)$ denote the ensuing estimate. Substituting $\hat{\delta}(T_j)$ in the objective function in Equation (2.14), and denoting the resulting sum of squared residuals as $S_T(T_1, \dots, T_m)$, the estimated break dates $(\hat{T}_1, \dots, \hat{T}_m)$ are such that:

$$(\hat{T}_1, \dots, \hat{T}_m) = \arg \min_{T_1, \dots, T_m} S_T(T_1, \dots, T_m) \quad (2.15)$$

where the minimisation of the objective function in Equation (2.16) is taken over all partitions $(\hat{T}_1, \dots, \hat{T}_m)$ ⁹. The break date point estimators are the global

⁹ $[\varepsilon T]$ denotes the minimal number of observations in each segment. According to Bai and Perron (2003), if the tests are not required and the estimation is the sole concern, then the

minimisers of the objective function. Finally, the regression parameter estimates are the associated least-squares estimates at the estimated m -partition T_j , i.e. $\hat{\delta} = \hat{\delta}(\hat{T}_j)$. For the empirical illustration, the researcher uses the efficient algorithm developed by Bai and Perron (2003) based on the principle of dynamic programming, which allows global minimisers to be obtained using a number of residuals that is of order $O(T^2)$ for any $m \geq 2$ (Ben and Jouini, 2003).

BP considered a test of the null hypothesis of l structural breaks against the alternative that an additional break exists. The test would be based on the difference between the sum of squared residuals obtained with l breaks and that obtained with $l+1$ breaks. The limiting distribution of this test statistic is, however, difficult to obtain hence we then pursue a different strategy. For a model with l breaks, the estimated break dates, denoted by T_1, \dots, T_l are obtained by a global minimisation of the sum of squared residuals. The adopted strategy proceeds by testing each $l+1$ segment (obtained using the estimated partition) T_1, \dots, T_l for the presence of an additional break (Ben and Jouini, 2003). The test amounts to the application of $l+1$ tests of the null hypothesis of a single break. It is applied to each segment $T_{i-1} + 1$, for $i = 1, \dots, l+1$ with $T_0 = 0$ and $T_{l+1} = T$. The estimates T_i need to be obtained by a global minimisation of the sum of squared residuals, and all that is required is that the break fraction $\lambda_i = \frac{T_i}{T}$ converge to the true values at a rate T .

We conclude for a rejection in favour of a model with $l+1$ breaks if the sum of squared residuals obtained from the estimated model with l changes is sufficiently larger than the overall minimal value of the sum of squared residuals (over all segments where an additional change is included), and the break point thus selected is the one associated with this overall minimum¹⁰. More precisely,

minimal number of observations in each segment can be set to any value greater than the trimming parameter, q .

¹⁰We can also use the sequential one-at-a-time estimates, which imply breaks fractions that converge at the rate T (Bai, 1997).

the test is defined by:

$$\sup F_T(l + \frac{1}{l}) = \left[S_T(\hat{T}_1, \dots, \hat{T}_l) - \min_{1 \leq t \leq l+1} \inf_{\tau \in \Lambda_{i\eta}} S_T(\frac{\hat{T}_i, \dots, \hat{T}_{I-1}, \hat{\tau}, \hat{T}_1, \hat{T}_l}{\hat{\sigma}_2}) \right] \quad (2.16)$$

where $\Lambda_{i\eta} = \tau, \hat{T}_{i-1} + (\hat{T}_i - \hat{T}_{i-1})\eta \leq \tau \leq \hat{T}_i - \hat{T}_{i-1}\eta$ $S_T(\hat{T}_i, \dots, \hat{T}_{I-1}, \eta, \hat{T}_1, \dots, \hat{T}_l)$ is the sum of squared residuals resulting from the least-squares estimation of each m -partition T_1, \dots, T_m , and $\hat{\sigma}^2$ is a consistent estimator of σ^2 under the null hypothesis. Asymptotic critical values were provided by BP for a trimming $\varepsilon = 0.05(m = 9)$ for $1 \leq q \leq 10$, and Bai and Perron (2003) presented additional critical values for $\varepsilon = 0.10(m = 8)$, $\varepsilon = 0.15(m = 5)$, $\varepsilon = 0.20(m = 3)$ and $\varepsilon = 0.25(m = 2)$ ¹¹.

To select the number of breaks and their locations, BP proposed a method based on the sequential application of the $\sup F_T(l + 1)$ test using the sequential estimation of breaks. The procedure to estimate the number of breaks is the following. Begin by estimating a model with a small number of breaks that are thought to be necessary (or with no break). Then execute a parameter-constancy test for every subsample (those obtained by cutting off at the estimated breaks), adding a break to a subsample associated with a rejection using the test $\sup F_T(l + 1)$. The process is repeated by increasing l sequentially until the test fails to reject the null hypothesis of no additional structural break. The final number of breaks is thus equal to the number of rejections obtained with the parameter-constancy test plus the number of breaks used in the initial round. Bai and Perron (2003) and Jouini and Boutahar (2005) are among the few scholars who have used this method of estimating multiple breaks. A distinct advantage of this procedure is that, compared to information criterion, it can directly take into consideration the effect of possible serial correlation in the errors and heterogeneous variances across segments (Ben and Jouini, 2003).

¹¹Note that m is the maximum possible number of breaks (Ben and Jouini, 2003).

2.7 Fractional Integration Method by Gil-Alana (2008)

As mentioned earlier, one of the drawbacks of the standard unit root methods is that they have very low power if the true data generating process is fractionally integrated. Thus, in this research we also examine the presence of unit roots in the Ghanaian and South African CPI inflation series, using a fractional integration framework developed by Gil-Alana (2008), and estimate the differencing parameter in the following set-up:

$$y_t = \alpha + \beta t + x_t, \quad (1 - L)^d x_t = \mu_t, \quad t = 1, 2, \dots, \quad (2.17)$$

From Equation (2.18), the researcher assumes that the error term μ_t is white noise and that it is autocorrelated. In the latter case, we assume that μ_t follows the exponential spectral model of Bloomfield (1973). This is a non-parametric approach that produces autocorrelations decaying exponentially as in the AR(MA) case. Its main advantage is that it mimics the behaviour of ARMA structures with a small number of parameters (Gil-Alana, 2004). Finally, noting that several authors argue that fractional integration may be an artificial artefact generated by the presence of breaks in the data, for example: Bai and Perron (2003), and Lumsdaine and Papell (1997), the framework of Gil-Alana (2008) is applied for testing for fractional integration in the context of unknown breaks.

2.8 Long Memory in Time Series

Long memory describes the correlation structure of a series at long lags. In the time domain, it is characterised by a hyperbolically decaying autocovariance function. This slow decay of the autocorrelation function is considered to be

the defining feature typical of a long memory process (Lo, 1991; Campbell et al., 1997). A stationary stochastic process y_t is called a long memory process if its autocovariance function ρ_τ is such that the autocorrelations are positive and decay monotonically and hyperbolically to zero. The asymptotic property can be expressed as:

$$\rho_t \approx |\tau|^{2d-1} \text{ as } |\tau| \rightarrow \infty \quad (2.18)$$

when $d \in (0, 0.5)$, the series is stationary and said to have long memory, while if $d > 0.5$, the series is non-stationary and hence unpredictable. For $d \in (-0.5, 0]$, the series is described as having short memory, which is a measure of the decline of statistical significance between distant observations¹².

Mcleod and Hipel (1978, p.492) have recommended that "it is often assumed that recent values of the series possess more information with regard to present and future values than the values in the distant past". They define the stationary process as having a long memory if its absolute autocorrelation function has an infinite sum. Consequently, the autocorrelation function ρ_j at lag j is defined according to:

$$\lim_{n \rightarrow \infty} \sum_{i=-n}^n |\rho_j| = \infty \quad (2.19)$$

where n is equal to the number of observations. This definition is in line with processes that do not have unit root, but whose autocorrelation function does not decay too fast. Brockwell and Davis (1995) strengthened this definition of long memory to $\rho_i \sim c|i|^{2d-1}$ as $n \rightarrow \infty$; $c \neq 0$; $0 < d < \frac{1}{2}$ where \sim means that the ratio on the RHS and LHS converges to unity as $i \rightarrow \infty$. Hence:

$$\lim_{n \rightarrow \infty} \sum_{i=-n}^n |\rho_j| = \pm\infty \quad (2.20)$$

¹²Long memory is also defined in terms of the Hurst, H exponent, which is simply related to d : i.e. $H = d - 0.5$ (see details in Chapter 4); Note that short memory is precisely the case of $d = 0$.

Beran (1994) also provided another definition based on the time domain, which involves the spectral density function. It states that, if an autocorrelation of a stationary time series is bounded geometrically, it has a short memory process:

$$|\rho_k| < cr^{-k}, \quad (k = 0, 1, 2, \dots) \quad (2.21)$$

for which $c > 0$ and $0 < r < 1$. A time series has long memory if a spectral density function is unbounded at low frequencies. As Beran (1994, p.7) has pointed out, "a stationary process with slowly decaying correlations is called a stationary process with long memory in contrast to a process with summable correlations which are also called processes with short memory". This implies that the process has a long memory, since the dependence between observations that are distant diminishes very slowly.

In literature, two classes of long memory stand out: continuous time process, such as the fractional Gaussian noise of Mandelbrot and Ness (1968), and discrete time series process, such as the autoregressive and fractionally integrated moving average (ARFIMA) of Granger and Joyeux (1980) and Hosking (1981).

2.8.1 Fractional Gaussian noise

The fractional Gaussian noise (FGN) process, first introduced by Mandelbrot and Ness (1968), is grounded on the Hurst phenomenon, which describes long memory between observations in time series of natural systems (Hurst, 1951). Mandelbrot and Wallis (1969a,b,c) also developed the concept of fractional Brownian motion (FBM) as a model of long memory. Standard Brownian motion (SBM) is a continuous time stochastic process, $B(t)$, composed of independent Gaussian increments (Baillie et al., 1996). FBM $B_H(t)$, can be derived from

SBM $B(t)$, using the following equation:

$$B_H t = \frac{1}{\Gamma(H + 0.5)} \int_{-\infty}^t (t - s)^{H-0.5} dB(s) \quad (2.22)$$

where $\Gamma(\cdot)$ denotes the gamma function and H is the Hurst exponent. When $H = 0.5$, FBM $B_H(t)$ reduces to white noise, i.e. SBM, $B(t)$. Nevertheless, relaxing the restriction of $H = 0.5$ necessary for SBM and allowing any $0 < H < 1$, the FGN process is defined as the increment of FBM, $B_H(t)$ at an integer t . The FGN process is expressed as:

$$\begin{aligned} \Delta B_H(t) &= (B_H(t) - B_H(t-1)) \\ &= \frac{1}{\Gamma(H + 0.5)} \left[\int_{-\infty}^t (t - s)^{H-0.5} dB(s) - \int_{-\infty}^{t-1} (t-1 - s)^{H-0.5} dB(s) \right] \end{aligned} \quad (2.23)$$

The autocorrelation function of the FGN process is given by¹³:

$$\rho_k = \frac{1}{2} [|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}] \approx H(2H-1)|k|^{2(H-1)} \quad (2.24)$$

2.8.2 The ARFIMA model

Even though the FGN process allows for modelling long memory processes using the Hurst exponent H , it does not discriminate between short and long memory processes. Lo and MacKinlay (1988, 1990) have established that time series display substantial short memory. Derived from the standard autoregressive moving average (ARMA) class of short memory models, the ARFIMA (autoregressive and fractionally integrated moving average) process of Granger and Joyeux (1980) and Hosking (1981) provides an alternative approach to modelling long memory process because it usually permits short and long memory to be modelled disjointedly. This gives the model a distinct advantage over the FGN process which only reflects the long memory process in the financial

¹³See Mandelbrot and Ness (1968) for proof.

time series. The ARFIMA model of order (p,d,q) , denoted by ARFIMA (p,d,q) , with mean μ , is expressed as follows:

$$\phi(L)(1-L)^d(y_t - \mu) = \theta(L)\varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2) \quad (2.25)$$

where L is the lag or the backshift operator, $\phi(L) = 1 - \phi_1(L) - \dots - \phi_p(L^p)$, $\theta(L) = 1 + \theta_1(L) + \dots + \theta_q(L^q)$ and $(1-d)^d$ is the fractional differencing operator defined by:

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(d+1)} \quad (2.26)$$

$$\begin{aligned} (1-L)^d &= \left(\sum_{k=0}^{\infty} (-L)^k \right) \\ &= 1 - dL - \frac{1}{2}d(1-d)L^2 - \frac{1}{6}d(1-d)(2-d)L^3 - \dots \end{aligned} \quad (2.27)$$

with $\Gamma(\cdot)$ denoting the gamma function. The parameter d is allowed to assume any real value. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. The stochastic process y_t is both stationary and invertible if all roots of $\phi(L)$ and $\theta(L)$ lie outside the unit circle and $|d| < 0.5$. The process is non-stationary for $d \geq 0.5$, as it possess infinite variance (Granger and Joyeux, 1980). Assuming that $d \in [0, 0.5)$ and $d \neq 0$, Hosking (1981) showed that the correlation function, $\rho(\cdot)$, of an ARFIMA process is proportional to k^{2d-1} as $k \rightarrow \infty$. As a result, the autocorrelation of an ARFIMA process decays hyperbolically to 0 as $k \rightarrow \infty$, which is dissimilar to the faster geometric decay of a stationary ARMA process.

Subsequent to Hosking (1981), a fractional difference time series shows different characteristics depending on the fractionally differenced or integrated parameter d . For instance, the fractional difference time series is stationary and invertible when $d \in (-0.5, 0.5)$. If $d = 0$, then the series is a short memory,

and $d = 1$ means that the series follows a unit root process. If $d = 0.5$, the series is invertible and non-stationary. If $d = -0.5$, then the series is stationary but not invertible. If $d \in (0, 0.5)$, then the series exhibits positive dependence between observations widely separated in time, suggesting long memory, and if $d \in (0.5, 1)$, then the process is mean reverting, even though it is not covariance stationary, as long as there is no long-run impact of innovation on future values of the process. If $d \in (-0.5, 0)$, then the series has a negative correlation, referred to as anti-persistence and its spectral density function is given by:

$$f_X(w) = f_U(w)[2\sin(\frac{w}{2})]^{-2d}, \text{ for } 0 < w < \pi \quad (2.28)$$

where $f_X(\cdot)$ is the spectral density function of the ARMA(p,q) process. One observes that $f_w \approx w^{-2d}$ when $w \rightarrow 0$.

2.9 Sources of Long Memory

There are a number of debatable issues regarding the source of long memory property of financial data. In particular, two common explanations for the presence of long memory in time series are contemporaneous and temporal aggregations in the volatility of financial time series returns (Balcilar, 2004; Lamoreaux and Lastrapes, 1990; Granger, 1980).

2.9.1 Contemporaneous aggregation

It is believed that long memory in financial time series could arise from concurrent aggregation (Granger, 1980; Barnes, 1998; Balcilar, 2004). Indeed, the construction of a financial time series sums up different persistence levels of specific economic variables, which would generate the long memory property in volatility. The theoretical dispute for aggregation biases resulting from concurrent aggregation was first proposed by Robinson (1978) and Granger

(1980). It has been recommended that "starting with individual independent first order AR(1) processes with random autoregressive coefficients, the aggregate series can exhibit long memory for certain specifications of the distribution function form which these coefficients are drawn" (? , p.265). This theoretical result shows that the sum of short memory processes produces a long memory process, which can in turn be characterised as an ARFIMA process (Granger, 1980). This discovery implies that aggregation of independent weakly dependent series can produce a strong dependent series.

2.9.2 Temporal aggregation

Following the suggestion by Lamoreaux and Lastrapes (1990), it is obvious that long memory may also be identified, thanks to the presence of structural breaks in the volatility of time series¹⁴. Modelling long memory time series has constituted a major research topic in the econometrics literature for a number of years. One of the most standard approaches for modelling the long memory properties of time series involves the use of the various ARFIMA models. Meanwhile, some econometric modelling have focused on the properties of stochastic processes with unit roots, which can be well-thought-out as a specific form of long memory. Perron (1997) has claimed that it is not easy to differentiate between unit root processes and long memory properties in the economic time series. In particular, structural breaks may give rise to persistence that is equivalent to a unit process (Banerjee and Urga, 2005). In financial economics, structural breaks may well be linked to certain economic variables that correspond to some specific monetary policy and market uncertainty levels, as laid out by various pieces of market news (Lui, 2000).

¹⁴Lamoreaux and Lastrapes (1990) demonstrated that if exogenous deterministic structural breaks are prevalent and ignored in estimating the GARCH (generalised autoregressive and conditional heteroscedascity) process, it could give rise to persistence corresponding to the IGARCH (integrated generalised autoregressive and conditional heteroscedascity) process.

Both long memory and structural breaks have been well-thought-out as independent phenomena in aforementioned literature. Diebold and Inoue (2001) indicated that structural breaks are easily confused with the long memory property in time series. Again, using Markov-switching model proposed by Hamilton (1989) and the stochastic permanent break model proposed by Engle and Smith (1999), Diebold and Inoue (2001) generated simulation data and estimated the long memory parameter using the GPH estimation technique. They found that the value of the parameter, d ranged between 0 and 1, signifying the existence of a long memory process in the data. Thus, structural breaks can be regarded as a source of long memory properties in a time series.

Some authors have also suggested that structural breaks generate slowly decaying autocorrelation and fractional differencing $I(d)$ processes. For example, Granger and Terasvirta (1999) have demonstrated that the value of the long memory parameter depends on the number of structural breaks and where they occur in a data. Cioczek-Georges and Mandelbrot (1995), Taqqu et al. (1997), Chambers (1998), Parke (1999), and Zaffaroni (2004). In their research, have also demonstrated the role of heterogeneous aggregation in the production of long memory.

In summary, there is little compromise in distinguishing between long memory and structural breaks. Even though some structural models and techniques are advanced, the debate cannot be resolved empirically because it is unclear whether long memory and structural breaks are part of the same issue or part of different issues in the series. In fact, it is often envisaged that they are identical phenomena in financial economics, but the two different terms, long memory and structural breaks, persist. Hence, it is important to establish a theory to explain why long memory and structural breaks have a relationship to each other in the volatility of inflation series.

2.10 Outline of Long Memory Models

This section outlines long memory models in financial time series, divided into four different methods of analysis. Section 2.10.1 introduces R/S analysis and discusses the empirical literature dealing with the Hurst exponent H . Section 2.10.2 presents modified R/S analysis and Section 2.10.3 presenting some empirical evidence of the Hurst exponent.

2.10.1 Rescaled range (R/S) analysis

R/S analysis was first developed by Hurst (1951), and was later re-defined by Mandelbrot and Wallis (1969a,b,c) and modified by Lo (1991). R/S analysis is constructed on the range of partial sums of deviations of a time series from its mean rescaled by its standard deviation. Following Hurst (1951), given a sample series $\{y_1, y_2, y_3, \dots, y_N\}$ of length T , the range of the cumulative deviations from sample mean or range R_N can be calculated as:

$$R_N = \text{Max}[X_{k,N}] - \text{Min}[X_{t,N}] \quad (2.29)$$

$$X_{t,N} = \sum_{t=1}^N (y_t - \bar{y}_N) \quad (2.30)$$

for $t \in [1, N]$, where $X_{t,N}$ is the cumulative deviation from the mean and \bar{y}_N is the sample mean over the series of length N . The range R_N will always be non-negative, because the maximum value of series y_N will always be greater than or equal to zero, and the minimum must be less than or equal to zero. Hurst (1951) rescaled the range R_N by its standard deviation over the series of length N , defined as follows:

$$(R/S)_N = \frac{R_N}{S_N} \quad (2.31)$$

$$S_N = \left\{ \frac{1}{N} \sum (y - \bar{y})^2 \right\}^{\frac{1}{2}} \quad (2.32)$$

The ratio R_N is called the R/S statistic. Under the assumption of a normal Gaussian distribution, Hurst (1951) defined the relationship between R/S statistic and the series length N . The R/S statistic is proportional to the time interval T with the Hurst exponent H , i.e.

$$(R/S)_N \approx c * T^H \quad (2.33)$$

where c is a constant. The R/S analysis can classify a time series into random or non-random components according to the estimated value of the Hurst exponent H (Mandelbrot and Wallis, 1969a,b,c; Mandelbrot, 1972). In particular, $H > 0.5$ indicates the Hurst phenomenon corresponding to a long memory process. Hurst (1951) observed that many natural series displayed values of $H > 0.7$, suggesting that natural phenomena show persistence or long memory. The fractional integration parameter is related to the Hurst exponent and it is given by $d = H - 0.5$.

Traditional approaches are incapable of differentiating between random and non-random behaviour. As a result, R/S analysis is superior to other traditional techniques such as autocorrelation, the variance ratio and spectral analysis models, in detecting the long memory in time series. For instance, unlike variance ratio test, R/S analysis is robust in capturing long memory even in the presence of non-Gaussian distributions with large skewness and kurtosis values (Mandelbrot and Wallis, 1969a,b,c). Another advantage of R/S analysis is that it provides an avenue to test for the existence of cycles (both periodic and non-periodic) within a data set. Such non-periodic cycles may be either a biased random walk or the result of non-linear dynamic system (Mckenzie, 2001).

The Hurst exponent of R/S analysis is generally estimated over a single period

length following Equation (2.30). This method has been modified to accommodate ordinary least squares (OLS) estimation techniques for estimating the Hurst exponent over several sub-series n (Mandelbrot and Wallis, 1969a,b,c). The Hurst exponent coefficient may be estimated using OLS regression according to the following form:

$$\log(R/S)_n = \log(c) + H * \log(n) + \varepsilon_t \quad (2.34)$$

where $\log(R/S)_n$ is the logarithm of the rescaled range for a sub-series length n , and $\log(n)$ is the logarithm of sub-series length. The Hurst exponent is estimated as the gradient of the equation. The OLS technique proposes that the R/S analysis be estimated over several sub-series contained within the entire series length. In this regard the choice of sub-series length is critical to the determination of the Hurst exponent in the time series¹⁵.

Interpretation of the Hurst exponent

There are three viable interpretations of the Hurst exponent (Peters, 1994). These are as follows:

- For $H = 0.5$, the time series are independent, random Gaussian processes (random walk), "it would also include non-Gaussian processes, such as the Student- t , or Gamma" (Peters, 1994, p.62).
- For $0.5 < H < 1$, the process is said to be long memory or long range dependence. The persistence means that if the time series has been up or down, it is likely to continue to be up or down in the future. This also suggests that large values tend to follow large values and small values tend to follow small values.

¹⁵There exist two different approaches for the determination of sub-series: overlapping sub-series and continuous sub-series.

- For $0 < H < 0.5$, the process is said to be anti-persistent. This indicates that large values and small values tend to alternate. This also implies that whenever the time series has been up in the last period, it is more likely to go down in the future, and vice-versa.

Weaknesses of the R/S analysis

Notwithstanding the apparent strength of R/S analysis there have been some criticism in a number of econometric studies (Anis and Lloyd, 1976; Lo, 1991; Peters, 1994). For instance, Peters (1994) identified the following weaknesses:

- R/S analysis needs a longer data span, because long period data provide a more realistic opportunity for non-periodic cycles to reveal themselves.
- R/S analysis has proven not to be suitable for testing volatility, since the value of the Hurst exponent is usually less than 0.5, indicating that the volatility process shows anti-persistence.
- R/S analysis favours low as opposed to high frequency data because high frequency data are known to display significant statistical correlation with each other.

Interestingly, the most well-known shortcoming of R/S analysis is that the results of the analysis are very sensitive to the presence of short-term dependence, and lack of a distribution theory with which to conduct general hypothesis tests for long memory under short memory null (Lo, 1991; Jacobsen, 1996). To account for short-term dependence, Lo (1991) sought to empirically demonstrate the robustness against short memory of a modified R/S analysis, deriving its limiting distribution under both long and short memory, and illustrating its power against certain long memory alternatives in terms of Monte-Carlo simulation (Ambrose et al., 1993).

2.10.2 Modified R/S analysis

To overcome the problem of short-term dependence, Lo (1991) proposed a modified R/S analysis, which incorporates a generalised model for short-term dependence, and permits an examination of the null hypothesis of short memory versus long memory alternative. The R/S statistic, Q_n , is given by the range of partial sums of deviations from the mean, rescaled by its standard deviation as follows:

$$Q_n = \left[\frac{1}{(\sigma_n(q))} \left\{ \max_{1 \leq j \leq N} \sum_{i=1}^j (y_i - \bar{y}_n)^2 - \min_{1 \leq j \leq N} \sum_{i=1}^j (y_i - \bar{y}_n) \right\} \right] \quad (2.35)$$

where

$$\sigma_n(q) = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y}_n)^2 + \frac{2}{n} \sum_{i=1}^q w_i(q) \left\{ \sum_{t=i+1}^n (y_t - \bar{y})(y_i - \bar{y}_n) \right\} = \hat{\sigma}_y^2 + 2 \sum_{i=2}^q w_i(q) \hat{v}_i \quad (2.36)$$

where q is the number of lag periods, and the weighted function is defined as $w_i = 1 - \frac{i}{q} + 1$, for $q < n$. If the series of sample returns is subject to short-term dependence, the variance term in the denominator includes some autocovariance terms which are weighted according to their lagged values $q = l$. From Equation (2.35), the choice of $q = l$ is arbitrary, and since there are no data driven guidance for the choice of q , we use the approach suggested by Zivot and Wang (1994), which is defined by $q = l = 4\left(\frac{T}{100}\right)^{0.25}$, where T is the sample size. At 5% significant level, the null hypothesis of no long memory can be rejected if the statistic does not fall within the confidence interval [0.809,1.862]. The interval for other significant levels can be verified from Lo's Table II (Lo, 1991, p.1288).

2.10.3 Empirical evidence of the Hurst phenomenon

Lately, there are recently dozens of estimation methods and tests for long memory models. Perhaps one of the reasons for the wide array of tools for estimation and testing is that the current consensus suggests that good estimation techniques remain elusive, and many tests of long memory have been shown, through finite sample experiments to perform poorly (Donald et al., 2003).

Much of this evidence has been reported in the context of comparing one or two classes of estimators, such as R/S analysis as introduced by Hurst (1951) and modified by R/S analysis Lo (1991) and log periodogram regression estimators by Geweke and Porter-Hudak (1983) used in the estimation of fractional integration parameter d . Sasikumar (2011) analysed the presence of long memory in Indian Forex market using a number of tests, including the Hurst exponent, Hurst-Mandelbrot R/S analysis, Lo's R/S analysis, Robinson's semi-parametric and Andrew-Guggenburger modified GPH estimators. The results depicted the existence of long memory in the Indian Forex market.

The empirical evidence of the Hurst exponent in literature have been mostly applied in stock market and exchange rates research, with little emphasis on inflation. Recent studies have sought to examine long memory properties in international stock markets using the R/S statistics. Huang and Yang (1995) applied both classical R/S and Lo's modified R/S statistics to nine Asian stock market indices together with USA and UK indices. The results from the classical R/S statistics suggested evidence of long memory in most emerging Asian markets. However, except for the Philippines, the evidence of long memory disappeared once the effects of short-term dependence were accounted for using Lo's modified R/S analysis. Huang and Yang (1995) however, indicated that the classical R/S analysis might lead to a spurious conclusion regarding evidence of long memory in the data, because it is biased by using short data span.

Golaknath and Reddy (2002) also employed the R/S analysis and a variance ratio (VR) analysis to analyse the effect of long memory on the Indian foreign exchange market. While the VR did not provide conclusive evidence of the presence of long memory, the R/S analysis indicated the presence of long memory with noise. Soofi and Zhang (2006) used plug-in and Whittle methods based on spectral regression analysis to test for long memory in 12 Asian currency daily exchange rates versus USD. They found that, except for the Chinese renminbi, the other 11 currencies displayed long memory characteristics.

In another study, Hsieh and Shyu (2009) examined the long-term dependency behaviour of Asian foreign exchange markets by using R/S analysis. Emerging markets in Korea, Taiwan, India and Thailand exhibited evidence of long memory in the exchange rate return series, whereas the exchange return persistence was not found in the markets of Japan, Australia, Hong Kong and Singapore. Their results suggested that the return-generating process and the presence of long memory depend on the degree of market development. Enet et al. (2010) examined the presence of long memory in nine selected currencies around the world in the context of Indian Rupee using R/S analysis together with Whittle test and Hurst exponent. They found significant presence of long memory in appreciation or depreciation in the nine exchange rates.

From the literature review, it is evident that there is a serious dearth of research in the area of testing long memory and modelling the conditional mean in the Ghanaian and South African inflation. This present research places itself in that context.

2.11 Estimation Methods for Fractionally Integrated Parameter and ARFIMA Model

This section reviews two estimation methods of ARFIMA models: parametric and semi-parametric approaches and their empirical evidence.

2.11.1 Parametric estimators

This section presents synopsis on parametric maximum likelihood estimators (MLEs), e.g. exact time domain MLE. For instance, the time domain estimators are based on the likelihood function of the ARFIMA(p,d,q) model with or without conditioning on initial observations, and the frequency domain estimator is based on Whittle's approximation to the likelihood function in the frequency domain.

Maximum likelihood in the time domain

The objective function of the exact maximum likelihood (EML), for the models in Equations (2.25), (2.69), (2.70) and (2.72), for $d \in (-0.5, 0.5)$ is given by:

$$L_E(d, \phi, \theta, \sigma^2, \mu) = -\frac{1}{2} \ln |\Omega| - \frac{1}{2} (X - \mu)' \Omega^{-1} (X - \mu l) \quad (2.37)$$

where $l = (1, \dots, 1)'$, $X = (x_1, \dots, x_T)$, ϕ and θ are the parameters of $\phi(L)$ and $\theta(L)$, μ is the mean of X , and Ω is the variance matrix of X , which is a complicated function of d and the remaining parameters of the model. Sowell (1992) derived an efficient procedure for solving this function in terms of hypergeometric functions. The main shortcoming of this model is that the roots of the autoregressive polynomial cannot be multiple. Hence, collecting the parameters in the vector $\gamma = (d, \phi', \theta', \sigma^2)'$ the EML estimator is obtained by maximising the likelihood function in Equation (2.37) with respect to γ . Sowell (1992)

showed that the EML estimator of d is consistent and asymptotically normal, i.e.

$$\sqrt{T} = (\hat{d}_{EML} - d) \rightarrow_d N(0, \frac{\sigma^2}{6} - c)^{-1} \quad (2.38)$$

where $c = 0$, when $p = q = 0$, and $c > 0$ otherwise. The variance of the EML may be obtained as $(1, 1)$ 'th element of the inverse of the matrix:

$$\frac{1}{4\pi} \int_0^{2\pi} \frac{\partial \ln f_y(\lambda)}{\partial \gamma} - \frac{\partial \ln f_y(\lambda)}{\partial \gamma'} d\lambda \quad (2.39)$$

Although the time and frequency domain (see next discussion) MLEs are asymptotically equivalent, their finite sample properties differ, and a small Monte Carlo study carried out by Sowell (1992) shows that the time domain estimator has better finite sample properties than the frequency domain estimator when the mean of the process is known. However, Cheung and Diebold (1994) showed that the finite sample efficiency of the discrete Whittle frequency domain MLE defined in Equation (2.39) relative to time domain EML, increases rapidly when the mean is unknown and has to be estimated.

Maximum likelihood in the frequency domain

An alternative approximate MLE of the ARFIMA(p,d,q) model follows the concept of Whittle (1951), who noted that for stationary models the covariance matrix Ω can be diagonalised by transforming the model into the frequency domain. Fox and Taqqu (1986) showed that when $d \in (-0.5, 0.5)$ the log likelihood (LL) can be approximated by:

$$L_F(d, \phi, \theta, \sigma^2) = - \sum_{j=1}^{\lfloor \frac{T}{2} \rfloor} \left[\ln f_y(\lambda_j) + \frac{I(\lambda_j)}{f_y(\lambda_j)} \right] \quad (2.40)$$

where $\lambda_j = \frac{2\pi j}{T}$ are the Fourier frequencies, $I(\lambda) = \frac{1}{2\pi T} |\sum_{t=1}^T y_t e^{it\lambda}|^2$ is the periodogram of y_t and $f_y(\lambda)$ is the spectral density of y_t , and it is given by:

$$f_y(\lambda) = \frac{\sigma^2}{2\pi} (2 \sin \frac{\lambda}{2})^{-2d} \frac{|\theta(e^{i\lambda})|^2}{|\phi(e^{i\lambda})|^2} \quad (2.41)$$

where $\lfloor x \rfloor$, denotes the largest integer not greater than x . The frequency domain estimator is invariant to the presence of non-zero mean, i.e. $\mu = 0$, since $j = 0$ (the zero frequency) is left out in the summation in Equation (2.40). The approximate frequency domain maximum likelihood (FDML) estimator is defined as the maximiser of Equation (2.40) proposed by Fox and Taquq (1986), who also proposed a continuously integrated version of Equation (2.41). Dahlhaus (1989) also assumed Gaussianity and considered the exact likelihood function in the frequency domain. The FDML estimator has the same asymptotic properties as the EML estimator, i.e. \sqrt{T} -consistency and asymptotic normality, when the process is Gaussian and asymptotically efficient. Giraitis and Surgailis (1990) relaxed the Gaussianity assumption and analysed the Whittle estimate for linear processes by showing that it is \sqrt{T} -consistent and asymptotically normal but no longer efficient, while Hosoya (1997) extended the previous analysis to a multivariate framework.

2.11.2 Semi-parametric estimators

The semi-parametric frequency domain estimators are based on the approximation of the periodogram, $f_y \sim g|\lambda|^{-2d}$, where $0 < g < \infty$ as $\lambda \rightarrow 0$ to the spectral density function. Two classes of semi-parametric estimators have become very popular in empirical work, the log-periodogram regression method introduced by Geweke and Porter-Hudak (1983) and the LW approach suggested by Robinson (1995). Again, the semi-parametric estimators enjoy robustness to short-run dynamics since they use only information from the periodogram ordinates in the vicinity of the origin. For instance, the short-run dynamics

of the ARFIMA model in Equation (2.25), i.e. the autoregressive and moving average polynomials, $\phi(L)$ and $\theta(L)$ respectively, do not have to be specified when a semi-parametric estimator is applied. The drawback is that only \sqrt{m} -consistency is achieved, where $m = m(T)$ is a user-chosen bandwidth parameter, in comparison to \sqrt{m} -consistency (and efficiency) in the parametric case. Thus, the semi-parametric approach is less efficient than the parametric one since it requires at least $\frac{m}{T} \rightarrow 0$. However, a problem with parametric approaches is that they require the model to be correctly specified, or else the estimates are liable to be inconsistent. In fact, misspecification of the short-run components of the series may invalidate the estimation of d . Hence, adopting semi-parametric procedures might be advantageous. Subsequent sections of this chapter present detailed description of GPH and LW estimators together with EML (see previous section).

Geweke and Porter-Hudak (1983) estimator

The Geweke and Porter-Hudak (1983) (GPH) estimation finds an estimate of the long memory parameter d in the frequency domain in which the periodogram is estimated first from the time series and its logarithm is subsequently regressed using a set of Fourier frequencies close to zero, where the gradients of the log spectrum relative to the frequency is dependent on d . The GPH estimation method can capture long memory processes, without any short memory effect being evident in the time series at higher frequencies. This distribution of the estimated d is assumed to be Gaussian, hence the null hypothesis of no long memory ($d = 0$) can be tested with t -test statistics, using the standard deviation given by the regression.

To permit a fractionally integrated process, GPH assumed that the spectral

density function $f(\omega)$ of the first difference $y_t = (1 - L)\varepsilon_t$ can be stated as:

$$f_y(\omega) = |1 - \exp(-\omega)|^{-2d} = \left[4 \sin^2\left(\frac{\omega}{2}\right)\right]^{-d} f_x(\omega) \quad (2.42)$$

where $f_x(\omega)$ is the spectral density of stationary ARMA process y_t . For processes with this spectral density, GPH proposed a fractionally integrated parameter $d \in (-0.5, 0.5)$. The sample size of the time series y_t is T and the spectral density function is evaluated at discrete harmonic frequencies $\omega_j = \frac{2\pi j}{T}$, ($j = 1, \dots, T$). Taking logarithms in Equation (2.42) gives:

$$\log\{(\omega_j)\} = \log\{f_x(0)\} - d \log\left\{4 \sin^2\left(\frac{\omega_j}{2}\right)\right\} + \log\left\{\frac{f_x(\omega_j)}{f_x(0)}\right\} \quad (2.43)$$

For sufficiently low frequency ordinates ω_j close to zero, the last term in Equation (2.43) is considered insignificant or constant compared to other terms, and can be treated as an error term. In practice, $f_y(\omega_j)$ is estimated by the periodogram $I(\omega_j)$ at ordinates j , so that Equation (2.43) can be rearranged as:

$$\log\{(\omega_j)\} = \log\{f_x(0)\} - d \log\left\{4 \sin^2\left(\frac{\omega_j}{2}\right)\right\} + \log\left\{\frac{f_x(\omega_j)}{f_x(0)}\right\} + \log\left\{\frac{I_y(\omega_j)}{f_y(\omega_j)}\right\} \quad (2.44)$$

As ω_j approaches the zero frequency $j \leq n \ll T$, the term $\frac{f_x(\omega_j)}{f_x(0)}$ also converges to zero (Geweke and Porter-Hudak, 1983). This implies that as the sample size grows, this term becomes negligible, with Equation (2.44) reducing to:

$$\log\{(\omega_j)\} = \log\{f_x(0)\} - d \log\left\{4 \sin^2\left(\frac{\omega_j}{2}\right)\right\} + \log\left\{\frac{I_y(\omega_j)}{f_y(\omega_j)}\right\} \quad (2.45)$$

which can be transformed as:

$$Y_j = A + BX_j + \varepsilon_j \quad (2.46)$$

where $Y_j = \log\{(\omega_j)\}$, $A = \log\{f_x(0)\}$, $X_j = \log\left\{4 \sin^2\left(\frac{\omega_j}{2}\right)\right\}$, $\varepsilon_j = \log\left\{\frac{I_y(\omega_j)}{f_y(\omega_j)}\right\}$,

$B = -d, j = 1, 2, 3, \dots, n$ and $n = g(T)$. To estimate the parameter d from Equation (2.46) using the ordinary least squares (OLS) regression, a value of $g(T)$, under certain specific conditions, is satisfied by T^τ for $0 < \tau < 1$ (Geweke and Porter-Hudak, 1983)¹⁶.

In practice, the fitting value of $g(T)$ is usually taken as $g(T) = T^{0.5}$. GPH recommended that the theoretical variance of ε_t in spectral regression is equal to $\frac{\pi}{6}$, which improves the efficiency of estimation. GPH also discovered that the null hypothesis test with respect to the value of d can be based on the t -statistics of the regression coefficient. Since GPH's work, a number of researchers have sought to modify the GPH estimation technique to correct its poor sample properties. As contended by Agiakloglou et al. (1993), GPH estimation has a considerable finite sample bias, and is inefficient when the error term is persistent AR or MA process. Choi and Wohar (1992) also indicated that the fractional difference parameter d can be seriously biased under a large value of the autoregressive parameter. Robinson (1995) redefined a proof and limiting distribution of the GPH estimation with the Gaussian estimator. The researcher also developed a LW estimator, in which a discrete form of an approximate frequency domain Gaussian likelihood can be centered around a neighbourhood of zero frequency. The neighbourhood grows to infinity (though more slowly than the sample size) and hence this estimator is asymptotically normal and more efficient than previous methods.

Local Whittle (LW) estimator

Robinson (1995) proposed the Gaussian semi-parametric (GSP) estimator using the LW estimator, $L_\omega(\theta)$. This estimator represents approximately a maxi-

¹⁶To test for the sensitivity of the estimates, many have attempted to determine a reasonable value of τ (Cheung and Lai, 1995; Barkoulas et al., 2000; So, 2000). If the choice of τ is less than 0.5, the null hypothesis of short memory cannot be rejected in applications. But Geweke and Porter-Hudak (1983) argue that the estimated results might be seriously altered by the short-term dependence when the value of τ becomes large.

mum likelihood estimator (MLE) in the frequency domain, since for large sample size T

$$I(\lambda_j) \sim \exp\{f(\lambda_j)^{-1}\} \quad (2.47)$$

Given Equation (2.47), the likelihood function is:

$$L\{I(\lambda_1), \dots, I(\lambda_j), \theta\} = \prod_{j=1}^m \frac{1}{f_\theta(\lambda_j)} \exp\left\{-\frac{I(\lambda_j)}{f_\theta(\lambda_j)}\right\} \quad (2.48)$$

where $\theta = (c, d)$ is the parameter vector. The likelihood function becomes:

$$l(\theta) = \sum_{j=1}^m \left\{ \log f_\theta(\lambda_j) - \frac{I(\lambda_j)}{f_\theta(\lambda_j)} \right\} \quad (2.49)$$

In the neighbourhood of $\lambda \simeq 0$ we obtain:

$$l(c, d) = \sum_{j=1}^m \left\{ -\log c - 2d \log(\lambda_j) + \frac{I(\lambda_j)}{c\lambda_j^{-2d}} \right\} \quad (2.50)$$

$$\frac{\partial l(c, d)}{\partial c} = \sum_{j=1}^m \left\{ \frac{1}{c} - \frac{I(\lambda_j)}{c^2 \lambda_j^{-2d}} \right\} \quad (2.51)$$

yielding $\hat{c} = m^{-1} \sum_{j=1}^m \left\{ \frac{I(\lambda_j)}{\lambda_j^{-2d}} \right\}$. Replacing c with \hat{c} in Equation (2.50) gives:

$$l(\hat{c}, d) = m \log \hat{c} - 2d \sum_{j=1}^m I(\lambda_j) + m^2 \quad (2.52)$$

yielding the minimisation with respect to d as:

$$\arg \min_d l(\hat{c}, d) = \log \hat{c} - \frac{2d}{m} \sum_{j=1}^m \log(\lambda_j) + m \quad (2.53)$$

which is equivalent to:

$$\arg \min_{d_{GSP}} \left[\log \left\{ \frac{1}{m} \sum_{j=1}^m \frac{I(\lambda_j)}{\lambda_j^{-2d}} \right\} - \frac{2d}{m} \sum_{j=1}^m \log(\lambda_j) \right] \quad (2.54)$$

For $m^* = \lfloor \frac{T}{2} \rfloor$, an approximation to the Gaussian LL is given by (Beran, 1994):

$$l(\theta) = -2\pi^{-1} \sum_{j=1}^{m^*} \log f_{\theta}(\lambda_j) + \frac{I(\lambda_j)}{f_{\theta}(\lambda_j)} \quad (2.55)$$

for a given spectral density $f_{\theta}(\lambda_j)$.

Robinson (1995) demonstrated that the LW estimator is consistent for $d \in (-0.5, 0.5)^{14}$. Nevertheless, its consistency depends on the bandwidth m , which must satisfy $\frac{1}{m+T} \rightarrow 0$, as $T \rightarrow \infty$. This estimator is attractive due its asymptotic properties, the mild assumptions underlying it and the likelihood interpretation. Indeed, Robinson (1995) showed that $\sqrt{m}(d_{GSP}^{\hat{}} - d) \rightarrow N(0, \frac{1}{4})$.

Empirical evidence of a fractionally integrated process

Inflation persistence or long memory contains essential information for the formulation of monetary policy. In particular, it helps in the decision making process towards adjusting the policy instrument to achieve the anticipated target and in general, constitutes an important element in the formulation of monetary policy. Again, persistence refers to an important statistical property of inflation, where the current value of the inflation is strongly influenced by its history. Despite extensive study on the dynamic properties of inflation rates, there is still no agreement about the key question of persistence in inflation.

Studies on inflation can be classified into two major groups. The first group of

¹⁴Velasco and Robinson (2000) demonstrated that the fractional differencing parameter can be consistently estimated semi-parametrically in a non-stationary context by means of tapering.

papers tests for the existence of unit root in the inflation series but disagreement still remains in these papers on the classification of inflation as stationary or non-stationary. Barsky (1987), MacDonald and Murphy (1989), and Ball and Cecchetti (1990) provided evidence in support of unit root in inflation rates. On the other hand, Rose (1988) found evidence of stationarity $I(0)$ in inflation rates. Brunner and Hess (1993) claimed that the inflation rate was stationary before 1960, but has become non-stationary since that time.

In response to this debate on stationarity of inflation series the second group of papers provided an explanation by modelling inflation rates as fractionally integrated processes $I(d)$ where the order of differencing d is a fractional value. The fractionally integrated model implies that the autocorrelations of inflation exhibit very slow hyperbolic decay. Baillie et al. (1996) used an ARFIMA model extended with GARCH errors to test for long memory in the inflation rate of the G7 countries, and found significant evidence. Similar evidence of strong long memory in the inflation rate of the USA, UK, Germany, France and Italy, was also provided by Hassler and Wolters (1994). Baum et al. (1999) found significant evidence of long memory in the inflation rates for the industrial as well as the developing countries. Baillie et al. (2002) also explored the long memory property in the first and second conditional moments of inflation rates simultaneously. Furthermore, Reisen et al. (2003) suggested that the inflationary dynamics of Brazil were better modelled by a long memory process than by a unit root mechanism.

Currently, there has been an increase in research concerning estimation of inflation persistence or long-range dependence in South Africa. Rangasamy (2009) employed the eminent AR(1) equation to measure inflation persistence at an aggregate and disaggregate levels of inflation. The method is heavily dependent on estimation of the AR terms where inflation is the dependent vari-

able, set out in a regression equation, which includes a constant term. The sum of the AR terms, ρ , where $\rho(0, 1)$, determines the extent of persistence, so that if $\rho \geq 1$ then inflation contains unit root and is volatile; and if $\rho < 1$, then inflation is mean reverting. His results revealed a decline in inflation persistence after the adoption of IT policy in February 2000, with respective estimates for before and after adoption of IT being 0.98 and 0.83.

Balcilar et al. (2016) used a fractionally integrated model in the context of regime switching set up to investigate inflation persistence. Balcilar et al. (2016) concluded that inflation persistence is much stronger in high inflation episodes compared to low inflation episodes, while controlling for high inflation volatile rates. Their results were similar to those of Rangasamy (2009). Another research conducted by Burger and Marinkov (2008), sort to obtain recursive estimates for inflation persistence before and after IT policy in South Africa. They established that inflation persistence has not decreased since IT policy (though mean reverting) but expressed concern about the SARB's ability to decrease inflation persistence at all with the new monetary policy regime.

In Ghana, not much has been done on the estimation of inflation persistence. However, Alagidede et al. (2014) examined the crucial issue of inflation persistence in Ghana. Specifically, their study investigated the existence of persistence at the aggregate, national and regional levels. The study included investigation of persistence across 13 sectors of the economy, hence covering both core and headline inflation persistence. Employing fractional integration methods, the study provided some important additions to the literature. Their results showed an empirical evidence suggesting (i) asymmetries in the degrees of inflation persistence at both regional and sectorial areas and (ii) high potential for significantly different conclusions about inflation persistence being drawn, depending on whether month-on-month inflation or year-on-year infla-

tion is assessed. Tweneboah et al. (2015) conducted a study on long memory behaviour of real interest rates in Ghana using ARFIMA and fractionally integrated generalised autoregressive and conditional heteroscedastic (FIGARCH) models and found them to exhibit an indistinguishable integration property.

Given the importance of inflation persistence for evaluating optimality of monetary policies, and the little consensus about the degree of inflation persistence in the empirical literature, our study strives to provide new evidence on persistency of dynamics of inflation for Ghana and South Africa. Econometric, statistical and mathematical theories and methods have been employed in this case.

2.12 Long Memory and Fractionally Integrated ARMA with Conditional Heteroscedastic Innovations

This section presents an extensive examination of modelling long memory in the conditional mean using the ARFIMA model with the sGARCH and 'gjr-GARCH processes to describe persistence and time-dependent heteroscedasticity under three different distributions. Subsequent to the work of Granger (1980), Granger and Joyeux (1980), Hosking (1981), and Baillie et al. (1996), several studies have dealt with the estimation of the ARFIMA process. For a time-dependent conditionally heteroscedastic data, one useful extension is to consider the ARFIMA model with GARCH-type innovations. These models can provide a useful way of analysing the relationships between the conditional mean and variance of a process displaying long memory and slow decay in its level, yet with time-varying volatility. This study models the fractional inte-

gration process or persistence of the CPI inflation series of Ghana and South Africa using the ARFIMA-sGARCH and ARFIMA-gjrGARCH under three distributional assumptions.

2.12.1 Autoregressive conditional heteroscedasticity model

Numerous efforts have been made to comprehend inflation dynamics in terms of linear models. However, linear structure models remain incapable to explain these important features of financial time series such as inflation. These include: (1) the unconditional distribution of data exhibiting fat tails and excess kurtosis or peakedness at the mean (leptokurtosis); (2) the conditional variance is not constant but changes over time (heteroscedasticity), and (3) large changes in the conditional variance tend to be followed by large changes of either sign and small changes in the conditional variance by small changes (volatility clustering). These phenomena are renowned stylised facts evident in most financial time series data (Mandelbrot, 1963). They are better captured by autoregressive and conditional heteroscedascity (ARCH) class of models introduced by Engle (1982).

The ARCH models are used to capture the return of an asset. Engle created the first heteroscedastic model in 1982 to capture the movements in UK inflation rates (Bollerslev, 1986). Owing to the characteristics of the volatility for any financial time series, the ARCH model was built on two assumptions. The first assumption is that high volatility appears in clusters and hence the movement of the assets return relies on the previous values, but for the whole time series it is uncorrelated.

Following Engle (1982), let ε_t be a discrete time real-valued stochastic process:

$$\varepsilon_t = z_t \sigma_t \tag{2.56}$$

where z_t is *iid* with $E(z_t) = 0$ and $var(z_t) = 1$ and σ_t is a positive time-varying and measurable function with reference to the information set available at time $t - 1$. The series ε_t is the deviation from the conditional mean for some other process y_t :

$$\varepsilon_t = y_t - E_{t-1}(y_t) \quad (2.57)$$

where $E_{t-1}(y_t)$ is the expectation of the conditional mean on the information set at $t - 1$. The conditional variance of ε_t , represented by σ_t^2 , is expressed as:

$$var(\varepsilon_t) = E_{t-1}(\varepsilon_t^2) - [E_{t-1}(\varepsilon_t)]^2 \quad (2.58)$$

σ_t^2 is equal to the conditional expected value of squared ε_t , which is changing over time. The conditional variance of the ARCH(q) model depends on q lags of square errors, defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad (2.59)$$

where all the coefficients would be expected to be positive: $\omega > 0$ and $\alpha_1, \alpha_2, \dots, \alpha_q \geq 0$. Indeed this is a sufficient, but not a necessary condition for the non-negativity of the conditional variance. In ARCH(q), the effect of a return shock i periods ago $i \leq q$ on current volatility is measured by the parameter α_i . For $i > j$, $\alpha_i > \alpha_j$. This means that the older the news, the less effect it has on current volatility. In general, old news which arrived at the market more than i periods ago has no effect at all on current volatility; more current information is likely to induce 'ARCH effect phenomena' in which volatility occurs in bursts (Engle and Ng, 1993).

The ARCH(q) model has a number of flaws that affect its empirical application. First, there is no clear 'best' method to determine the value of q , the number of lags of the squared residuals. Second, the number of lags required to capture all of the dependence in the conditional variance might be very large, which

would result in a large conditional variance model that was not parsimonious. Third, since σ_t^2 is the conditional variance, its value must always be positive. Nonetheless, the more parameters there are in the conditional variance equation, the more likely it is that one or more of them will violate non-negativity constraints.

2.12.2 The standard autoregressive conditional heteroscedasticity model

The standard autoregressive conditional heteroscedasticity (sGARCH) model (Bollerslev and Mikkelsen, 1996) may be defined as:

$$\sigma_t^2 = (\omega + \sum_{j=1}^m \varsigma_j \nu_{jt}) + \sum_{j=1}^q \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (2.60)$$

where σ_t^2 denotes the conditional variance; ω the intercept; ε_t^2 the residuals from the mean filtration process, and ν_j the deterministic coefficient. The sGARCH order is defined by (q, p) (ARCH, GARCH) with possibly m external regressors ν_j , which are passed pre-lagged. If variance targeting is used, then ω is replaced by:

$$\bar{\sigma}^2(1 - \hat{P}) - \sum_{j=1}^m \varsigma_j \bar{\nu}_j \quad (2.61)$$

where $\bar{\sigma}^2$ is the unconditional variance of ε^2 which is consistently estimated by its sample counterpart at every iteration of the solver following the mean equation filtration, and $\bar{\nu}_t$ denotes the sample mean of the j^{th} external regressors in the variance equation (assuming stationarity); \hat{P} is the persistence, defined as:

$$\hat{P} = \sum_{j=1}^q \alpha_j + \sum_{j=1}^p \beta_j \quad (2.62)$$

One of the key features of the observed behaviour of financial data such as inflation series which GARCH models capture, is volatility clustering, which may be quantified in the persistence parameter \hat{P} . For the standard GARCH model, this may be calculated as in Equation (2.61). Related to this measure of persistence is the ‘half-life’ (say $h2l$) defined as the number of days it takes for half of the expected reversion back towards $E(\sigma^2)$ to occur, and this is given by:

$$h2l = \frac{-\log_e 2}{\log_e \hat{P}} \quad (2.63)$$

Again, the unconditional variance $\bar{\sigma}^2$, is related to the persistence as $\hat{\sigma}^2 = \frac{\hat{\omega}}{1-\hat{P}}$, where $\hat{\omega}$ is the estimated value of the intercept from the GARCH model.

2.12.3 Glosten-Jagannathan-Runkle generalised autoregressive conditionally heteroscedasticity model

The Glosten-Jagannathan-Runkle generalised autoregressive conditionally heteroscedasticity (‘gjr-GARCH’) model by Glosten et al. (1993) models positive and negative shocks on the conditional variance asymmetrically via the use of the indicator function I ,

$$\sigma_t^2 = (\omega + \sum_{j=1}^m \varsigma_j \nu_{j_t}) + \sum_{j=1}^q (\alpha_j \varepsilon_{t-j}^2 + \gamma_j I_{t-j} \varepsilon_{t-j}^2) + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (2.64)$$

where γ_j denotes the leverage term. The indicator function I takes on value 1 for $\varepsilon \leq 0$ and 0 otherwise. Because of the presence of indicator function, the persistence of the model crucially depends on the asymmetry of the conditional distribution used. The persistence of the model \hat{P} is:

$$\hat{P} = \sum_{j=1}^q \alpha_j + \sum_{j=1}^p \beta_j + \sum_{j=1}^p \gamma_j k \quad (2.65)$$

where k is the expected value of the standardised residuals z_t below zero (effectively the probability of being below zero):

$$\kappa = E(I_{t-j} z_{t-j}^2) = \int_{-\infty}^0 f(z, 0, 1, \dots) dz \quad (2.66)$$

where f is the standardised conditional density with any additional skew and shape parameters. In the case of symmetry distributions the value of k is simply equal to 0.5.

2.12.4 ARFIMA(p,d,q)-sGARCH(P,Q) model

The fractionally integrated ARFIMA(p,d,q)-sGARCH(P,Q) process is given by:

$$\phi(L)(1-L)^d(y_t - \mu - bx_{1t} - \delta\sigma_t) = \theta(L)\varepsilon_t \quad (2.67)$$

$$\varepsilon_t | \Omega_{t-1} \sim D(0, \sigma_t^2) \quad (2.68)$$

$$\beta(L)\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \gamma'x_{2t} \quad (2.69)$$

where $y_t = 100\Delta \log CPI_t$ is the CPI inflation, x_{1t} and x_{2t} are vectors of predetermined variables, μ is the mean of the process, $\phi(L) = 1 - \phi_1(L) - \dots - \phi_p(L)^p$, $\theta(L) = 1 + \theta_1(L) + \dots + \theta_q(L)^q$, $\beta(L) = 1 - \beta_1(L) - \dots - \beta_P(L)^P$, $\alpha(L) = 1 + \alpha_1(L) + \dots + \alpha_Q(L)^Q$ and all the roots of $\phi(L)$, $\theta(L)$, $\beta(L)$, and $\alpha(L)$ lie outside the unit circle.

With $\delta = 0$ and $b = 0$, Equations (2.67) and (2.68) describe the ARFIMA process introduced by Granger (1980), and Granger and Joyeux (1980). With $\delta \neq 0$, the model is extended to allow volatility to influence mean inflation. The innovations ε_t are assumed to follow a conditional density D , which is either Norm, STD or GED, and the time-dependent heteroscedasticity σ_t^2 follows sGARCH

model of Bollerslev and Mikkelsen (1996). Lagged inflation, which is predetermined, is allowed to possibly enter the conditional variance Equation (2.69) through being included in x_{2t} . The population features of ARFIMA process have been broadly studied by Granger and Joyeux (1980), and Hosking (1981). For $d \in (-0.5, 0.5)$ the process y_t in Equation (2.25) is covariance stationary and moving average coefficients decay at a relatively slow hyperbolic rate compared with the stationary and invertible ARMA process where the moving average coefficients decline exponentially with increasing lag¹⁷.

2.12.5 ARFIMA(p,d,q)-'gjrGARCH(P,Q) model

The fractionally integrated ARFIMA(p,d,q)-'gjrGARCH(P,Q) process is given by:

$$\phi(L)(1-L)^d(y_t - \mu - bx_{1t} - \delta\sigma_t) = \theta(L)\varepsilon_t \quad (2.70)$$

$$\varepsilon_t | \Omega_{t-1} \sim D(0, \sigma_t^2) \quad (2.71)$$

$$\sigma_t^2 = (\omega + \sum_{j=1}^m \varsigma_j \nu_{jt}) + \sum_{j=1}^q (\alpha_j \varepsilon_{t-1}^2 + \gamma_j I_{t-1} \varepsilon_{t-j}^2) + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (2.72)$$

where $y_t = 100\Delta \log CPI_t$ is CPI inflation, x_{1t} and x_{2t} are vectors of predetermined variables, μ is the mean of the process, $\phi(L) = 1 - \phi_1(L) - \dots - \phi_p(L)^p$, $\theta(L) = 1 + \theta_1(L) + \dots + \theta_q(L)^q$, $\beta(L) = 1 - \beta_1(L) - \dots - \beta_p(L)^p$, $\alpha(L) = 1 + \alpha_1(L) + \dots + \alpha_q(L)^q$ and all the roots of $\phi(L)$, $\theta(L)$, $\beta(L)$, and $\alpha(L)$ lie outside the unit circle.

When $\delta = 0$ and $b = 0$, Equations (2.70) and (2.71) describe the ARFIMA process introduced by Granger (1980), and Granger and Joyeux (1980). With $\delta \neq 0$, the model is extended to allow volatility to influence mean inflation. The innovations ε_t are assumed to follow a conditional density D , which is either Norm, STD or GED and the time-dependent heteroscedasticity σ_t^2 follows 'gjrGARCH

¹⁷see Baillie et al. (1996) for details.

model by Glosten et al. (1993)¹⁸.

The 'gjrGARCH presented in Equation (2.72) models the positive and the negative shocks on the conditional variance asymmetrically through the use of an indicator function I , where γ_j denotes the leverage term. The indicator function I takes on value of 1 for $\varepsilon \leq 0$ or 0 otherwise. Because of the presence of the indicator function, the persistence of the model in Equation (2.74) critically depends on the asymmetry of the conditional distribution used. The persistence of the model \hat{P} is given by:

$$\hat{P} = \sum_{j=1}^q \alpha_j + \sum_{j=1}^p \beta_j + \sum_{j=1}^q \gamma_j \kappa \quad (2.73)$$

where κ is the expected value of the standardised residuals $z(t)$ below zero (effectively the probability of being below zero):

$$\kappa = E(I_{t-j} z_{t-j}^2) = \int_{-\infty}^0 f(z, 0, 1, \dots) dz \quad (2.74)$$

where f is the standardised conditional density with any additional skew and shape parameters.

2.13 The Framework of Cuestas and Gil-Alana (2015) based on Chebyshev Polynomial

This empirical section starts by conducting a test for potential presence of nonlinearities in the context of fractional integration in CPI inflation series of Ghana and South Africa. In particular, the researcher assumes a non-linear trend model based on the Chebyshev polynomials in time, where the errors are

¹⁸Details of this model is provided in Chapter 5.

fractionally integrated, $I(d)$. The specific model is:

$$y_t = \sum_{i=0}^m \theta_i P_{i,T}(t) + x_t, \quad t = 1, 2, \dots, \quad (2.75)$$

where y_t denotes the observed time series, m indicates the order of the Chebyshev polynomial and x_t is fractionally integrated $I(d)$, such that:

$$(1 - L)^d x_t = \mu_t, \quad t = 1, 2, \dots, \quad (2.76)$$

where μ_t is a covariance stationary $I(0)$ process with a spectral density function that is positive and finite at the long-run or zero frequency. The Chebyshev polynomials $P(i, T)$ in Equation (2.78) is defined by:

$$P_{0,T}(t) = 1 \quad (2.77)$$

$$P_{i,T}(t) = \sqrt{2} \cos \frac{i\pi(1 - 0.5)}{T}, \quad t = 1, 2, \dots, T, \quad i = 1, 2, \dots, \quad (2.78)$$

For a detailed description of these polynomials, see Hamming (1973), and Smyth (1998). If $m = 0$, the model contains an intercept; if $m = 1$, a linear trend is also included; and if $m > 1$, the model becomes non-linear; and the higher the value of m , the less linear the approximated deterministic model becomes (Caporale et al., 2015). Cuestas and Gil-Alana (2016) recommended a simple method that is basically a slight modification of Robinson (1994). They considered a set-up in Equations (2.80) and (2.81) testing the null hypothesis:

$$H_0 : d = d_0 \quad (2.79)$$

for any real value d_0 . Under H_0 :

$$y_t^* = \sum_{i=0}^m \theta_i P_{i,T}^* + x_t = (1 - L)^{d_0} y_t, \quad t = 1, 2, \dots, \quad (2.80)$$

where

$$P_{i,T}^* = (1 - L)^{d_0}(t) \quad (2.81)$$

and given the linear structure of the relationship and the $I(0)$ nature of the error term x_t , the coefficients in Equation (2.80) can be estimated by standard least square methods (OLS/GLS). Cuestas and Gil-Alana (2016) proposed a Lagrange Multiplier test of H_0 in Equations (2.80) and (2.81) that is asymptotically standard normal, $N(0, 1)$.

Linear models have been adopted, in case of rejection of the non-linear hypothesis. We have considered a model of the form:

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1 - L)^d x_t = \mu_t, \quad t = 1, 2, \dots, \quad (2.82)$$

where y_t refers to the inflation series, β_0 and β_1 are the coefficients on the intercept and a linear trend respectively, x_t is assumed to be an $I(d)$ process, and μ_t is $I(0)$. Given the parametric nature of this method its functional form must be specified. We considered two cases: uncorrelated errors (white noise) and autocorrelated errors. In the latter case we use the exponential spectral model of Bloomfield (1973), which is a non-parametric approach of modelling the $I(0)$ errors that produces autocorrelations that decay exponentially as in the AR case (Gil-Alana, 2004).

2.14 State Space Modelling and Threshold effect of Inflation-Growth Nexus

Policy reliability, commitment, reputation and time stability are currently some of the disputed issues both in the developed and developing countries. In the

case of Ghana and South Africa, missing the targets of the macroeconomic variables such as inflation and economic growth raises the issue of policy credibility¹⁹. Again, missing the targets of variables of interest such as inflation, while framing monetary and fiscal policies, creates obligation crisis of the macroeconomic policies and therefore the potency of monetary or fiscal policy. Usually, the reason could be attributed to the behaviour of the various economic agents that informs policy formulation since most of them are time-varying. Hence, policy prescription based on constant estimated parameters could be misleading²⁰.

Reaching sustainable rapid economic growth is the objective of most countries. However, it has always been problematic to achieve such an objective as a result of many factors that affect economic growth. Economic growth and the rate of inflation are critical for macroeconomic policies. Among many variables that can be stated as determinants of economic growth is inflation (Borro, 1995). Nevertheless, there is no precise decision about the relationship between economic growth and inflation.

Research on inflation and economic growth has often led to debatable and different conclusions in both theory and empirical findings. The first contentious issue about economic growth and inflation is the relationship between them. Theories and former studies about the relationship between these two economic issues have shown that there might be no relationship (Sidrauski, 1967), or there could be a negative relationship (Fisher, 1993), or a positive relationship (Mallik and Chowdhury, 2001). Today we are not only confronted with a simple question of relationship, but also the level of inflation that can influ-

¹⁹Recently South African inflation exceeded its target of 3%-6% recording 7% in March 2016. Ghana on the other hand had never achieved its target of 6%-9% since 2007, even though some pockets of success were achieved during 2010-2012.

²⁰The issue of parameters that are invariant over time in modelling has come under scrutiny with the introduction of Lucas's critique, who argued that expectations of economic agents are determined by the policy changes (monetary/fiscal policy rules) (Lucas, 1976).

ence economic growth. From the structuralist's point of view, inflation has a positive effect on economic growth, whereas monetarists perceive inflation as detrimental to economic growth. Indeed, both views have their own explanation for the possible reason why inflation has a positive effect or negative effect on economic growth. For example, the neo-classical theory believes that inflation can increase economic growth by shifting the income distribution in favour of higher saving capitalists. But Keynesians are of the opinion that inflation may increase growth by raising the rate of profit, hence increasing private investment. The second controversial issue is the causal relationship between inflation and economic growth.

The issue of the forecasting power of inflation for economic growth and vice-versa has also been debatable. Granger causality assesses whether there is any potential predictive power for one indicator (inflation) against the other (GDP). For instance, inflation Granger cause economic growth means inflation contains information about future economic growth. Owing to the controversial issues about economic growth and inflation, our study examines the relationship between inflation and other macroeconomic variables such as growth rate per capita measured by the GDP, total exports of goods and services, total imports of goods and services, total investments and the population, among others, using the state space model of unobserved component with Kalman Filter from two perspectives. First, the study investigates the stability of the time-varying parameters of the basic structural model (BSM) of Ghana and South Africa. Second, the study estimates the threshold effect of inflation on economic growth using state space model.

2.14.1 State space model of unobserved components with Kalman filter

The state space model offers a flexible tool to estimate the coefficients of the model that are inherently time-varying, making economic relationships potentially unstable (Koopman et al., 2006). This model allows the researcher to model an observed time series as being explained by a vector of (possibly unobserved) state variables which are driven by a probabilistic process. The model can also be utilised in the following situations: (i) to determine the relationship between growth rate and inflation changing over time (as the case may be in this research) and (ii) to establish whether inflation has a permanent or transitory components, among others. Real Business Cycle (RBC) model and Neo-Keynesian models that come under Dynamic Stochastic General Equilibrium (DSGE) model are also built under state space framework. These models actually apply different rules in policy design to address time-varying relationship in the model so that the estimated parameters are considered structural and profound. The Kalman filter, named after Kalman (1960), is a particular algorithm that is used to solve state space models in linear and non-linear cases.

2.14.2 Estimation of state space model

State space modelling by Koopman et al. (2006) consists of a measurement (observation) equation and a state (transition) equation where the latter expresses the dynamics of the state variables while the former relates the observed variables to the unobserved state vector²¹. Let y_t denote an $N \times 1$ time series of observations whose development over time can be characterised in terms of an

²¹State space model in addition, allows for the inclusion of the AR, $\sum_{i=1}^p \phi_i y_{t-i}$ terms and explanatory variables, $\sum_{i=1}^m \beta_j y_{tjt}$, where the former represents the “momentum” of the time series as it relates to its past observations and the latter denotes the causal factors that are supposed to affect the time series in question. For example, regressing the causal relation between GDP and inflation, imports, etc.

unobserved state vector β_t of dimension $M \times 1$. Based on this, a standard state space formulation can be expressed as follows²²:

$$y_t = Z_t' \beta_t + d_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, H_t) \quad (2.83)$$

$$\beta_t = T_t \beta_{t-1} + c_t + R_t \nu_t, \quad \nu_t \sim N(0, Q_t) \quad (2.84)$$

Equation (2.83) denotes the measurement equation, where y_t represents an $n \times 1$ vector of measured variables, β_t is the state vector of unobserved variables (assumed unobserved components such as trend, season, cyclical and irregular components) of dimension $n \times p$ and $\varepsilon_t \sim N(0, H_t)$. Similarly, Equation (2.84) represents the state equation, where R_t denotes the matrix of parameters and $\nu_t \sim N(0, Q_t)$ with H_t , and Q_t referring to the hyper-parameters of the model. The $m \times 1$ vector of c_t and $n \times 1$ of d_t are deterministic parts of the state and observation model equations, in that order.

The innovations ε_t and ν_t are assumed to be uncorrelated with each other in all time periods, i.e. $\forall(s, t), E(\varepsilon_t, \nu_s) = 0$ and $\forall(\varepsilon_t, A'_0) = 0$ is uncorrelated with initial state. Once the model is set in the state space form, the Kalman filter can be used to estimate state vector by filtering. The Kalman filter provides estimates of the unobserved variable which plays a central part in estimating changes. Indeed, the reason for filtering is to update one's knowledge of the state vector as soon as a new observation y_t arrives. Kalman filter can be described as an algorithm for the unobserved components at time t based on the availability of information at the same period. The estimates of any other desired parameters together with hyper-parameters can be obtained by EML algorithm adopted by Shumway and Stoffer (1982). Estimations of the states through Kalman filter process involve three stages: the Initialisation Stage

²²We refer the reader to state space models by Hamilton (1989) Handbook on Econometrics, volume IV for more information.

(InS), Prediction Stage (PrS) and Updating Stage (UpS) defined as follows:

$$\text{InS} : \beta_{0 \setminus 0}, P_{0 \setminus 0} \quad (2.85)$$

$$\text{PrS} : \hat{\beta}_{t \setminus t-1} = \mu + F\hat{\beta}_{t-1 \setminus t-1} \quad (2.86)$$

$$P_{t \setminus t-1} = FP_{t-1 \setminus t-1}F^T + Q_t \quad (2.87)$$

$$\text{UpS} : K_t = P_{t \setminus t-1}H_t^T((H_t)P_{t \setminus t-1}H_t^T + R_t)^{-1} \quad (2.88)$$

$$\hat{\beta}_{t \setminus t} = K_t(y_t - H_t\hat{\beta}_{t \setminus t-1}) \quad (2.89)$$

$$P_{t \setminus t} = (I - K_tH_t)P_{t \setminus t-1} \quad (2.90)$$

where $\hat{\beta}$ estimates state, F is the state transition matrix, P is the state variance (i.e. errors emanating from the process) matrix, y_t represent the measurement variable, H in the measurement matrix (i.e. mapping measurements onto state), K is the Kalman gain and R is the measurement variance matrix (i.e. error from measurements). The subscripts $t \setminus t$ denote current time periods, with $t \setminus t - 1$ representing the previous time period and $t - 1 \setminus t - 1$ denoting intermediate steps. $\beta_{0 \setminus 0}$ and $P_{0 \setminus 0}$ are the vectors of initial states and covariance matrix, in that order. The covariance matrix $P_{0 \setminus 0}$ portrays the noise of $\beta_{0 \setminus 0}$. If the vector $\beta_{0 \setminus 0}$ and the covariance matrix $P_{0 \setminus 0}$ are not given as prior, $\beta_{0 \setminus 0}$ is assumed to be zero, and with a larger number of diagonal elements of matrix $P_{0 \setminus 0}$.

Equations (2.83) and (2.84) are a set of prediction equations with the former being the expected value of the transition equation $E(\mu + F\hat{\beta}_{t-1}\nu_t)$ and the latter described as $VAR(\mu + F\hat{\beta}_{t-1}\nu_t)$. Equations (2.88), (2.89) and (2.90) are a set of update equations, with K_t representing the Kalman gain which represents the weight given to the new information. In parenthesis are the prediction errors, which contain new information relative to the old news. When K_t increases

as a result of uncertainty regarding the state (model noise), it is said to have heavy weight on new information. On the contrary, if the value of K_t falls as result of R_t , the shock is said to be less informative. Again, Equation (2.88) updates information of $t - 1$ adjusted by K_t which is determined by the equation of the prediction error ($y_t - H_t \hat{\beta}_t \setminus t - 1$).

The second part addresses the core issue of our study, which is the threshold effect of inflation on economic growth of Ghana and South Africa. Indeed several studies have been conducted in this area with a number of them using Khan and Senhadji (2001) approach. Our study applies state space model capable of estimating the threshold effect of inflation together with unobserved components with Kalman filter, i.e:

$$y_{GDP_t}^* = y_t^T + \beta_1 inf_t + \beta_2 D_t (inf_t - \delta) + z_{it} + \varepsilon_t \quad (2.91)$$

where $y_{GDP_t}^*$ denotes growth rate per capita measured by GDP, y_t^T represents trend component, inf_t denotes the CPI inflation series, z_{it} represents exogenous or explanatory variables including: total investments, total exports of goods and services, and total imports of goods and services and growth rate in population²³, ε_t is the error term and D_t is the dummy variable which defines the threshold level δ of inflation as:

$$D_t = \begin{cases} 1, & \text{if } inf_t > \delta, \\ 0, & \text{if } inf_t \leq \delta \end{cases}$$

²³Population, total investments, total imports of goods and services, and total exports are used as control variables. The reason for choosing these variables is their authenticity in empirical literature on growth. Samuelson and Solow (1956) who developed the first neo-classical models for growth, took the rate of population growth as one of the exogenous variables in their model to show that the faster the rate of population growth, the poorer the country. Fisher (1993) included investment in his model to show that inflation reduces growth by reducing investment and productivity growth. In another research, Mankiw et al. (2004) also added investments and population growth in their model.

2.15 Empirical Evidence of Inflation and Economic Growth Nexus

This section presents some empirical research on inflation-growth nexus taking two items into consideration. These are the issues of stability of time-varying parameters and threshold effect on economic growth of Ghana and South Africa. Early applications of Kalman filter to solve state space models in economics dates back to 1982, when Fama and Gibbons (1982) modelled the unobserved ex-ante real interest rate as a state variable that follows an AR(1) process. Clark (1987) used an unobserved components model to decompose quarterly real Gross National Product (GNP) data into two independent components: stochastic trend component and cyclical component. Stock and Watson (1993) used state space model to identify unobserved variables that represent the state of business cycle. Koirala (2013) applied the state space model to examine the time-varying parameters of inflation in Nepal. After applying the Kalman filter technique for the estimation of coefficients of the random walk model they found non-constant time-varying parameters of the constant (level) and AR(1) coefficients of inflation over the long-run. To our knowledge, in the Ghanaian and South African contexts, there is no research conducted on the stability of time-varying parameters and threshold effect of inflation on economic growth using state space model of unobserved components with Kalman filter. This research is essential, especially in the area of policy formulation, the gap of which is filled by our study.

On the issue of inflation-growth nexus, Fisher (1993) studied the role of macroeconomic factor in growth for 93 countries using macroeconomic variables, including inflation. He applied a simple alternative to the mixed regression model and found that inflation affects economic growth and that inflation negatively affects growth by reducing the investment, and by reducing the rate of

productivity. Fisher (1993) established that inflation could distort price mechanism, which will interfere with efficiency of resource allocation and thus stimulate economic growth. Another study by Borro (1995) established the relationship between inflation and economic growth using data from 100 countries covering the period of 30 years. In that study, Borro (1995) included other determinants of economic growth, in addition to inflation, using a system of regression equation model. The results of his study indicated that an increase in inflation by 10% per year led to a reduction in growth rate of real per capita GDP by 0.2%-0.3% per year, and a decrease in the ratio of investment to GDP by 0.4%-0.6%. However, his results only became significant when high inflation experiences were included in the sample.

Motley (1998) examined the effect of inflation on economic growth rate of real GDP. He extended the model of Mankiw et al. (2004) which is based on Solow growth model, by allowing for the possibility that inflation tends to reduce the rate of technical change. The results of Motley's study indicated a negative relationship between inflation and growth rate of real GDP. Khan and Senhadji (2001) investigated the relationship between inflation and economic growth separately for industrial and developing countries. The researcher employed new econometric techniques initially developed by Chan and Tsay (1998) and Hansen (1999), to show the existence of threshold effects in the relationship between inflation and economic growth. They actually used an unbalanced panel data containing 140 countries for the period 1960-1998. The estimated value of threshold was found to be 1%-3% and 11%-12% for developed and developing countries, respectively. They also found that the threshold level for industrialised countries is lower than that of developing countries. They concluded that inflation level below the threshold level (of inflation) has no influence on growth, but inflation rates above the threshold level have a significant negative effect on growth.

Mubarik (2005) examined the causal relationship between inflation and economic growth. The test result indicated that the causality between the two variables is unidirectional, i.e. inflation causes GDP growth, but not vice-versa. By employing VAR Granger causality test, Chimobi (2010) studied the relationship between inflation and economic growth in Nigeria and found a unidirectional causality from inflation to growth. Odhiambo (2012) also examined the short-run or long-run causal relationship between inflation, investment and economic growth in Tanzania using autoregressive and distributed lag (ARDL) bounding testing approach and found a unidirectional causal relationship running from inflation to economic growth. Mamo (2012) also conducted a study on economic growth and inflation using a panel data analysis for Sub-Saharan African countries from 1969 to 2009. His results revealed a negative relationship between inflation and economic growth. He then examined the causal relationship between inflation and economic growth using Granger causality test. The results of his study indicated that inflation can be used to predict economic growth for all the countries, except the Democratic Republic of Congo and Zimbabwe.

In Ghana, studies on the inflation threshold effect on growth are limited and virtually non-existent. This is surprising given the attention and importance inflation and growth issues are to policymakers and economists. Also, given the fact that BoG (Bank of Ghana), has been pursuing an IT framework since 2007, it would have been expected as a sequel to the monetary policy framework by the BoG, and such studies should be carried out to guide decision-making and to support the BoG in setting its inflation targets. Quartey (2010) used the Johansen co-integration methodology to examine whether the revenue maximising rate of inflation is in itself growth maximising, and found that there is a negative relationship between inflation and growth. His study also depicted an interesting result of a revenue maximising rate of 9.14% over the period

1970-2006 using the Laffer curve. He further established that the rate of inflation that is growth maximising is not a single digit one.

In another study, Frimpong and Oteng-Abayie (2010), found a threshold effect of inflation on economic growth of 11% for Ghana over the period 1960 to 2008. By estimating the threshold model using the OLS, the authors estimated a robust 11% threshold inflation level with close coefficients, after dropping growth rate of aggregate labour force and money supply growth, which were found to be insignificant in the OLS models. A closer examination of their results revealed that even at relatively lower threshold levels, inflation is still significant. The study, however, failed to check for sensitivity of the estimated coefficients across sub-samples of the full sample period to establish new evidence of the threshold effect. The study concluded by highlighting the need to extend the context of analysis to deal with lower threshold levels in search of that evidence.

Although there are earlier empirical studies on inflation in Ghana, for instance, Sowah (1994), Sowah (1996), Ocran (2007), and Asafu-Adjaye (2008), there is still scope for further empirical investigations on drivers of inflation on economic growth in Ghana. First, earlier studies have approached the subject using error correction techniques which can only predict short-run impact of the regressors. In cases where an attempt has been made to capture long-run effects, OLS estimators are used (Sowah, 1994, 1996). Ocran (2007) used the Johansen multivariate method to estimate the long-run parameters. However, the small sample properties of this estimator are not known with any certainty since simulation studies are usually based on sample sizes of 500 or more.

There are, but few research studies on inflation and economic growth in South Africa. For instance, Hodge (2006) used South African data to test whether

the data supported the findings of other cross-section studies that inflation has a negative effect on growth over the longer term. He further investigated whether higher economic growth can be gained at the cost of higher inflation in the short-run. The study made use of annual data from 1950 to 2002. The findings of the study revealed that, in South Africa, higher inflation could retard economic growth in the long-run.

Another study conducted by Phiri (2010) on inflation and economic growth, estimated the threshold level that is detrimental to finance-growth activity for the South African economy. He used quarterly data for the period of 2000-2010, and the results revealed an inflation threshold level of 8%. Later, Leshoro (2012) re-examined the inflation-growth relationship in South Africa using quarterly data from 1980 to 2010. He adopted the threshold regression model developed by Khan and Senhadji (2001) and estimated an inflation threshold level of 4% below which there is a positive, but statistically insignificant relationship between inflation and growth, and above which the relationship becomes negative and significant. Our study adopts a state space model of unobserved components to address the issue of threshold effect of inflation on economic growth. In order to bring clarity to persistency of inflation dynamics in relation economic growth, the first part of the analysis employs monthly CPI (Consumer Price Index) inflation series for the period January 1971 to October 2014 obtained from the Bank of Ghana (BoG), and for the period January 1995 to December 2014 obtained from Statistics South Africa. The second part of the study involves the estimation of threshold effect of inflation on economic growth using annual data obtained from the IMF (International Monetary Fund) database for the period 1981 to 2014, for both countries.

2.16 Summary

Inflation is often modelled as a non-stationary unit root process (MacDonald and Murphy, 1989; Ball and Cecchetti, 1990; Brunner and Hess, 1993). Moreover, if inflation contains a unit root, shocks to the series will have a long lasting effect (Greene, 2003). In such circumstances, successful inflation IT monetary policy enacted to control the path of inflation will be difficult to achieve for central banks (Gregoriou and Kontonikas, 2006). In order to avoid misalignments from the anticipated inflation level, central banks require fitting policy rules. The design and execution of economic policy rules depend on characteristics or properties of the inflation series (Dias and Marques, 2010). Hence, within the ambiance of this study, three time series properties which are subjected to investigation are structural breaks, long memory and non-linearity.

In order to determine whether or not CPI inflation follows a unit root or a stationary process, our study has subjected the CPI inflation series of Ghana and South Africa to stationarity tests using ADF, PP, KPSS, ZA, BP and the fractional integration method developed by Gil-Alana (2008). Using two or more tests helps to ascertain the true underlying process of the series, and is also useful for consolidation and confirmatory purposes.

The concept of integration order, which measures the time it takes inflation series to revert to its mean level after a shock, provides policymakers with valuable information. Our study, therefore, examines the presence of long memory (via fractional integration) in CPI inflation of Ghana and South Africa. The study employs semi-parametric methods such as LW method by Robinson (1995). This method has advantages over the (GPH) test by Geweke and Porter-Hudak (1983) (also a semi-parametric method), because it can detect the presence of memory even in the presence of structural breaks. Our study also applies modified R/S analysis (non-parametric) by Lo (1991) and EML (a

parametric method) by Sowell (1992) in order to compare results. From the literature, semi-parametric estimators are considered to be more robust compared to the parametric and non-parametric estimators because they do not require exact specification of the short memory parameters (i.e. AR and MA) and the Gaussian assumption.

Our study attempts to model persistence in the conditional mean by extending the ARFIMA process with sGARCH and 'gjrGARCH conditional heteroscedastic models to describe time-varying dependent heteroscedasticity under three different distributional assumptions and then compare results thereof. The extension of the ARFIMA model helps in capturing the remaining ARCH effects (if any) in the residuals in order to obtain a correct estimation of the fractional differencing parameter, and also aids in statistical inference and forecasting.

This study also endeavours to model the CPI inflation series of both countries with non-linear deterministic trends coupled with fractionally integrated errors based on Chebyshev polynomial using the framework of Cuestas and Gil-Alana (2016). The choice of this model is based on the fact that many of the classical models in the literature are based on linear trends, intercepts and/or at most structural breaks at fixed points in time.

The primary objective of macroeconomic policies is to attain high and sustainable output growth rates together with low and stable inflation rates (Kan and Omay, 2010), meaning that a certain degree of inflation is necessary to grease the wheels of the economy (Tempel, 2000). Hence, policymakers find it expedient to understand the relationship between inflation and economic growth, in order to ensure sound policy formulation. As a result, our study makes use of the state space model to examine the relationship between inflation and economic growth from two perspectives: (a) to investigate the stability of time-

varying parameters (such as the mean level, shocks etc.) of inflation, and (b) to estimate the threshold effect of inflation on economic growth of Ghana and South Africa. The empirical chapters that follow use longer and broader data periods than previously used, employ various long memory models and propose new models to analyse the persistency of inflation dynamics of Ghana and South Africa.

Chapter 3

Structural Breaks and Testing for Random Walk Hypothesis in Inflation Series

3.1 Introduction

As is well-known, the issue of structural change and its consequential implications for structural breaks in macroeconomic time series data, must be robustly addressed in order to ensure non-spurious results of unit root tests. There can be many reasons for structural breaks, and these can include such diverse circumstances as economic crises, policy changes or regime shifts. For this reason it is extremely imperative to test the null hypothesis of structural stability against the alternative of no break, one-time break and multiple structural breaks. Failure to allow for structural breaks in the specification of an econometric model, may lead to misleading results because they can be biased to-

wards the erroneous non-rejection of the non-stationary hypothesis (Perron, 1989, 1997; Leybourne and Newbold, 2003; Pahlavani et al., 2005; Harvie and Pahlavani, 2006).

3.2 Synopsis of Inflation Dynamics of Ghana and South Africa

Ghana, which attained its independence in March 1957, is one of the countries whose economy has experienced the devastating effects of high inflation. Having recorded some success by registering low rates of inflation in the years immediately after independence, the country had its first taste of double-digit inflation in 1964. This was followed by a brief period of relief during 1967 to 1971, with inflation below 10% per annum. Since 1972, however, inflation levels have remained generally high, ranging between 10% in 1972 and 123% in 1983 (Mackay and Sowah, 2014). The high rates of inflation have weakened the otherwise impressive macro performance since the introduction of the Economic Recovery Programme (ERP). For instance, consumer prices in Ghana increased by 17.7% year-on-year in December 2015, following a 17.6% rise in November, 2015. This was the highest inflation rate since July 2015. The biggest upward pressure came from the non-food group, partially fuelled by previous month's rise in utility prices and transport fares. The rate of inflation has been in the upward trend for the last three years (i.e. 2013-2015), as lower commodity prices and fiscal policy crisis led to a sharp drop in the Ghanaian currency, cedi, against the United States Dollar (USD), raising import prices despite the country's adoption of IT policy in May 2007 by the BoG. Inflation rate in Ghana averaged 17.12% from 1998 until 2015, reaching an all-time high of 63% in March of 2001 and a record low of 0.40% in May of 1999 (Trading Economics Report, 2016a).

Inflation in South Africa, like any other developing country, has been a major concern, with inflation rates consistently being double digit (Amusa et al., 2013). Nevertheless, through an informal IT regime during the decade of 1990-2000, considerable success was achieved in bringing down the inflation rate to lower levels. Given this success of informal IT, the SARB decided to move to a formal IT framework in February 2000, with the then Finance Minister declaring that the sole objective of the SARB would be to maintain inflation within a target band of 3%–6% (Mboweni, 2003; der Merwe, 2004). The inflation rate averaged 9.5% from 1968 to 2014, with a maximum of 20.9% in January 1986 and a minimum of 0.2% in January 2004. As at May 2014, the inflation rate was 6.6%, the highest increase since July 2009 when it stood at 6.8% (Statistics South Africa, 2014; Trading Economics Report, 2016b). Subsequently, due to monetary policy intervention, price increases have slowed down to 6.4% in August 2014, but still remain outside the target band of 3%-6%.

Inflation and its volatility (e.g. China' market volatility) can impose costs on the real economic output, and hence the welfare of the citizens of any nation (Chowdhury, 2014). Minimising the adverse economic consequences and welfare costs of increases in the inflation rate would require policy makers to have a clear understanding of the major channels through which inflation may affect the real economy. Consumer prices in South Africa improved by 4.8% year-on-year in November 2015, up from 4.7% reported in the previous month and matching market forecasts. Again, the cost of household contents and services rose at a faster pace while transport prices were flat after falling for three consecutive months (Trading Economics Report, 2016b).

The rest of the chapter is structured as follows: Section 3.3 provides descriptive statistics for the two inflation series and reviews the methodology used in the stationarity process. Section 3.4 presents the evidence of structural breaks,

while Section 3.5 provides the chapter summary and concluding remarks.

3.3 Data and Methodology

3.3.1 Descriptive statistics

The data employed in the study is the log monthly CPI inflation series of Ghana and South Africa. The data for Ghana spans from January 1971 to October 2014 and that of South Africa spans from January 1995 to December 2014. Both data sets were respectively obtained from the BoG (Bank of Ghana, 2014) and Stats SA (Statistics South Africa, 2014). Figures 3.1 and 3.2 depict the behaviour of GHCPI and SACPI, respectively²⁴.

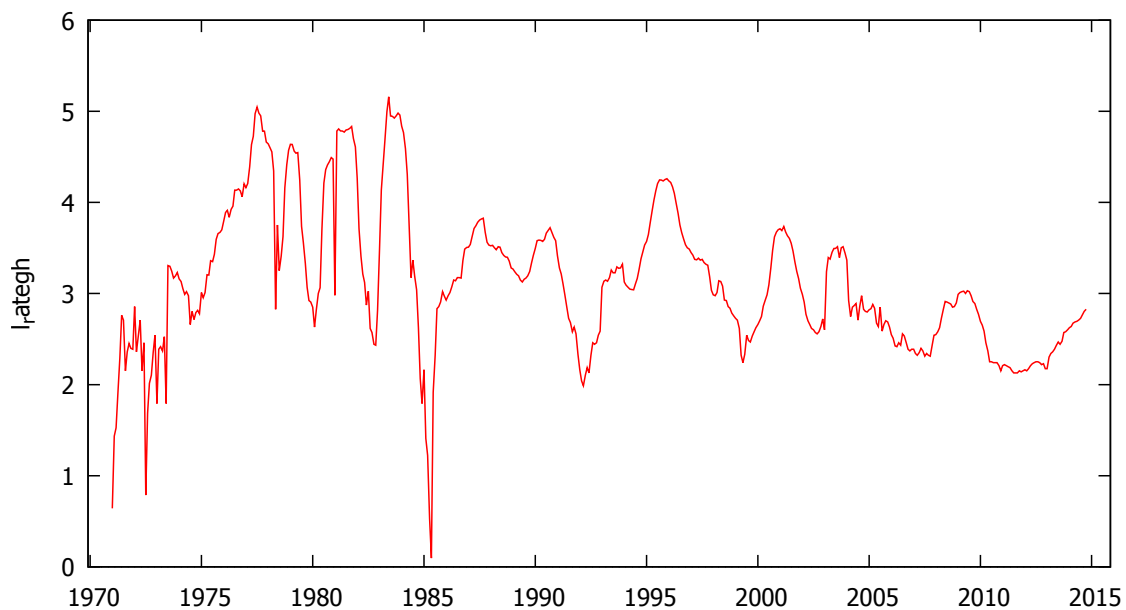


Figure 3.1: Monthly CPI Inflation series of Ghana (1971:M01-2014:M10)

²⁴The study utilised headline inflation rates from both countries, i.e. Ghana Consumer Price Index (GHCPI) and South Africa Consumer Price Index (SACPI)

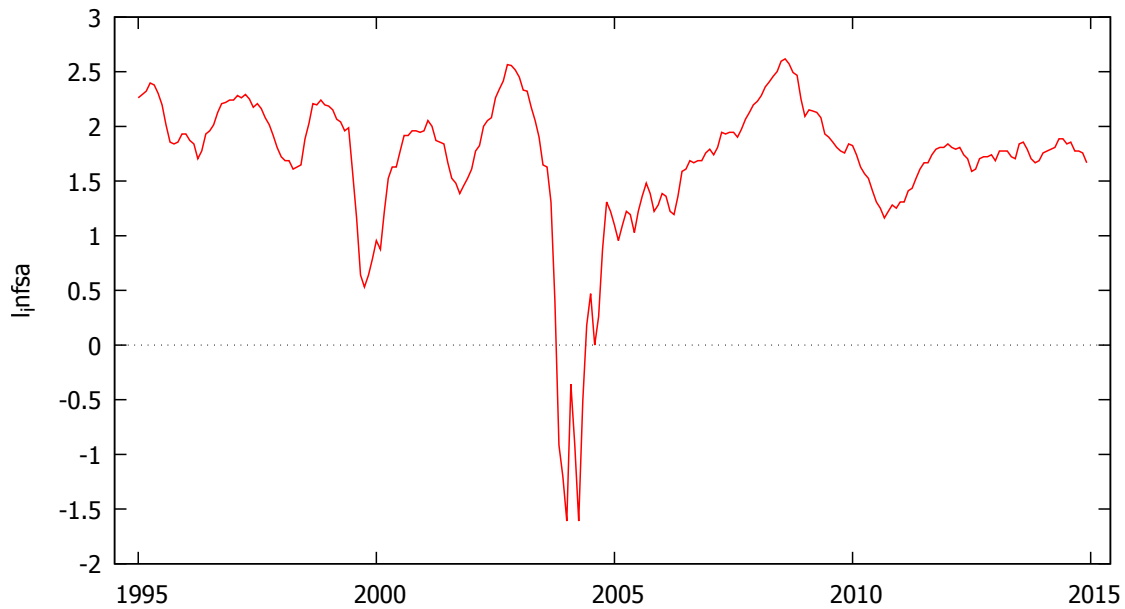


Figure 3.2: Monthly Inflation series of South Africa (1995:M01-2014:M12)

From Figures 3.1 and 3.2, it can be seen that inflation series for both countries exhibit some form of volatility and structural breaks. For instance, a close inspection of GHCPI in Figure 3.1 reveals a sharp downward trend around 1977 and 1983 to 1984, but becomes more volatile around 1993. Similarly, SACPI (see Figure 3.2) has also been subjected to some form of constraints, notably structural breaks around 2000 and 2003 to 2005 and volatility, especially between 2000 and 2009.

Table 3.1 presents the descriptive statistics of the data which include sample mean, median, maximum, minimum, variance, standard deviation, skewness and kurtosis. The highest mean return of CPI inflation is 3.12% in Ghana and the lowest is 1.68% in South Africa. The standard deviation ranges from 0.65% (SACPI the least volatile) to 0.78% (GHCPI the most volatile). Both series have excess kurtosis exceeding 3%, implying that they have thicker tails (leptokurtosis) and a higher peak than a normal distribution.

Table 3.1: Descriptive and Summary Statistics of CPI Inflation of Ghana and South Africa

Statistic	GHCPI Inflation	SACPI Inflation
Min	0.09	-1.61
Max	5.15	2.61
Mean	3.12	1.68
Median	3.02	1.79
Variance	0.61	0.42
Std.Dev	0.78	0.65
Skewness	0.29	-2.41
Kurtosis	3.49	11.27

Note: This table describes several descriptive and summary statistics, including the mean (%), standard deviation (Std.Dev), skewness and kurtosis.

3.3.2 Methodology

Dickey and Fuller (1979), Phillips and Perron (1988), and Kwiatkowski et al. (1992) tests have been used to determine whether the series is level stationary or differenced stationary. We note, however that, results from stationarity tests could be biased if structural breaks are not taken into account. Therefore, our study made use of Zivot and Andrews (1992), and Bai and Perron (1998) tests that respectively allow for single and multiple breaks to be determined endogenously. As cited earlier, one of the weaknesses of the standard unit root methods is that they have very low power, especially if the true data generating process is fractionally integrated. Hence, this thesis examines the presence of unit roots in the Ghanaian and South African inflation series, using a fractional integration framework developed by Gil-Alana (2008).

3.4 Empirical Results

Tables 3.2 to 3.5 report the results of the unit root tests with and without breaks and Tables 3.6 to 3.9 present the estimates of the fractional differencing parameter, with or without structural breaks with the framework of Gil-Alana

(2008), since the unit root results confirmed the existence of non-stationarity in the GHCPI and SACPI inflation series.

Table 3.2: Unit Root Test for GHCPI inflation series

Unit Root Tests		
Test Type	Test Statistic	
	Cont.	Cont. and Trend
ADF	-3.29(-2.86)*	-4.17(-3.41)*
PP	-4.92(-2.86)*	-5.55(-3.41)*
KPSS	0.76(0.46)*	0.16(0.14)*
ZA	-5.70(-4.80)*	-5.81(-5.08)*

Note: *In all cases of constant, and constant and trend, the hypothesis of stationarity is rejected at 5% level of significance for ADF, PP, KPSS and ZA tests since the test statistics fall outside the critical regions, hence the GHCPI is non-stationary. In parenthesis are the critical values.

Table 3.3: Unit Root Test for SACPI inflation series

Unit Root Tests		
Test Type	Test Statistic	
	Cont.	Cont. and Trend
ADF	-3.57(-2.87)*	-3.57(-3.42)*
PP	-3.37(-2.87)*	-3.40(-3.42)*
KPSS	0.14(0.46)*	0.12(0.14)*
ZA	-5.70(-4.80)*	-5.81(-5.08)*

Note: *In all cases of constant, and constant and trend, the hypothesis of stationarity is rejected at 5% level of significance for ADF, PP, KPSS and ZA tests since the test statistics fall outside the critical regions, hence the GHCPI is non-stationary. In parenthesis are the critical values.

From Tables 3.2 and 3.3, the hypothesis of stationarity is rejected at 5% significance level in all cases after applying ADF, PP and KPSS tests. After allowing for one and multiple structural break(s) in the unit root null hypothesis, the ZA and BP tests were applied. Similarly, the hypothesis of stationarity was rejected. Hence, both GHCPI and SACPI inflation series are non-stationary.

Next, we investigate the presence or absence of multiple breaks in GHCPI and

SACPI using Bai and Perron (1998)(BP) test. The BP test was actually applied to investigate multiple breaks in the level and persistence in the series with a trimming parameter arbitrarily permitted to be 0.15 and a maximum number of breaks set to $m = 5$ through the sequential procedure of 5% significance level. To impose the minimum structure on the data, the researcher allowed for different distributions of regressors and the errors in different sub-samples. The results are depicted in Table 3.4 and Table 3.5 for GHCPI and SACPI inflation series, respectively.

Table 3.4: Multiple breakpoint tests of $L + 1$ vs L sequentially determined breaks for GHCPI inflation series

Sequential F-statistic determined break: 3			
	Break Date	F-stats	CR*
	0 vs 1*	115.03	8.58
	1 vs 2*	34.12	10.13
	2 vs 3*	120.67	11.14
	3 vs 4	8.40	11.83
	Break Date	Sequential	Repartition
	1	2004M02	1977M07
	2	1984M06	1984M06
	3	1977M07	2004M05

Note: Breaking variable: GHCPI inflation series for the sample period between 1971M1-2014M12. Breaking point options for the trimming parameter: 0.15, maximum breaks=5. *Significant at the 5% level. Bai and Perron (2003) critical values (CR).

In Table 3.4, it is observed that GHCPI inflation series had been subjected to three significant breaks corresponding to 1977M07 (i.e. July 1977), 1984M06 (i.e. June 1984) and 2004M02 (i.e. February 2004), respectively. These break dates coincided with some difficulties the Ghanaian economy experienced as a result of inflation. For example, after experiencing some price stability over a five-year period after independence (i.e., 1957-1961) due to inflation hovering around a single digit in July 1977, the annual inflation rate exceeded 100%

Table 3.5: Multiple breakpoint tests of $L + 1$ vs L sequentially determined breaks for SACPI inflation series

Sequential F-statistic determined break: 4			
	Break Date	F-stats	CR*
	0 vs 1 *	59.72	8.58
	1 vs 2 *	24.85	10.13
	2 vs 3 *	54.53	11.14
	3 vs 4 *	35.41	11.83
	4 vs 5	0.00	12.50
	Break Date	Sequential	Repartition
	1	1999M07	1999M07
	2	2006M06	2003M06
	3	2009M08	2006M06
	4	2003M06	2009M08

Note: Breaking variable: SACPI inflation series for the sample period between 1995M1-2014M12. Breaking point options for the trimming parameter: 0.15, maximum breaks=5. *Significant at the 5% level. Bai and Perron (2003) critical values (CR).

(Sowah and Kwakye, 1993). In the period 1983-1984, Ghana continued to experience high inflation due to external shocks, unsustainable macroeconomic policies and exchange rate depreciation. The last significant break of January 2004 is associated with the period when Ghana started to experience a disinflation from 27.1% to about 10% (Sowah and Kwakye, 1993).

Similarly a close inspection of Table 3.5, shows that SACPI inflation series was also subjected to four significant breaks corresponding to 1999M07, 2003M06, 2006M06 and 2009M08. These break dates also coincided with significant events in the South African economy as a result of inflation. From Figure 3.2 and Table 3.5, it is evident that SACPI inflation has undergone severe fluctuations, especially during 2000Q3 to 2009Q3, with two big shocks in 2004Q4 (as a result of massive depreciation of the South African rand) and 2008Q3 (as a result of global increase in food price together with a rise in oil prices, and a positive shock which took place in 2004Q1 (as a result of South African rand

appreciation), before stabilising near the upper bound of 6% during the financial crisis (Kabundi et al., 2014)²⁵.

In the literature, it is well-known that ADF, PP, KPSS, ZA and BP tests exhibit low power and can lead to a wrong acceptance of the hypothesis of non-stationarity (Diebold and Ruderbusch, 1991). Our study makes a contribution to literature, especially to GHCPI and SACPI inflation studies, by applying a recently robust framework developed by Gil-Alana (2008) to determine the true order of integration. This contribution could be cascaded to other African countries and developing countries, in general.

Table 3.6: Estimates of the fractional differencing, d parameter for GHCPI inflation series

Std.cases	No regressors	Intercept	Linear time trend
White Noise	0.95(0.89,1.01)	0.93(0.88,0.99)	0.93(0.88,0.99)
Bloomfield	1.16(1.01,1.31)	1.15(0.98,1.31)	1.14(0.98,1.30)

Note: In parenthesis, are the 95% confidence band; In bold are the selected specifications; Bloomfield autocorrelation.

Table 3.6 displays the results for the three standard cases examined in the literature, that is (i) the case of no deterministic terms, $\beta_0 = \beta_1 = 0$, a priori in the undifferenced Equation (2.18); (ii) including only a constant term, β_0 unknown, $\beta_1 = 0$ a priori, and (iii) the case of a constant with linear trend, β_0 and β_1 unknown. The results were fairly similar in all three cases. However, if we assume that the errors are white noise, the estimated value of d is found to be slightly below 1 (0.93) and the 95% confidence interval leads to the rejection of the null hypothesis of unit root in favour of mean reversion $d < 1$. On the other hand, assuming autocorrelation, the estimated value of d is found to be 1.15, hence the unit root null cannot be rejected.

²⁵ Q_i , $i = 1, 2, 3, 4$ denotes the quarters in a year, e.g. Q_1 refers to January to March.

Table 3.7: Estimates of d in the context of multiple breaks for GHCPI inflation series

White noise			
Sample	No regressors	Intercept	Linear time trend
1st sub-sample	0.76(0.61,0.93)	0.51(0.44,0.69)	0.57(0.37,0.81)
2nd sub-sample	0.95(0.81,1.14)	1.17(1.04,1.34)	1.17(1.04,1.34)
3rd sub-sample	0.87(0.78,0.97)	0.84(0.76,0.93)	0.84(0.76,0.93)
4th sub-sample	0.97(0.88,1.10)	1.06(0.97,1.17)	1.06(0.97,1.17)
Bloomfield			
1st sub-sample	1.11(0.77,1.54)	0.96(0.39,1.53)	0.98(0.18,1.45)
2nd sub-sample	0.88(0.59,1.37)	1.53(0.99,2.14)	1.52(0.99,2.16)
3rd sub-sample	0.86(0.73,1.04)	0.95(0.75,1.21)	0.95(0.77,1.21)
4th sub-sample	1.02(0.85,1.29)	1.15(0.98,1.35)	1.14(0.98,1.34)

Note: The break dates are 1977M7, 1984M4 and 2004M01; In parenthesis, are the 95% confidence band; In bold are the selected specifications; Bloomfield autocorrelation.

Again, testing for the existence of multiple breaks in GHCPI inflation series in the context of fractional integration using the approach developed by Gil-Alana (2008), revealed three breaks (see. Table 3.7), occurring at 1977M7, 1984M4 and 2004M01, which are almost the same as in the unit root case (see Table 3.4). Focusing on the degrees of integration, the results changed once more depending on the specification of the error term. Thus, if μ_t is white noise, we observe mean reversion $d < 1$ in the first and third sub-samples; the estimated value of d is statistically higher than 1 in the second sub-sample, and the unit root hypothesis $d = 1$ cannot be rejected in the fourth sub-sample. However, for autocorrelated errors, the unit root null cannot be rejected in any of the four sub-samples, due to the wide confidence intervals of the estimated values, though these values substantially change from one sub-series to another.

Similarly, Table 3.8 displays the results of the three standard cases examined in the literature for SACPI inflation series, that is (i) the case of no deterministic terms, $\beta_0 = \beta_1 = 0$ a priori in the undifferenced Equation (2.18), (ii) includ-

Table 3.8: Estimates of the fractional differencing, d parameter for SACPI inflation series

Std.cases	No regressors	Intercept	Linear time trend
White Noise	1.21(1.12,1.33)	1.45(1.34,1.59)	1.45(1.34,1.59)
Bloomfield	1.11(1.89,1.39)	1.18(0.96,1.45)	1.14(0.97,1.45)

Note: In parenthesis, are the 95% confidence band; In bold are the selected specification; Bloomfield autocorrelation.

ing only a constant, β_0 unknown and β_1 a priori and (iii) the case of a constant with a linear time trend, β_0 and β_1 unknown, under the two assumptions of uncorrelated and correlated errors. It can be observed that if μ_t is white noise, the estimates of d are significantly higher than 1. However, under the most realistic assumption of autocorrelation, the $I(1)$ hypothesis cannot be rejected, and the estimated value of d is found to be 1.18 for the case of an intercept.

Next, the method proposed by Gil-Alana (2008) is applied on the most significant break found to be August 2003 in SACPI inflation series. The estimated values of d for each of the two sub-samples are displayed in Table 3.9.

Table 3.9: Estimates of the fractional differencing, d parameter for SACPI inflation series

1st sub-sample			
Std.cases	No regressors	Intercept	Linear time trend
White Noise	1.15(0.98,1.36)	1.51(1.36,1.69)	1.51(1.36,1.69)
Bloomfield	0.79(0.50,1.32)	1.39(0.88,1.89)	1.39(0.89,1.87)
2nd sub-sample			
Std.cases	No regressors	Intercept	Linear time trend
White Noise	1.07(0.97,1.21)	1.37(1.22,1.60)	1.38(1.22,1.62)
Bloomfield	1.02(0.85,1.22)	1.01(0.83,1.23)	1.01(0.83,1.23)

Note: In parenthesis, are the 95% confidence band; In bold the selected specification; Bloomfield autocorrelation; 1st and 2nd sub-samples correspond to 1995M01-2003M8 and 2003M9-2014M12 respectively.

From Table 3.9, there is evidence of unit roots in the two sub-samples. However, a substantial reduction in the degree of integration, d in the second sub-sample from 1.37 to 1.01 can also be observed, implying evidence of no mean reversion, especially during the first sub-sample and permanent effects of shocks coupled with evidence of long memory in the CPI inflation rates of South Africa. Investigating deeper established if there has been an abrupt change in the estimation of the differencing parameter d , a recursive estimate of d was conducted, starting with a sample of 60 observations, and adding one observation each time. Based on the results reported in Tables 3.8 and 3.9, the model with an intercept and Bloomfield disturbances could be adopted for SACPI inflation.

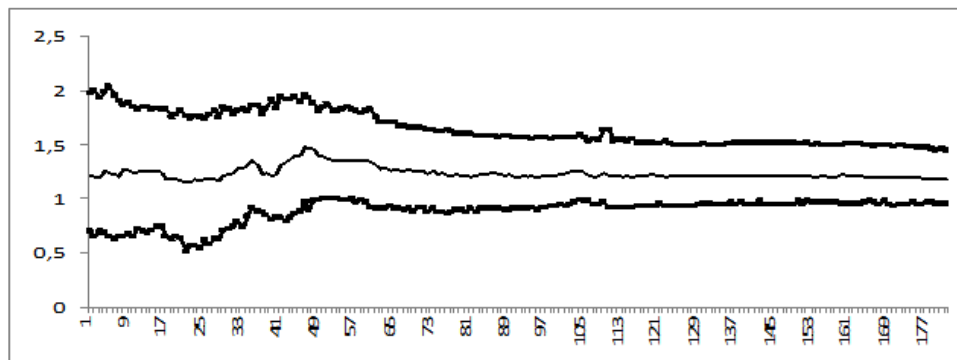


Figure 3.3: Recursive estimates of d for SACPI inflation series

Figure 3.3 shows that the estimated values of d behave relatively stable across the sample, with some increase around the 46th estimate (that corresponds to October 2003), thereafter declining very slowly. The unit root null hypothesis cannot, therefore, be rejected in any single case supporting the hypothesis of unit root and permanency of the shocks.

3.5 Summary

The unit root hypothesis is an important method for testing whether shocks (e.g. demand and supply, market volatility, etc.) to a particular series (e.g. GHCPI/SACPI inflation series) have permanent or transitory effect, also known as persistence. The presence of a unit root is indicative of the fact that a series has no tendency to recover equilibrium after experiencing an economic shock. Nonetheless, if it is possible to reject the unit root hypothesis, then this is equivalent to saying that shocks to a series only have a transitory effect.

In this chapter the unit root behaviour of GHCPI and SACPI inflation series has been examined. Results from the conventional unit root (ADF, PP and KPSS) approaches suggested that the inflation for GHCPI and SACPI is non-stationary. After allowing for structural breaks in the null hypothesis and applying ZA and BP tests, similar results were obtained. This means that, shocks to GHCPI and SACPI inflation will persist for a long time (or may also possess long memory) and this may give the central banks of these two countries an additional incentive to intervene by designing and enacting policies that could steer inflation back to an equilibrium path. It is important since the cost of inaction on the part of monetary authorities could diverge inflation further from its long equilibrium value. Results from this chapter also revealed that both series had been subjected to multiple breaks (evidence of shocks) with estimated breaks for GHCPI found to be 1977M07, 1984M06 and 2004M05, and for SACPI found to be 1999M07, 2003M06, 2006M06 and 2009M08. All these estimated break dates coincided with major economic shocks in the respective economies.

Using fractional integration methods, the results indicate strong evidence of non-stationarity and persistence, and the unit root is rejected in favour of higher orders of integration under uncorrelated errors, but evidence of unit

roots is found under the more realistic case of autocorrelated errors for GHCPI and SACPI inflation series. Again, in the context of fractional integration, we obtain similar results, supporting the hypothesis of unit root with multiple breaks in the two inflation series.

The most important implication of this study is that random shocks have permanent effects on the long-run level of inflation series and that fluctuations are not transitory, confirming the research by Nelson and Plosser (1982), and Arize and Malindrtos (2012). Indeed, the fractional integration parameter d , obtained for GHCPI inflation series, appears to have increased from 0.51 to 1.17 and also an upsurge from 0.96 to 1.15 respectively under white noise, Bloomfield errors and also in the context of multiple breaks. These differences in the value of d might be due to monetary policies that were in place during that dispensation, hence the need to introduce IT monetary policy in March 2007. The same cannot be said for SACPI inflation series since its fractional integration parameter d , a measure of inflation persistence decreased from 1.51 to 1.39 and 1.37 to 1.01 respectively, under white noise, Bloomfield errors and also taking the multiple structural breaks into account. This reduction in inflation persistence can be attributed to the introduction of IT monetary policy in February 2000.

Subsequent chapters delve deep into the nature of inflation persistence, induced by presence of long memory, structural breaks, volatility and non-linearity (or regime shifts) captured in the results. This will assist in ascertaining the likely impact of shocks and the future trajectory of inflation in the two countries in relation to IT monetary policy, since the efficacy of monetary policy depends on inflation persistence.

Chapter 4

Analysis of Long Memory and ARFIMA Modelling of Inflation Series

4.1 Introduction

In macroeconomic theory and economic policy, changes in the general price level or the rate of inflation, play a critical role. For example, one of the reasons behind the adoption of IT policy by Ghana and South Africa and the treaty espoused by the European Monetary Union, known as the Maastricht Treaty, was to improve convergence of inflation. Moreover, the last two decades of macro and financial economic research have resulted in a huge array of important contributions in the area of long memory modelling, both from a theoretical and an empirical viewpoint. From a theoretical perspective, considerable effort has been focused in the areas of testing and estimation, with significant contribu-

tions by Granger (1980), Granger and Joyeux (1980), Hosking (1981), Geweke and Porter-Hudak (1983), Lo (1991), Sowell (1992), Ding et al. (1993), Cheung and Diebold (1994), and Robinson (1995), among others. The empirical analysis of long memory models has equally seen a remarkable treatment, including studies conducted by Diebold and Rudebusch (1989), Diebold and Ruderbusch (1991), Hassler and Wolters (1994), Gil-Alana and Robinson (1997), Bos et al. (2002), and Hyung et al. (2006). Hence, a considerable number of publications on the subject is not surprising, given the importance of long memory models in economics (Granger, 1980)²⁶.

Recent statistical literature has concerned itself with a study of long memory models which go beyond the presence of random walks and unit roots in univariate time series representations, and ARFIMA(p,d,q) processes are known to be capable of modelling long-run persistence. This was introduced by Granger and Joyeux (1980), who generalised Box-Jenkins models, when the order of integration d was allowed to be fractional. Since modelling inflation series as a fractionally integrated process appears to improve our understanding of inflationary dynamics, this thesis extends the existing long memory (discovered in Chapter 3) analysis of Ghana and South Africa along two lines.

First, a number of empirical studies have suggested that the use of multiple methodologies can improve the robustness of long memory results. Using four different techniques, namely non-parametric (modified R/S analysis), parametric (EML) and semi-parametric (GPH and LW) tests, we can improve the robustness of statistical results of short memory against long memory alternative²⁷.

²⁶A time series exhibits long memory or long range dependence when there is significant dependence between observations that are separated by a long period of time. Characteristics of a long memory time series are an autocorrelation function ρ_k that decays hyperbolically to zero and a spectral density function $f_x(\cdot)$ that is unbounded in the neighbourhood of zero frequency.

²⁷See Sections 2.5.1 and 2.5.2 of Chapter 2 for more details on these methodologies.

Second, this chapter models the long memory process in the conditional mean using the ARFIMA model, in order to capture the short and long-range effects of the inflation response to shocks (e.g. demand/supply shocks). Interestingly, what distinguishes this study from others is the emphasis on the method of estimating the fractional differencing parameter and our quest to obtain an appropriate model to describe the inflationary dynamics of the two countries. This is because a clear appreciation of inflation behaviour will inform policymakers to consolidate the adoption of IT policy or adapt other policies to control inflation. The motivation behind this thesis is purely on the grounds of economic policy and statistical inference. This study makes a contribution to the existing literature on inflation in the following ways: (1) establishing the existence or non-existence of long memory in the inflation of Ghana and South Africa, and (2) knowing the properties of long memory in inflation may provide useful information to policymakers as well as to investors' decisions on investment and risk management since Ghana and South Africa are the only two countries in Sub-Saharan Africa with IT policy.

4.2 Empirical Evidence of Long Memory and ARFIMA Modelling

In Ghana, a number of studies have been done on inflation modelling and forecasting. However, the use of the fractionally integrated approach in describing inflation dynamics is very limited. For instance, Tweneboah et al. (2015) conducted a study on long memory behaviour of real interest rates in Ghana using ARFIMA and FIGARCH models, and found them to exhibit an indistinguishable integration property. Alagidede et al. (2014) also conducted a study on a regional analysis of inflation dynamics in Ghana with respect to persis-

tence, causes and policy implications, using fractional integration. Notable evidence of asymmetries in the degree of persistence was found in both regional and sectorial areas of the economy. Atta-Mensah and Bawumia (2003) presented a Vector Error Correction Forecasting Mode (VECFM), based on broad money to forecast some selected Ghanaian macroeconomic variables such as money, growth, inflation, output growth, treasury-bill rate and exchange rate. Their results revealed that the VECFM model performed well around the turning points. In another study, Omane-Ajepong et al. (2013) examined a better approach for short-term forecasting of Ghanaian inflation between seasonal-ARIMA and Holt-Winters. Their study concluded that Ghana's inflation could be described by seasonal-ARIMA process, especially for short-term forecasting.

There are, but few studies in the area of long memory conducted in South Africa. Gil-Alana (2011) analysed South African inflation between the period of 1970M1–2008M12 using long range dependence techniques and found inflation to be a covariance stationary process with long range dependence, with an order of integration ranging in the interval $(0, 0.5)$. Rangasamy (2009) studied inflation persistence and used an ARMA type model that identifies persistence as the time it takes inflation to return to a time-varying inflation mean. To estimate persistence, Rangasamy (2009) used inflation deviations from a time-varying inflation mean calculated using an HP filter. This was to ensure that inflation is stationary and to avoid the possibility of estimating a unit root variable. The researcher showed that inflation had been persistent until the implementation of inflation targeting in February 2000. These results were found to be robust at a disaggregated level. He then recommended that future research should take structural breaks into account since that could bias persistence measures downward.

Other methods of core inflation also suggested that inflation is more persistent

in a high inflation environment. For example, Blignaut et al. (2009) measured core inflation by using a trimmed mean measure. The trimmed-mean measure ignores short-run volatility aspects of inflation and focuses on individual components that have a strong bearing on the current and future trend of inflation. The distribution of CPI components is positively skewed (trim 24% off the lower tail and only 17% from the upper tail). Ruch and Dirk (2013) identified core inflation by isolating its trend from various cyclical components using Singular Spectral Analysis. It became evident in their studies that most of the noise was removed after eliminating the high frequency components from the headline inflation. The scholars showed that a model with trend coupled with inflation cycles at 65 months, 42 months and 24, months does well at explaining inflation. Overall, their findings were similar to those of Gupta and Josine (2012), confirming that the long-run cyclical components of inflation volatility have increased since IT. They, however, showed that volatility had been decreasing since 2008.

In another study, Balcilar et al. (2016) analysed South African inflation persistence using an ARFIMA model with Markov-switching fractional differencing parameter. Their results revealed that inflation is more volatile and persistent during high inflation episodes relative to low inflation episodes. They estimated that it will take 70 months for 50% of the shocks to dissipate in a high inflation regime compared to 10 months in a lower inflation regime.

The above review provides evidence that some research has been done in South Africa, especially in the area of long memory (long-range persistence) of inflation. However, we seek to add to the literature on long memory, especially in estimating the fractional integration parameter using semi-parametric estimators. In Ghana, this will probably be the first time to address issues around the concept of long memory through a fractionally integrated approach and mod-

elling the process using an ARFIMA model. It is evident that there is a serious lack of research in the area of long memory. The present study places itself in that context. The rest of the chapter is organised as follows: Section 4.3 briefly describes the data and methodology used. Section 4.4 presents the empirical results and some policy implications, and Section 4.5 provides the chapter summary.

4.3 Data and Methodology

4.3.1 Descriptive statistics

The data set analysed in this chapter consists of log monthly CPI inflation series of Ghana and South Africa. The data span for Ghana is from January 1971 to October 2014 and that of South Africa is from January 1995 to December 2014. Both data sets were respectively obtained from the BoG and Stats SA.

Table 4.1: Descriptive and Summary Statistics of CPI Inflation of Ghana and South Africa

Statistic	GHCPI Inflation	SACPI Inflation
Mean	3.12	1.68
Std.Dev	0.78	0.65
Skewness	0.29	-2.41
Kurtosis	3.49	11.27
J-B Test	12.94(0.00)*	921.84(0.00)*
ARCH-LM Test	357.006(0.00)*	164.918(0.00)*
Box-Pierce Q(18) Test	2985.64(0.00)*	916.25(0.00)*

Note: This table describes several descriptive and summary statistics: the mean (%), standard deviation (Std.Dev), skewness, kurtosis, Jarque-Bera (J-B) test, ARCH-LM test and Box-Pierce test Q(18) for GHCPI and SACPI inflation. *Indicates significance at the 5% level.

The distributional characteristics of GHCPI and SACPI inflation presented in Figures 4.1 and 4.2 and also Figures 4.3 and 4.4 respectively, can be investigated further by analysing the behaviour of their autocorrelation functions. The autocorrelation function of GHCPI and SACPI decreases slowly at a hyperbolic rate, a clear indication of long memory (or long-range dependence)²⁸.

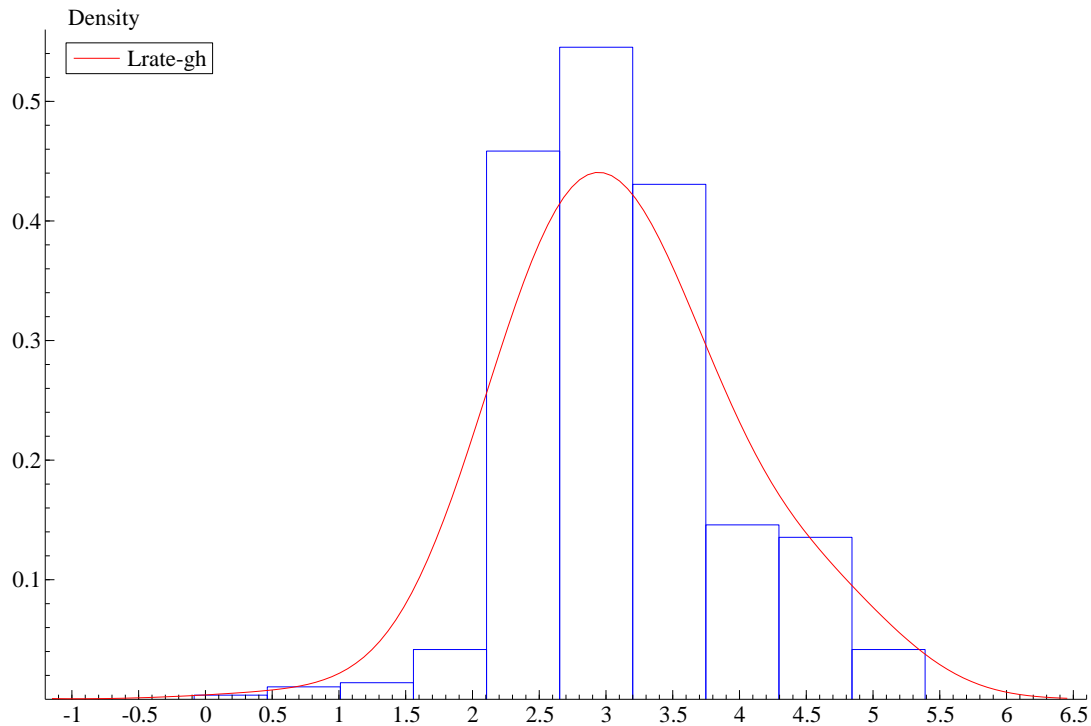


Figure 4.1: Density graph for normality for GHCPI inflation series

From Table 4.1, it is evident that the distribution of inflation series is fat tailed since kurtosis is greater than 3 (higher peak). The coefficient of skewness is 0.29 which shows that inflation of Ghana is skewed to the right, whereas that of South Africa is skewed to the left with a coefficient of -2.41 . These show that the distributions for both countries are non-normal and leptokurtic. The Jarque-Bera (J-B) test confirms these findings since it rejects normality as-

²⁸The behaviour of GHCPI and SACPI inflation series conforms to the usual characteristics of a fractionally integrated series (Haslett and Raftery, 1989; Gil-Alana, 2008).

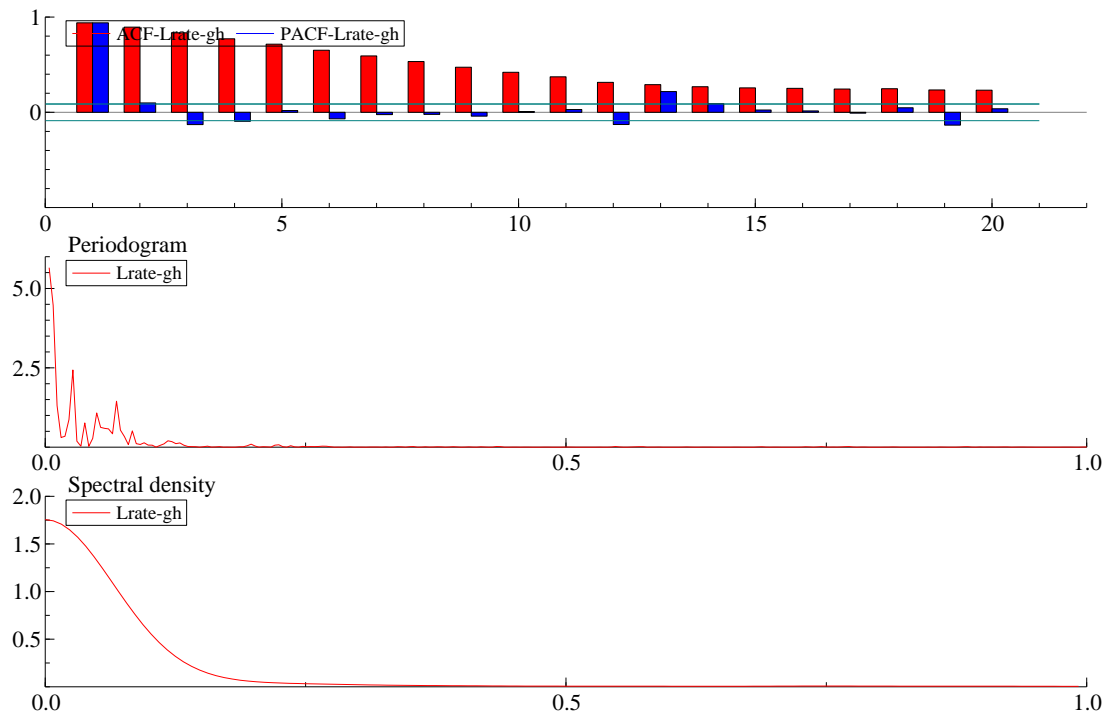


Figure 4.2: The spectral density, periodogram, ACF and PACF of GHCPi inflation series

sumption. Results from the ARCH-LM test for conditional heteroscedasticity up to lag 18, provide strong evidence of ARCH effects in the inflation series of both countries. The presence of a significant non-zero autocorrelation can also be seen in Table 4.1, with the Box-Pierce Q-statistic coefficients of 2985.64 and 916.25 for Ghana and South Africa, respectively. These results suggest that both series can be represented by long memory processes. The presence of long memory can also be confirmed in the spectral density and periodograms for GHCPi and SACPI inflation series (see Figures 4.2 and 4.4).

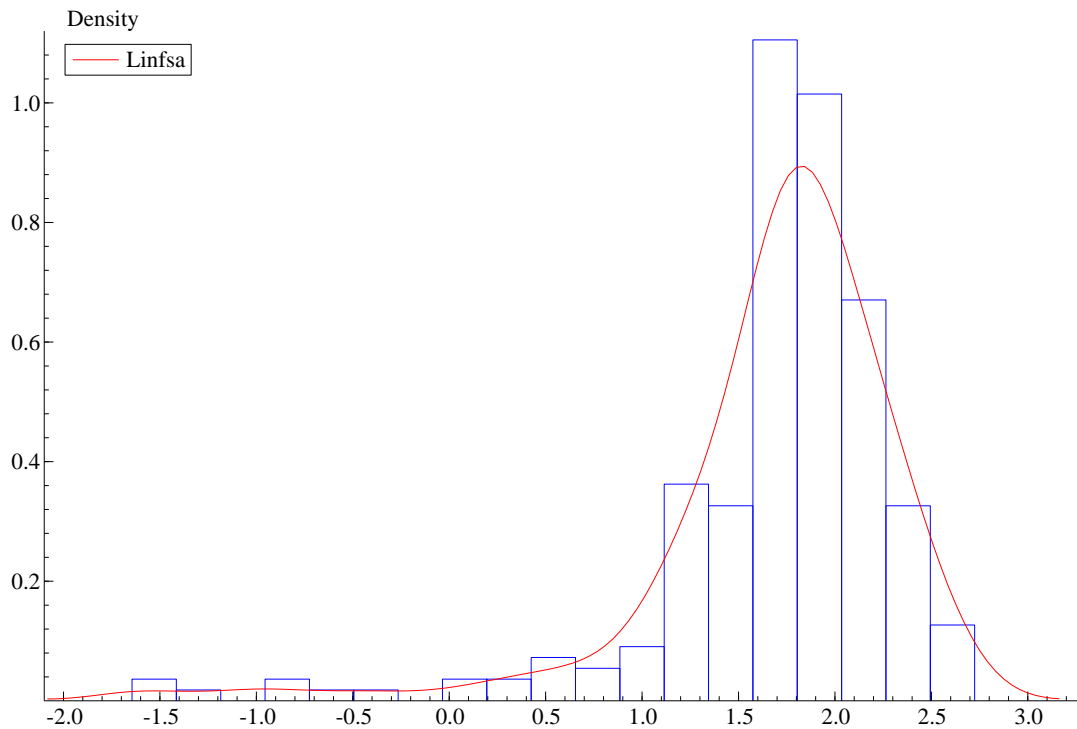


Figure 4.3: Density graph for normality for SACPI inflation series

4.3.2 Methodology

Testing for long memory is an essential task since evidence of long memory, which induces persistence, supports the use of fractionally integrated ARIMA model. This chapter makes use of four estimators: R/S by Lo (1991), EML by Sowell (1992), GPH by Geweke and Porter-Hudak (1983), and LW by Robinson (1995), to affirm the presence of long memory in the two IT countries.

From Table 4.2, it is observed that the GHCPI and SACPI inflation series are fractionally integrated, hence the existence of long memory. The null hypothesis of no long memory is, therefore, rejected in favour of the alternative hypothesis across all the sub-samples. For purposes of confirmation and also ascertaining the true long memory process, three more methods were employed to assess the series, and the results are displayed in Table 4.3.

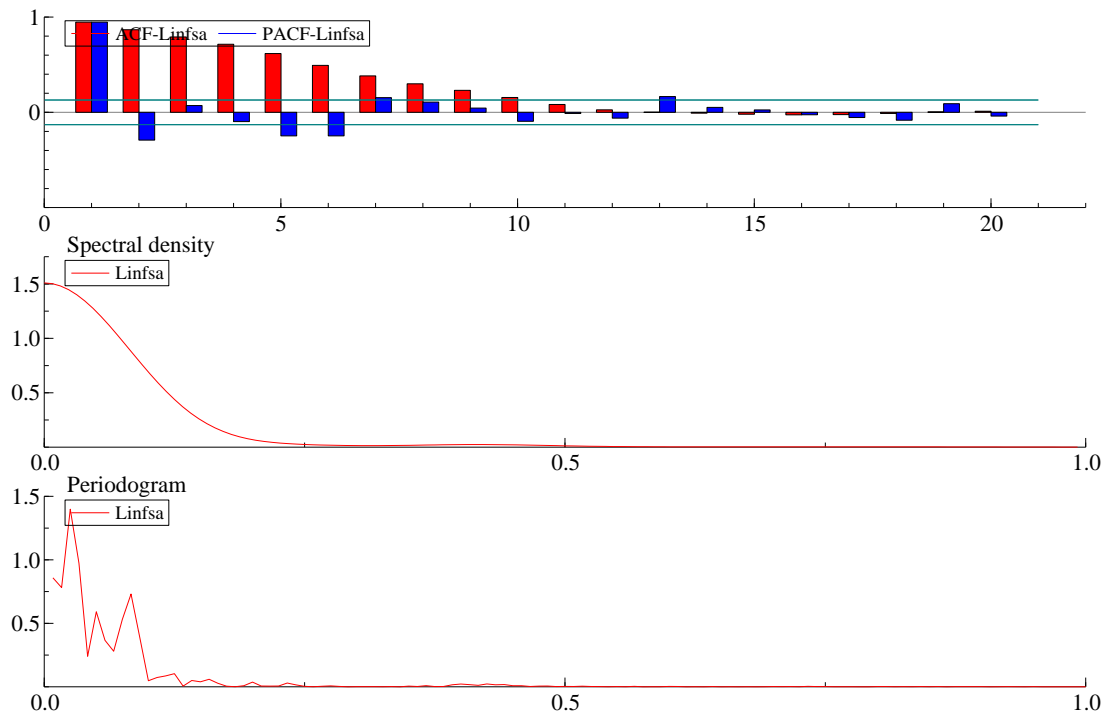


Figure 4.4: The spectral density, periodogram, ACF and PACF of SACPI inflation series

Table 4.3 also confirms the existence of long memory in the inflation series of the two countries with the values of d found to be 0.49 for all sub-samples across the two series, with respect to EML. The two semi-parametric (GPH and LW) estimators were also applied, and the results were not different from the EML parametric estimator. The GHCPI and SACPI inflation series were characterised by non-stationarity with persistence in the second and third sub-samples. The sub-samples were considered in order to investigate the influence of structural breaks in estimating the long memory parameter. The variations in the values of d across the various sub-samples could be due to the presence of structural breaks.

Table 4.2: Hurst estimates of modified R/S analysis for GHCPI and SACPI inflation series

Series	Hurst estimate (H)	Std. Err.	$d = H - 0.5$
1st sub-sample 1971M1-1984M12			
GHCPI Inflation	0.900	0.05*	0.40
2nd sub-sample 1985M1-2014M10			
GHCPI Inflation	0.87	0.03*	0.37
3rd full sample 1971M1-2014M10			
GHCPI Inflation	0.90	0.01*	0.40
1st sub-sample 1995M1-2003M12			
SACPI Inflation	0.57	0.09*	0.07
2nd Sub-sample 2004M1-2014M12			
SACPI Inflation	0.87	0.00*	0.37
3rd Full sample 1995M1-2014M12			
SACPI Inflation	0.79	0.03*	0.03

Note: Lo (1991) uses the interval $[0.809, 1.862]$ as the 95% (asymptotic) acceptance region for testing the null hypothesis; H_0 : No long memory, i.e. $H = 0.5$ against H_1 : long memory, i.e. $H = 0.5 < H < 1$; *Denotes statistical significance for all samples, hence a rejection of the null hypothesis of no long memory.

4.4 Empirical Results

This section models the persistence, induced by the presence of long memory, in the conditional mean by estimating alternative specification for ARFIMA(p,d,q) models for different orders of p and q under the Gaussian distribution assumption, and compares the performance of these ARFIMA models to determine the orders of p and q appropriate for the detection and modelling of long memory process of GHCPI and SACPI inflation series. Each series proceeds as follows: First, we estimate different ARFIMA(p,d,q) models with both p and q less than or equal to 3. Second, for each of the series, a number of tests are performed on the residuals to ensure that they are white noise²⁹. These include tests for normality, heteroscedasticity, ARCH-LM, Box-Pierce and Ljung-Box tests. The LL along with AIC were applied to choose the correct specification for each se-

²⁹This study follows the idea espoused by Gil-Alana and Toro (2002).

Table 4.3: Parameters estimates of long memory for GHCPI and SACPI inflation series

Series		GHCPI Inflation		
Test		d	SE	P-val.
1st sub-sample	1971M1-1984M12			
EML		0.49	0.45	-
GPH		1.02	0.19	0.00*
LW		0.99	0.11	0.00*
2nd sub-sample	1985M1-2014M12			
EML		0.49	0.23	-
GPH		0.68	0.15	0.00*
LW		0.66	0.11	0.00*
3rd sub-sample	1971M1-2014M10			
EML		0.49	0.32	-
GPH		0.39	0.19	0.05
LW		0.35	0.01	0.01*
Series		SACPI Inflation		
Test		d	SE	P-val.
1st sub-sample	1995M1-2003M12			
EML		0.49	0.39	-
GPH		0.32	0.12	0.02*
LW		0.48	0.11	0.00*
2nd sub-sample	2004M1-2014M12			
EML		0.49	0.30	-
GPH		0.83	0.09	0.00*
LW		0.84	0.11	0.00*
3rd sub-sample	1995M1-2014M12			
EML		0.49	0.28	-
GPH		0.66	0.15	0.00*
LW		0.60	0.15	0.00*

Note: *The results obtained from EML (parametric estimator) and GPH and LW (semi-parametric estimators) confirmed the existence of long memory in GHCPI and SACPI inflation series. This is in accordance with properties of fractional integration propounded by Granger (1980), Granger and Joyeux (1980), Hosking (1981), and Tkacz (2001) (see. Appendix 4.1); SE denotes standard error.

ries. To ensure stationarity, both series were fractionally differenced before the model building process. This chapter considers all possible combinations of the ARMA(p,q) part of the model for $p = 0, 1, 2, 3$ and $q = 0, 1, 2, 3$. The estimated results of the parameters in ARFIMA(p,d,q) models for GHCPI and SACPI in-

flation series are presented in Tables 4.4 and 4.5.

Table 4.4: Parameter estimates of ARFIMA models for GHCPI inflation series

ARMA	LL	d	Cont.	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC
(1,1)	430.96	0.32	1.26	0.88	-	-	-0.30	-	-	-851.93
SE	-	0.10	0.26	0.03	-	-	0.08	-	-	-
P-val	-	0.00	0.00	0.00	-	-	0.00	-	-	-
(1,2)	435.30	0.27	1.28	0.87	-	-	-0.27	0.13	-	-858.60
SE	-	0.14	0.18	0.05	-	-	0.11	0.04	-	-
P-val	-	0.05	0.00	0.00	-	-	0.01	0.00	-	-
(1,3)	435.39	0.24	1.29	0.87	0.27	-	-0.24	0.13	0.02	-856.78
SE	-	0.16	0.16	0.05	0.10	-	0.12	0.04	0.16	-
P-val	-	0.11	0.00	0.00	0.00	-	0.04	0.00	0.11	-
(2,2)	436.67	0.31	1.29	1.74	-0.78	-	-1.17	0.35	-	-859.35
SE	-	0.20	0.17	0.10	0.10	-	0.28	0.15	-	-
P-val	-	0.13	0.00	0.00	0.00	-	0.00	0.02	-	-
(3,1)	436.81	0.38	1.25	-0.37	0.70	0.31	0.87	-	-	-859.63
SE	-	0.08	0.35	0.09	0.07	0.04	0.06	-	-	-
P-val	-	0.00	0.00	0.00	0.00	0.00	0.00	-	-	-

Note: Model selection was based on AIC and the LL; SE denotes the standard error.

Using the significance of the parameter estimates, the LL and the AIC, ARFIMA(3,d,1) and ARFIMA(3,d,2) models were found to provide the best fit for GHCPI and SACPI inflation series, respectively. From the ARFIMA(3,d,1) model, the value of the long memory parameter $d = 0.38$ was found to be positive and statistically significant at 5% level. This evidence from ARFIMA(3,d,1) displayed in Table 4.4 strongly supports the presence of long memory, hence persistence, in the conditional mean of GHCPI inflation. Similarly, ARFIMA(3,d,2) was found to be appropriate for modelling persistence in the conditional mean of SACPI inflation series, based on the significance of the parameter estimates, AIC and the LL (see Table 4.5). The estimate of the fractional differencing parameter $d = 0.43$ was also significant, implying the existence of long memory in the conditional mean of the SACPI inflation series. The diagnostics of these models

Table 4.5: Parameter estimates for ARFIMA models for SACPI inflation series

ARMA	LL	d	Cont.	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC
(1,1)	250.68	0.06	0.74	0.89	-	-	0.29	-	-	-491.36
SE	-	0.11	0.09	0.04	-	-	0.09	-	-	-
P-val	-	0.57	0.00	0.00	-	-	0.00	-	-	-
(1,2)	250.76	0.13	0.74	0.88	-	-	0.22	-	-	-489.52
SE	-	0.20	0.11	0.05	-	-	0.18	-	-	-
P-val	-	0.51	0.00	0.00	-	-	0.21	-	-	-
(2,2)	250.77	0.13	0.74	0.96	-0.06	-	0.15	-0.07	-	-487.54
SE	-	0.19	0.11	0.50	0.44	-	0.51	0.14	-	-
P-val	-	0.50	0.00	0.06	0.88	-	0.76	0.63	-	-
(3,2)	264.38	0.43	0.76	1.40	-1.36	0.76	-0.67	0.78	-	-512.77
SE	-	0.06	0.38	0.07	0.07	0.06	0.09	0.61	-	-
P-val	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-	-
(1,3)	250.82	0.13	0.74	0.89	-	-	0.23	-0.04	-0.02	-487.64
SE	-	0.18	0.11	0.22	-	-	0.17	0.11	0.07	-
P-val	-	0.48	0.00	0.00	-	-	0.17	0.70	0.70	-

Note: Model selection was based on AIC and the LL. SE denotes the standard error.

are displayed in Table 4.6.

While there exists significant evidence of long memory, which induces persistence in the conditional means of both GHCPI and SACPI inflation series, the results of diagnostic statistics for all ARFIMA models exhibit some shortcomings in the model building process. For instance, larger values of the J-B statistics of 12260.80 and 8302.04, respectively for GHCPI and SACPI inflation series, confirm the non-normality of the residuals. In addition, the values of the ARCH-LM test statistics 8302.04 and 12260.80, respectively for GHCPI and SACPI inflation series, are statistically significant, leading to a rejection of the null hypothesis of no ARCH effects in the residuals. However, the Ljung-Box test for autocorrelation seems to be statistically different from zero (i.e. 7.82 for GHCPI and 8.15 for SACPI), hence we fail to reject the null hypothesis of no autocorrelation. In the next chapter, the aspect of volatility is incorporated in the

Table 4.6: Residual diagnostics test for ARFIMA (p,d,q) models selected for GHCPI and SACPI inflation series

Type of Test	Test Statistic	P-val.
ARFIMA (3,0.38,1) for GHCPI		
Ljung-Box Test for Resid. Corr	7.82	0.25
ARCH LM Test	60.18	0.00*
J-B test for Normality	8302.04	0.00*
ARFIMA (3,0.48,2) for SACPI		
Ljung-Box Test for Resid. Corr	8.154	0.14
ARCH LM Test	112.02	0.00*
J-B test for Normality	12260.80	0.00*

Note: *The Ljung-Box tests the null hypothesis of no autocorrelation against the alternative of the existence of autocorrelation; The ARCH-LM tests the null hypothesis of no ARCH effect (heteroscedascity) against an alternative of the presence of ARCH effect; The Jarque-Bera (J-B) tests the null hypothesis that errors are normally distributed against and alternative of errors not normally distributed. *Indicates significance at all levels.

ARFIMA models specified for GHCPI and SACPI inflation, meant to cater for the remaining ARCH effects in modelling persistence in the conditional mean for the two African countries. For instance, ARFIMA-sGARCH and ARFIMA-gjrGARCH among others, shall be fitted to the GHCPI and SACPI inflation data in order to improve the performance of the models³⁰.

4.5 Summary

The long memory phenomenon in inflation series is an intriguing subject in the financial literature, since the presence of long memory contradicts the assumption of the Efficient Market Hypothesis (EMH). This chapter investigates the presence or absence of long memory property in the GHCPI and SACPI inflation series. The chapter also models the conditional mean of the two inflation series, after discovering persistence in GHCPI and SACPI inflation series in

³⁰Details of these models are provided in Chapter 5.

Chapter 3. Indeed, the distributional properties of GHCPI and SACPI are essential, not only for implementation of linear models, but also for determining the memory characteristics of GHCPI and SACPI inflationary processes. The GHCPI and SACPI inflation series were found to have violated the normality assumption, implying that the two series do not follow a random walk process, but rather a deterministic non-linear process associated with the presence of long memory. Hence linear statistical models based on normality assumption, such as the Capital Asset Pricing Model and the Black-Scholes option pricing models (normally used in stock market studies), will require some modification in order to model such data.

The study has applied two semi-parametric estimators: LW by Robinson (1995) and the log-periodogram regression model of Geweke and Porter-Hudak (1983), together with R/S analysis by Lo (1991), and EML by Sowell (1992), to investigate the presence of long memory in Ghanaian and South African CPI inflation series. Much emphasis was laid on semi-parametric estimators, especially LW estimator by Robinson (1995) because it can detect long memory, even in the presence of structural breaks.

The main findings are that the monthly CPI inflation series of Ghana and South Africa exhibit the properties of long memory (with varied degree of persistence), and this is consistent with the results obtained by Hassler and Wolters (1994), and Doornik and Ooms (2004). This study also examined the GHCPI and SACPI inflation by modelling the conditional mean by means of an ARFIMA model. In the first place, different ARFIMA models were specified for the two inflation series following Sowell (1992) EML estimation method. Appropriate and competing models were selected through a model selection criterion based on diagnostic tests on the residuals together with the LL test statistic and AIC. ARFIMA(3,0.38,1) and ARFIMA (3,0.43,2), were respectively

specified for GHCPI and SACPI inflation series. To a large extent, these results agree with those of Gil-Alana and Toro (2002).

The next chapter provides more information on modelling persistence in the conditional mean of GHCPI and SACPI inflation series by taking care of the ARCH effects under three distributional assumptions, and also assess the impact of IT monetary policy in dealing with persistence, which is an important phenomena in monetary formulation by policymakers.

Chapter 5

Modelling Persistence in the Conditional Mean of Inflation using ARFIMA-sGARCH and ARFIMA-‘gjrGARCH Models

5.1 Introduction

Following the work of Granger (1980), Granger and Joyeux (1980), and Hosking (1981), several recent studies have dealt with the estimation of ARFIMA process. In fact one convenient way to deal with data with time-dependent conditional heteroscedasticity is to consider an ARFIMA model with GARCH-type innovations. This model can provide a useful way to examine the relationship between the conditional mean and variance of a process exhibiting long memory and the slow decay in its level, hitherto with time-varying volatility. The

aggregate price level is obviously one of the crucial variables in understanding the macroeconomy. Another point worthy of note is the issue of how aggregate prices adjust to shocks. For instance, in their research, Barsky (1987), Ball and Cecchetti (1990), Kim (1993), and Nelson and Schwert (1997), have established evidence of two unit roots in prices, implying that any shock has a permanent effect on inflation. Similarly, Brunner and Hess (1993) modelled US inflation as an $I(0)$ process before 1960, but it has become non-stationary, $I(1)$ since that time.

The perceived non-stationarity in inflation is far from harmless in terms of economic implications. For example, inflation is the sole relevant measure of the opportunity cost of holding money in most developing countries like Ghana and South Africa. Again, knowledge of whether or not the inflation rate is non-stationary is also important (given emerging efforts towards financial liberalisation in some developing countries) to the empirical estimation of the long-run relationship between the ex-ante inflation rate and the nominal interest rates.

The primary goal of this chapter is to extend the ARFIMA model, which has a fractionally integrated conditional mean with the standard generalised sGARCH by Bollerslev (1986) and 'gjrGARCH by Glosten et al. (1993) processes, to describe time-dependent heteroscedascity. Among other things, this chapter employs ARFIMA-sGARCH together with ARFIMA-'gjrGARCH, after having established the presence of non-stationary long memory with varied degrees of persistence in GHCPI and SACPI inflation series. One advantage of the ARFIMA model is that it has the fractional integration parameter explicitly incorporated into the model, allowing one to estimate it jointly with other parameters. In this respect, the main contributions of this chapter are twofold. First, the ARFIMA model is known to be able to capture the long memory property in the conditional mean, in this case the GHCPI and SACPI inflation series.

However, the diagnostic residuals from the ARFIMA model revealed the presence of ARCH effects (see Chapter 4). Thus, it is evident that the long memory property in the conditional mean of CPI inflation may not be modelled sufficiently by an ARFIMA model. Second, this chapter extends the ARFIMA process, which has a fractionally integrated conditional mean with the sGARCH and 'gjrGARCH processes to describe time-dependent heteroscedasticity under three distributional assumptions.

The remainder of this chapter is presented as follows: Section 5.2 provides characteristics of volatility process using the residual plots, data and methodology. Section 5.3 displays the empirical results of ARFIMA-sGARCH and ARFIMA-'gjrGARCH processes under three different distributional assumptions. Finally, Section 5.4 outlines the chapter summary.

5.2 Data and Methodology

5.2.1 Descriptive statistics

The data used in this study comprise of log monthly CPI inflation of Ghana and South Africa, respectively covering the period January 1971 to October 2014 and January 1995 to December 2014. Both data sets were respectively obtained from the BoG and Stats SA. Figures 5.1 and 5.2 demonstrate evidence of volatility clustering, i.e. burst of high volatility separated by periods of relative tranquility. Hence, returns of GHCPI and SACPI inflation are not normally distributed with mean, $\mu = 0$ and variance, $\sigma^2 = 1$ at each point in time³¹. Instead, it is fair to establish that σ_t^2 changes with time t .

³¹Details of descriptive statistics are presented in Table 4.1. (Chapter 4).

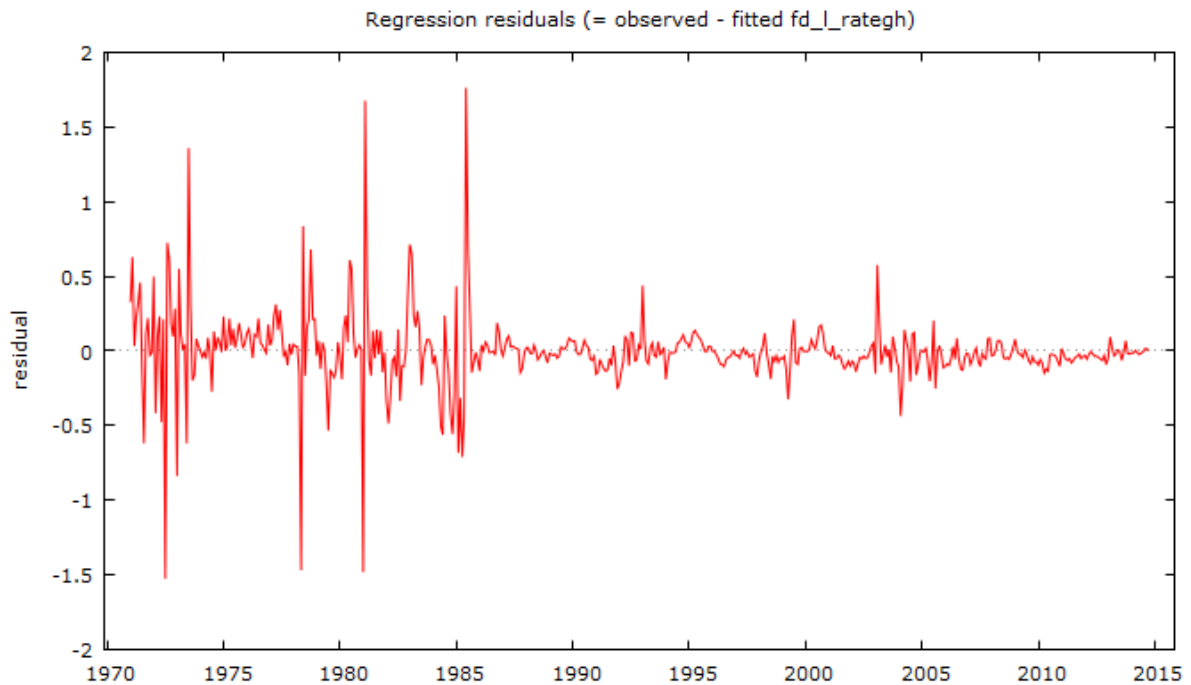


Figure 5.1: Residual plots for GH CPI inflation for ARFIMA(3,0.38,1)

The time-varying nature observed in Figures 5.1 and 5.2 is that which is referred to as heteroscedasticity, where periods of high volatility are followed by periods of high volatility, and also periods of low volatility are followed by periods of low volatility for a prolonged period of time. Again, Figures 5.1 and 5.2 confirm the presence of ARCH effects reported in the previous chapter, hence the need to extend the ARFIMA model with time-varying GARCH components, specifically sGARCH and 'gjrGARCH under three distributional assumptions. By extending the ARFIMA model with time-varying volatility components, the remaining ARCH effects will be dealt with, hence resulting in obtaining a good model for the conditional mean for the two countries.

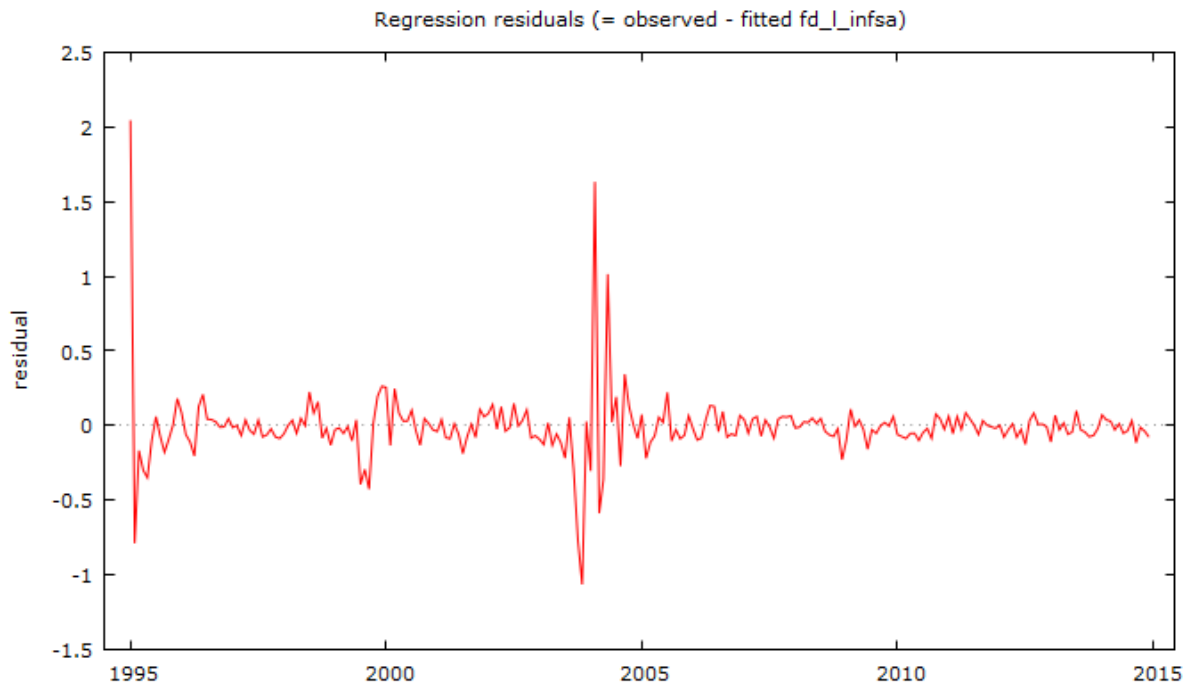


Figure 5.2: Residual plots for SACPI inflation for ARFIMA(3,0.43,2)

5.2.2 Methodology

Following Equations (2.67), (2.69) and (2.72), ARFIMA-sGARCH and ARFIMA-'gjrGARCH specifications have been employed because of their greater flexibility in dealing with the remaining ARCH effects in the conditional mean modelling process for GHCPI and SACPI inflation under three distributional assumptions³².

5.3 Empirical Results

To get the parameters p and q of an ARMA to fit the GHCPI and SACPI inflation series, an Akaike Information Criterion (AIC), was applied to evaluate the ARMA models and the results are displayed in Table 5.1. Based on the LL an ARMA(3,1) and ARMA(3,2) models were selected for GHCPI and SACPI

³²Details of these models are provided in Chapter 2.

inflation series, confirming the results in Chapter 4.

Table 5.1: Criteria for ARMA(p,q) selection

GHCPI inflation series		
ARMA	LL	AIC
(1,1)	430.96	-851.93
(1,2)	435.30	-858.60
(1,3)	435.39	-856.78
(2,2)	436.67	-859.35
(3,1)	436.81	-859.81
SACPI inflation series		
(1,1)	250.68	-491.36
(1,2)	250.76	-489.52
(2,2)	250.77	-487.54
(3,2)	264.38	-512.77
(1,3)	250.82	-487.64

Note: Criteria for ARMA(p,q) selection is based on the LL and AIC (Akaike, 1974).

Table 5.2 presents the extension of the ARFIMA model with sGARCH and 'gjrGARCH volatility components under three distributional assumptions. These models were estimated using the EML by Sowell (1992).

In Table 5.2, ARFIMA(3,d,1)-sGARCH(1,1) and ARFIMA(3,d,1)-'gjrGARCH(1,1) under GED provided the best fit for modelling persistence in the conditional mean of GHCPI inflation series. Correspondingly, the fitted models for SACPI inflation are ARFIMA(3,d,2)- sGARCH(1,1) and ARFIMA(3,d,2)-'gjrGARCH(1,1), all under STD. The selection of these models were based on the LL and AIC. Tables 5.3 and 5.4 present the two competing models for goodness of fit, for GHCPI inflation series under GED.

Tables 5.3 and 5.4 display the results for modelling the persistence in the conditional mean of GHCPI inflation series from the two selected models, which are ARFIMA(3,0.37,1)-sGARCH(1,1) and ARFIMA(3,0.35,1)-'gjrGARCH(1,1), under GED assumption. A close inspection of Tables 5.3 and 5.4 reveals the signif-

Table 5.2: Fitted models for GHCPI and SACPI inflation series

Criteria	Norm	STD	GED
GHCPI inflation series			
ARFIMA(3,d,1)-sGARCH(1,1)			
Log.lik	255.02	429.73	430.61
AIC	-1.95	-1.62	-1.63
ARFIMA(3,d,1)-'gjrGARCH(1,1)			
LL	282.21	429.12	434.66
AIC	-1.05	-1.62	-1.64
SACPI inflation series			
ARFIMA(3,d,2)-sGARCH(1,1)			
LL	197.58	210.60	206.19
AIC	-1.63	-1.74	-1.69
ARFIMA(3,d,2)-'gjrGARCH(1,1)			
LL	206.39	215.05	212.30
AIC	-1.69	-1.76	-1.74

Note: Model selection was based on the LL and AIC.

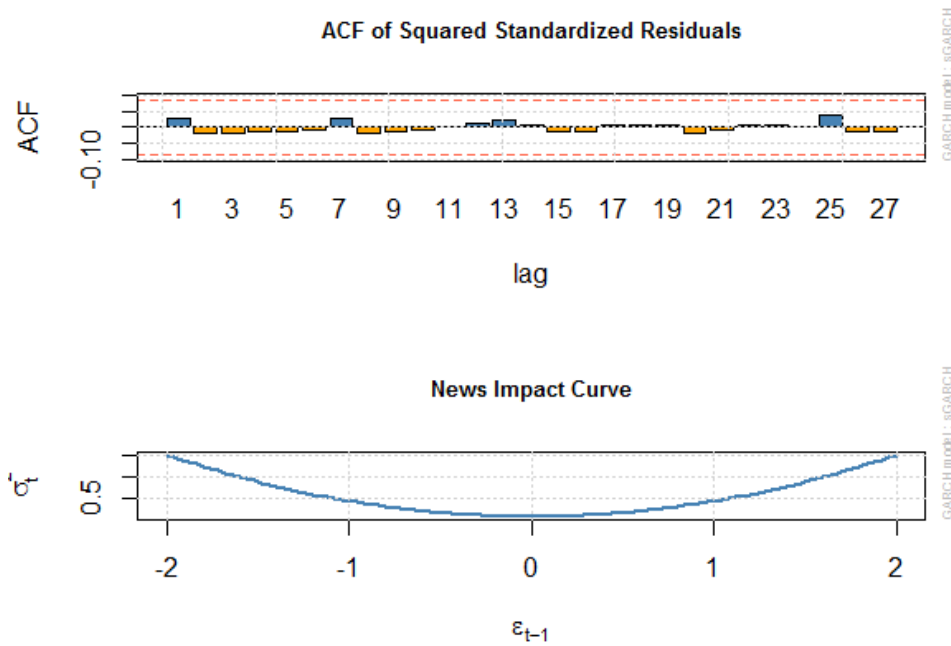


Figure 5.3: ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.37,1)-sGARCH(1,1) of GHCPI inflation series

Table 5.3: ARFIMA(3,d,1)-sGARCH(1,1) under GED for GHCPI inflation series

Variables	Estimates	SE	t-val.	P-val.	Nyblom stats.
μ	2.79	0.00	2566.11	0.00*	0.73
ϕ_1	1.69	0.00	1775.59	0.00*	0.07
ϕ_2	-0.64	0.00	-836.66	0.00*	0.06
ϕ_3	-0.08	0.00	-345.75	0.00*	0.06
θ_1	-0.78	0.00	-269.22	0.00*	0.94
d	0.37	0.00	224.93	0.00*	1.17
ω	0.00	0.00	1.64	0.10	0.82
α	0.37	0.11	3.42	0.00*	2.59
β	0.62	0.11	5.67	0.00*	0.83
κ	0.66	0.04	14.66	0.00*	0.20
LL	430.61	-	-	-	-
AIC	-1.63	-	-	-	-
$Q^2(\cdot)$	1.64	-	-	0.89	-
ARCH-LM Test	1.64	-	-	0.99	-
Sign Bias Test	t-val.	Prob. sig.			
Sign Bias	1.58	0.11			
Negative Sign Bias	0.61	0.55			
Positive Sign Bias	0.09	0.93			
Joint Effect	4.10	0.25			

Note: *Denotes the significance of all parameters; Box-Pierce $Q^2(\cdot)$ statistics fail to reject the null hypothesis of no serial correlation; ARCH-LM test also fails to reject the null hypothesis of no ARCH effects. d denotes the fractional differencing parameter.

ificance of all parameter estimates, except the intercept ω in the variance equation from both models and the ARCH, α component of the ARFIMA(3,0.35,1)-'gjrGARCH(1,1) model. All parameters were found to be stable (according to Nyblom statistics) and the sign bias test presented also revealed a non-rejection of no sign effect. There also appears to be a symmetric effect of inflationary shocks (negative and positive) (see Figure 5.3) in ARFIMA(3,0.37,1)-sGARCH(1,1) model compared to an asymmetric effect of these inflationary shocks (negative and positive) in ARFIMA(3,0.35,1)-'gjrGARCH(1,1) on GHCPI inflation series (see Figure 5.4). The results further show a considerable high volatility and persistence ($\alpha + \beta$) of 0.98 for ARFIMA(3,0.37,1)-sGARCH(1,1)

Table 5.4: ARFIMA(3,d,1)-'gjrGARCH(1,1) under GED for GHCPI inflation series

Variables	Estimates	SE	t-val.	P-val.	Nyblom stats.
μ	2.67	0.00	974.10	0.00*	0.65
ϕ_1	1.72	0.00	1561.47	0.00*	0.08
ϕ_2	-0.66	0.00	-913.48	0.00*	0.19
ϕ_3	-0.07	0.00	-1481.65	0.00*	0.62
θ_1	-0.79	0.00	-1952.12	0.00*	1.09
d	0.35	0.00	777.47	0.00*	1.48
ω	0.00	0.00	1.38	0.17	1.09
α	0.14	0.10	1.33	0.18*	1.59
γ	0.66	0.14	4.73	0.00*	1.07
β	0.41	0.15	2.65	0.01*	2.67
κ	0.68	0.05	14.39	0.00*	0.15
LL	434.67	-	-	-	-
AIC	-1.64	-	-	-	-
$Q^2(\cdot)$	2.96	-	-	0.71	-
ARCH-LM Test	3.01	-	-	0.98	-
Sign Bias Test	t-val.	Prob. sig.			
Sign Bias	1.01	0.32			
Negative Sign Bias	0.34	0.73			
Positive Sign Bias	0.08	0.93			
Joint Effect	1.58	0.66			

Note: *Denotes the significance of all parameters; Box-Pierce $Q^2(\cdot)$ statistics all fail to reject the null hypothesis of no serial correlation; ARCH-LM test also fails to reject the null hypothesis of no ARCH effects; γ denotes the leverage effect.

and 0.54 for ARFIMA(3,0.35,1)-'gjrGARCH(1,1), which are in line with the expectations.

Again, from Tables 5.3 and 5.4, the ARCH-LM test statistics from both models indicate that there is no ARCH effects up to lag 27 in the standardised residuals for GHCPI. Box-Pierce Q-statistics of standardised squared residuals were found to be serially uncorrelated. Taking the overall model assessment displayed in Table 5.5 into consideration, ARFIMA(3,0.35,1)-'gjrGARCH(1,1) appears to provide a good fit for GHCPI inflation series, since it has the highest

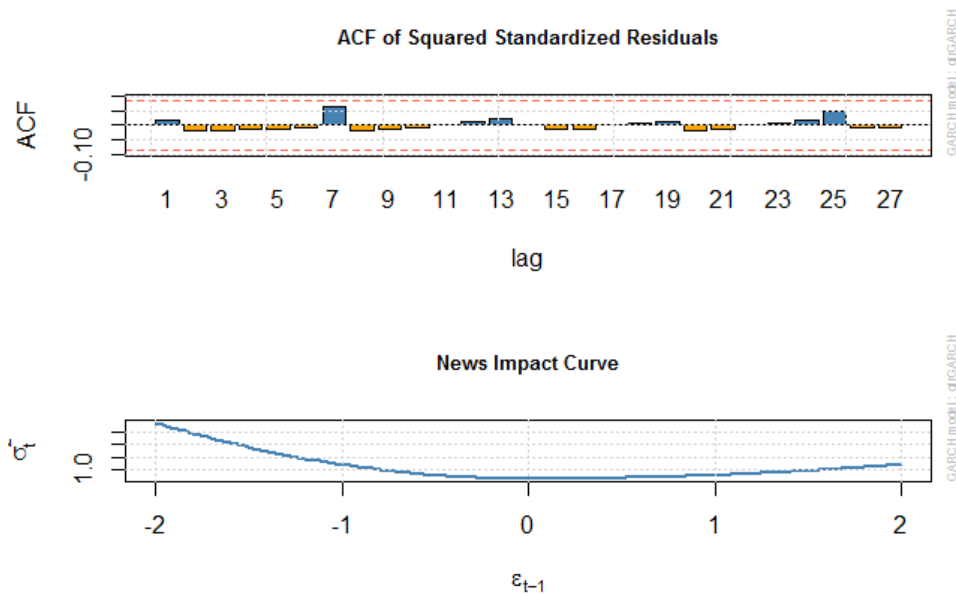


Figure 5.4: ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.35,1)-'gjrGARCH(1,1) of GH CPI inflation

LL (434.67) and the lowest AIC (-1.64). This model has been used to forecast ten-month inflation rates (see Appendices 5.1 and 5.2) with residual analysis provided in the appendix, where the forecast or predicted rates appear to be increasing into the future.

Table 5.5: Assessment of goodness of fit under GED for GH CPI inflation series

Criteria	ARFIMA(3,0.37,1)-sGARCH(1,1)	ARFIMA(3,0.35,1)-'gjrGARCH(1,1)
LL	430.61	434.66
MSE	0.00	0.00
MAE	0.04	0.04
ARCH-LM	1.64	3.00
$Q^2(\cdot)$	1.64	2.95
AIC	-1.63	-1.64

Note: Goodness of fit was based on the LL and AIC.

The results of ARFIMA(3,0.50,2)-sGARCH(1,1) and ARFIMA(3,0.50,2)-'gjrGARCH(1,1)

Table 5.6: ARFIMA(3,d,2)-sGARCH(1,1) under STD for SACPI inflation series

Variables	Estimates	SE	t-val.	P-val.	Nyblom stats.
μ	2.06	0.13	16.35	0.00*	0.14
ϕ_1	0.94	0.10	9.28	0.00*	0.06
ϕ_2	0.73	0.09	7.87	0.00*	0.06
ϕ_3	-0.69	0.07	-10.54	0.00*	0.07
θ_1	-0.02	0.02	-1.07	0.29	0.07
θ_2	-0.94	0.02	-40.39	0.00*	0.06
d	0.50	0.12	4.03	0.00*	0.10
ω	0.00	0.00	2.23	0.03	0.17
α	0.33	0.11	2.86	0.00*	0.26
β	0.59	0.10	5.89	0.00*	0.14
ν	4.86	0.55	3.13	0.00*	0.16
LL	210.60	-	-	-	-
AIC	-1.74	-	-	-	-
$Q^2(\cdot)$	1.98	-	-	0.851	-
ARCH-LM Test	2.47	-	-	0.991	-
Sign Bias Test	t-val.	Prob. sig.			
Sign Bias	0.63	0.53			
Negative Sign Bias	1.20	0.23			
Positive Sign Bias	0.44	0.66			
Joint Effect	1.64	0.65			

Note: *Denotes the significance of all parameters; Box-Pierce $Q^2(\cdot)$ statistics fail to reject the null hypothesis of no serial correlation; ARCH-LM test also fails to reject the null hypothesis of no ARCH effects.

specified for modelling persistence in the conditional mean of SACPI inflation series are presented in Tables 5.6 and 5.7. The parameter estimates for both models are also presented in Tables 5.6 and 5.7 for modelling persistence in the conditional mean of SACPI inflation series. Specifically, ARFIMA(3,0.50,2)-sGARCH(1,1) and ARFIMA(3,0.50,2)-'gjrGARCH(1,1) were specified to describe persistence in the conditional mean under STD assumption. All the parameter estimates from the two models were found to be significant with the exception of MA(1) in both models and the ARCH α component in ARFIMA(3,0.50,2)-'gjrGARCH(1,1). All parameters were found to be stable (according to Nyblom statistics) and the sign bias test presented in both tables reveals a non-

Table 5.7: ARFIMA(3,d,2)-'gjrGARCH(1,1) under STD distribution for SACPI inflation series

Variables	Estimates	SE	t-val.	P-val.	Nyblom stats.
μ	2.02	0.16	12.88	0.00*	0.13
ϕ_1	0.91	0.12	7.76	0.00*	0.09
ϕ_2	0.76	0.09	8.51	0.00*	0.09
ϕ_3	-0.69	0.08	-8.87	0.00*	0.09
θ_1	-0.02	0.02	-0.79	0.43	0.11
θ_2	-0.94	0.02	-38.53	0.00*	0.08
d	0.50	0.15	3.32	0.00*	0.07
ω	0.00	0.00	2.47	0.01	0.25
α	0.04	0.07	0.51	0.61	0.19
γ	0.61	0.09	6.91	0.00*	0.21
β	0.48	0.18	2.57	0.01*	0.28
ν	5.81	2.12	2.74	0.01*	0.15
LL	215.06	-	-	-	-
AIC	-1.77	-	-	-	-
$Q^2(\cdot)$	0.47	-	-	0.99	-
ARCH-LM Test	3.01	-	-	1.00	-
Sign Bias Test	t-val.	Prob. sig.			
Sign Bias	0.76	0.45			
Negative Sign Bias	0.57	0.57			
Positive Sign Bias	0.42	0.68			
Joint Effect	1.16	0.76			

Note: *Denotes the significance of all parameters; Box-Pierce $Q^2(\cdot)$ statistics all fail to reject the null hypothesis of no serial correlation; ARCH-LM test also fails to reject the null hypothesis of no ARCH effects; γ denotes the leverage effect.

rejection of no sign bias effect. There appears to be a symmetric effect of inflationary shocks (negative and positive) (see Figure 5.5) in ARFIMA(3,0.50,2)-sGARCH(1,1) model compared to the asymmetric effect of these inflationary shocks (negative and positive) in ARFIMA(3,0.50,2)-'gjrGARCH(1,1) on SACPI inflation series³³. The results, in addition, show a considerable high volatility and persistence ($\alpha + \beta$) of 0.91 for ARFIMA(3,0.50,2)-sGARCH(1,1) and 0.50 for ARFIMA(3,0.50,2)-'gjrGARCH(1,1). Similarly, the ARCH-LM test statistics

³³see Engle and Ng (1993), and Chiang and Doong (2001) for details of positive and negative shocks on volatility.

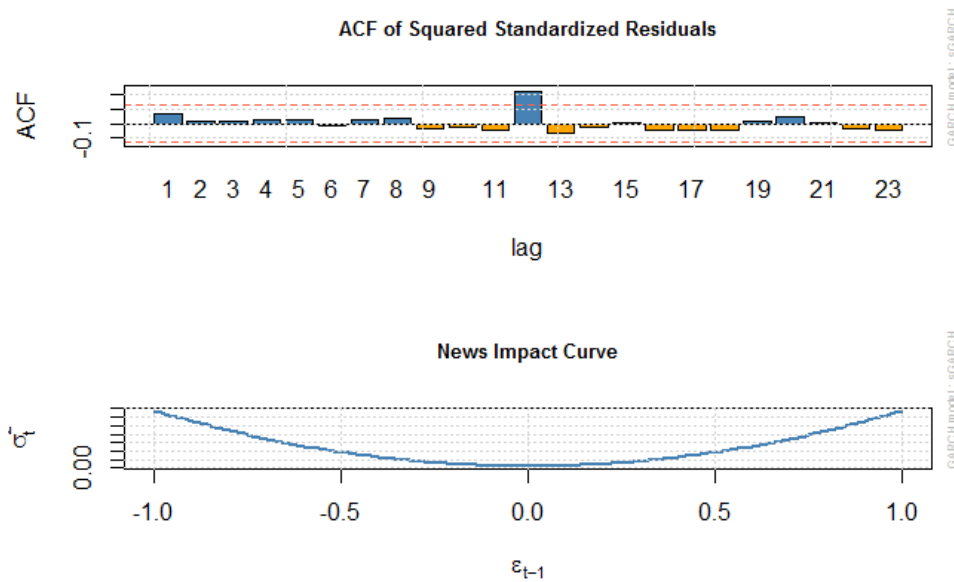


Figure 5.5: ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.50,2)-sGARCH(1,1) of SACPI inflation series

from both models indicate that there is no ARCH effect up to lag 27 in the standardised residuals. Box-Pierce Q-statistics of standardised squared residuals are serially uncorrelated. In Table 5.8, ARFIMA(3,0.50,2)-'gjrGARCH(1,1) seems to provide a good fit for SACPI inflation, based on the LL (215.06) and lowest AIC (-1.77). This model was therefore used to forecast a ten-month inflation rates for South Africa with the residuals analysis provided in the appendix, where the predicted rates appear to be increasing into the future (see Appendices 5.3, 5.4 and 5.5).

5.4 Summary

Chapter 5 has explored the extension of the ARFIMA process with sGARCH and 'gjrGARCH models to describe the time-dependent heteroscedasticity and persistence in the conditional mean of CPI inflation series of Ghana and South

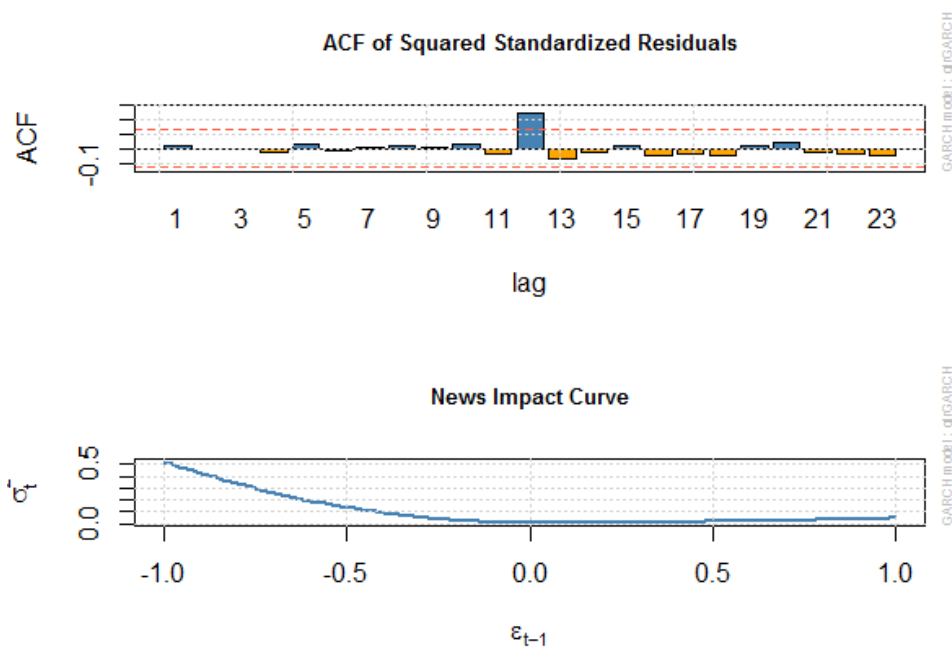


Figure 5.6: ACF of squared standardised residuals and news impact curve for ARFIMA(3,0.50,2)-gjrGARCH(1,1) of SACPI inflation series

Table 5.8: Assessment of goodness of fit under STD distribution for SACPI inflation series

Criteria	ARFIMA(3,0.50,2)-sGARCH(1,1)	ARFIMA(3,0.50,2)-gjrGARCH(1,1)
LL	210.60	215.06
MSE	0.01	0.01
MAE	0.01	0.01
ARCH-LM	2.47	3.01
$Q^2(\cdot)$	1.99	0.47
AIC	-1.74	-1.77

Note: Goodness of fit was based on the LL and AIC.

Africa under three distributional assumptions (i.e., Norm, STD, and GED). ARFIMA(3,0.35,1)-gjrGARCH(1,1) under GED and ARFIMA(3,0.50,2)-gjrGARCH(1,1) under STD provided the best fit for modelling the time-dependent heteroscedasticity and persistence in the conditional mean of CPI inflation series of Ghana and South Africa. Results from this chapter have provided a strong evidence

of persistence in relation to asymmetric effect of inflationary shocks in the CPI inflation series of Ghana and South Africa, thus confirming the studies conducted by Caporale and Caporale (1998).

Although ARFIMA(3,0.35,1)-'gjrGARCH(1,1) under the GED assumption and ARFIMA(3,0.50,2)-'gjrGARCH(1,1) under the STD, seem to provide a good fit to GHCPI and SACPI inflation series respectively, but the mere presence of long memory raises an issue of regime shifts or non-linearities. In particular, overlooking non-linearities can result in misleading in-sample diagnostics (Potter, 1995). The next chapter introduces non-linear specifications in the context of structural breaks and fractional integration. The aim is to obtain the true inflation persistence and also ascertain the impact of IT monetary policy in managing inflation persistence since its introduction by Ghana and South Africa (Parker and Rothman, 1997; Montgomery et al., 1998; Rothman, 1998).

Chapter 6

Non-Linearity, Structural Breaks and Fractional Integration in Inflation Series

6.1 Introduction

Keeping a low steady rate of inflation is one of the government's most important responsibilities. Inflation is also related to the real interest rate, output and unemployment through various economic models. Indeed, economists have shown continued interest in this essential economic variable. The most important question related to inflation is: Does non-linearity exist in inflation? The answer to this question, which has important policy implications, can support or endanger the validity of several important economic models. Hence, a clear understanding of the changing aspects of inflation is crucial to any economy because it is regarded as a significant variable in a number of economic models,

whose legitimacy critically relies on whether or not this variable is stationary. In practice, many economic time series models rely on linearity. Nonetheless, it has often been found that simple linear time series regularly leaves certain aspects of economic and financial data inexplicable. Again, economic and financial systems are often subjected to structural and behavioural changes and thus, it is rational to assume that different time series models may be required to explain empirical data such as inflation (Orphanides and Wieland, 2000).

Non-linearity in inflation may reflect different speeds of adjustment toward equilibrium or the level set by policymakers. This implies that the speed of adjustment increases as inflation rate deviates from the equilibrium position (Orphanides and Wieland, 2000). For countries such as Ghana and South Africa, whose central banks (i.e. BoG and SARB) tend to keep inflation within a target range, non-linearity in inflation may come from the response of monetary policy to inflation. As it has been pointed out in Orphanides and Wieland (2000), when there are more objectives than just inflation stabilisation, central banks may have to shift their attention to other objectives, such as output stabilisation or low unemployment, as inflation may be near or within the target range.

Recent empirical literature shows that the dynamic generating mechanism of inflation series is asymmetric, discovered in the two series in Chapter 5, meaning that the behaviour is different during different phases of business cycle. This, therefore, implies the possibility of non-linearity in inflation series. For instance, Shyh (2010) provided evidence of non-linearity in inflation rate in OECD countries. In his research, Yildirim (2004) also provided evidence of non-linearity in Turkish inflation series after fitting logistic smooth transition autoregressive model (LSTAR). The issue of testing and modelling non-linearity in inflation series continues to attract attention because they outperform their linear counterparts in terms of inference and forecasting.

In the existing literature, few studies have empirically examined the possibility of non-linearity in inflation, especially in Sub-Saharan Africa, and those that are available are based on linear trends, intercepts, and at most, structural breaks at fixed points in time. This study is an attempt to bridge this gap. We have sought to make a contribution to the existing literature on inflation studies by proposing a model with potential non-linear trends and structural breaks, where the errors are assumed to be fractionally integrated, $I(d)$. This framework is applied to two Sub-Saharan countries: Ghana and South Africa, which are countries with an IT policy. Indeed, this chapter combines fractional integration with non-linear deterministic terms based on the Chebyshev polynomials in time for the analysis of inflation series of Ghana and South Africa in Sub-Saharan Africa. This analysis requires non-linear deterministic terms in the context of fractional integration using a recently developed approach by Cuestas and Gil-Alana (2016).

This thesis seeks to address the issue of non-linearities in the context of structural breaks and fractional integration. To our knowledge, modelling non-linearity in the context of fractional integration and structural breaks has not been done, hence an opportunity to fill this gap.

In the case of South Africa, Cuestas and Gil-Alana (2016) estimated a fractionally integrated model with non-linear deterministic trends of inflation series in five countries (Angola, Botswana, Lesotho, Namibia and South Africa) using the 2002 to 2013 data span. The results of their study revealed the existence of non-linearities in Angolan and Lesotho's inflation series, whereas inflation series of Botswana, Namibia and South Africa had no non-linearities. This study extends the concept of non-linearities by including structural breaks in the context of fractional integration together with deterministic trends employed by Caporale et al. (2015). Largely, the inclusion of structural breaks and an in-

creased sample size may give new evidence on non-linearities of South African inflation series.

The rest of the chapter is organised as follows: Section 6.2 provides some empirical evidence, while Section 6.3 focuses on data and research methodology. Discussion of empirical results are presented in Section 6.4, while a summary of concluding remarks are presented in Section 6.5.

6.2 Empirical Evidence

Properties of inflation rates have been extensively explored in the literature. A common feature that runs through most related research is the issue of the degree of persistence of shocks, which is related to the unending debate about the possible existence of unit roots in inflation series. For instance, Baba et al. (1988), King et al. (1991), Jahansen (1992), Ericsson and Irons (1994), Evans and Lewis (1995), Crowder and Hoffman (1996), Ericsson et al. (1998), Hendry (2001), Ng and Perron (2001), Rapach and Weber (2004), Lee (2005), and Russell and Banerjee (2008), have presented an argument that inflation contains a unit root, meaning that shocks to inflation are completely persistent. On the other hand, other researchers such as Rose (1988), Culver and Papell (1997), Papell and Prodan (2004), and Noriega et al. (2014), argued that inflation is a stationary variable, and therefore suggested that the impact of economic shocks are transitory.

A number of studies have also claimed that inflation is a fractionally integrated process, with a differencing parameter significantly different from zero and unity. For example, Backus and Zin (1993), found the USA monthly data to be fractionally integrated; Hassler (1993), and Delgado and Robinson (1994) pro-

vided evidence of long memory in the Swiss and Spanish inflation in that order, and Hassler and Wolters (1994), Baillie et al. (1996), and Baum et al. (1999) all also found inflation to be fractionally integrated. However, the failure of such studies to consider the issue of structural breaks impaired their findings (Diebold and Inoue, 2001; Granger and Hyung, 2004).

In recent times, non-linearities have also attracted interest among economic researchers. Although research in this area has provided contradictory or mixed results, depending on the methodology employed (Potter, 1995; Granger and Terasvirta, 1999; Ng and Perron, 2001). Most of these non-linear models are based on smooth transition autoregressive (STAR) models and Markov-switching regime models. These models were actually set for different linear AR models based on linear trends, intercepts, or at most structural breaks at fixed points in time. This thesis adopts a completely different approach to address the problem of non-linearities in the context of fractional integration.

6.3 Data and Methodology

6.3.1 Descriptive statistics

The data considered in this section characterises the log monthly inflation series of Ghana and South Africa, covering the period January 1971 to October 2014 and January 1995 to December 2014, respectively. The data sets were obtained from the BoG and Stats SA. Figures 6.1 and 6.2 describe the behaviour of these two inflation series.

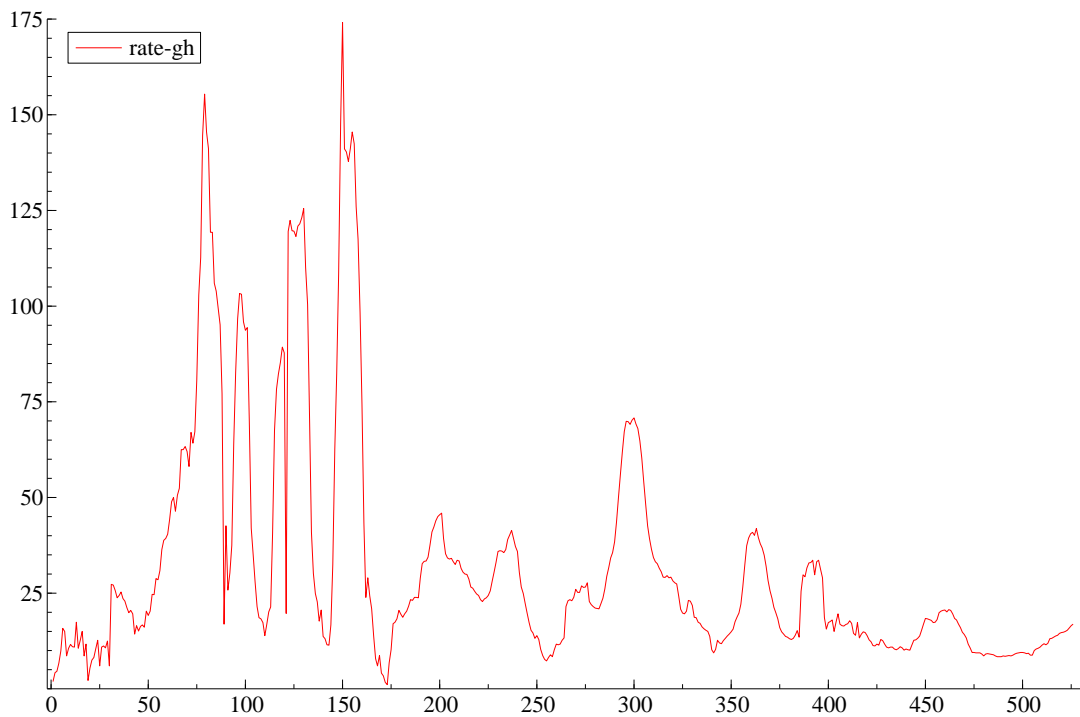


Figure 6.1: The behaviour of GHCPI inflation series

6.3.2 Methodology

This study employs the methodology recently developed by Cuestas and Gil-Alana (2016) which assumes a non-linear trend based on Chebyshev polynomial in time, together with fractionally integrated errors³⁴.

6.4 Empirical Results

The empirical section began by conducting a test proposed by Cuestas and Gil-Alana (2016) to test for fractional integration in the context of non-linear deterministic trends. The results are displayed in Table 6.1. It can be observed that the estimated values of d are 1.11 and 1.32 respectively, for GHCPI and

³⁴Details of this methodology is provided in Chapter 2.

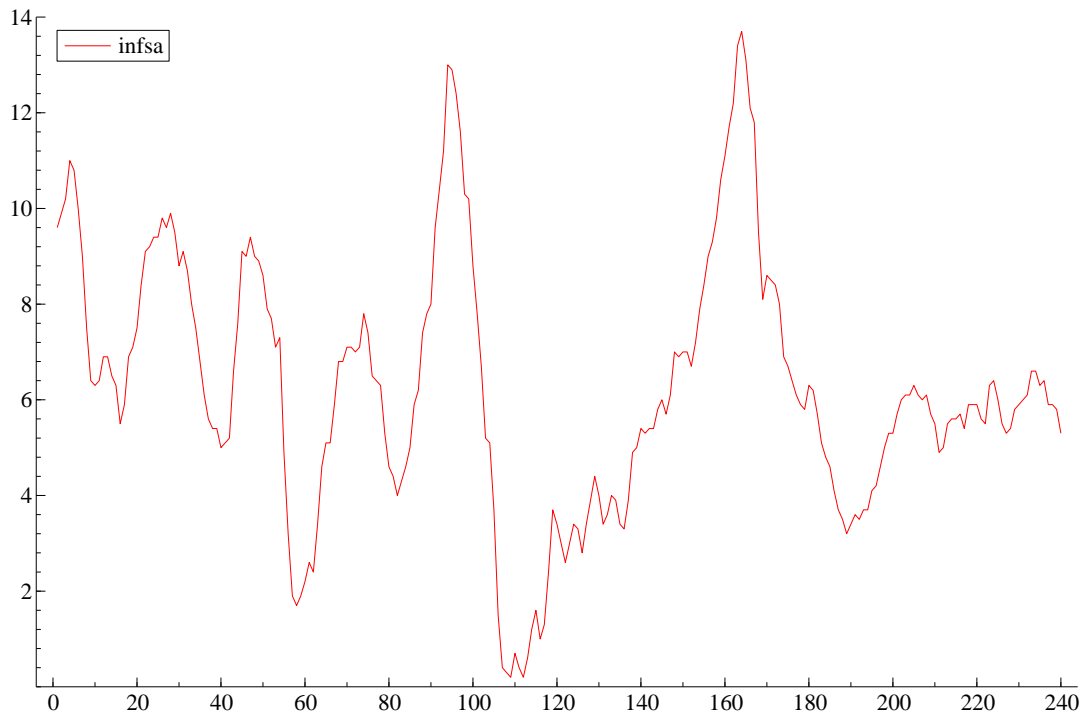


Figure 6.2: The behaviour of SACPI inflation series

SACPI inflation series. However, focusing on the estimated coefficients for the deterministic terms, it can be perceived that d for GHCPI and SACPI inflation series remain statistically insignificant, hence rejecting the hypothesis of non-linearities in the two series examined.

Table 6.1: Estimates of d in the context of non-linear trends for GHCPI and SACPI inflation series

Series	d	θ_1	θ_2	θ_3	θ_3
GHCPI inflation series	1.11 (1.04, 1.19)	-14.32 (-0.07)	16.43 (0.13)	-1.91 (-0.03)	-3.46 (-0.10)
SACPI inflation series	1.32 (1.21, 1.42)	9.09 (0.47)	-0.14 (-0.01)	4.42 (1.01)	-7.38 (-0.90)

Note: The values in parenthesis in the second column refer to the 95% confidence band. In the remaining columns are the t -values.

Owing to the rejection of the null hypothesis of non-linearities in the two inflation series, we then assume a linear model of the form given in Equation (2.76), and the results are presented in Table 6.2.

Table 6.2: Estimates of d with a linear model for GHCPI and SACPI inflation series

Series	No regressors	An intercept	A linear time trend
White noise errors			
GHCPI inflation series	1,11 (1.05, 1.19)	1,11 (1.05, 1.19)	1,11 (1.05, 1.19)
SACPI inflation series	1,21 (1.11, 1.33)	1,45 (1.34, 1.59)	1,45 (1.34, 1.59)
Autocorrelated (Bloomfield) errors			
GHCPI inflation series	1,22 (1.01, 1.45)	1,22 (1.01, 1.45)	1,22 (1.01, 1.45)
SACPI inflation series	1,10 (0.89, 1.40)	1,18 (0.96, 1.46)	1,18 (0.96, 1.46)

Note: The values in parenthesis refer to the 95% confidence bands.

Table 6.2 displays the estimates of d in Equation (2.76) for the three cases of (a) no regressors, $\beta_0 = \beta_1 = 0$ in Equation (2.76), (b) an intercept, $\beta_0 = 0$ and β_1 unknown, and (c) an intercept with a linear time trend, β_0 and β_1 unknown, for the two cases of uncorrelated (white noise) and autocorrelated (Bloomfield) errors. These results indicate that the intercept is sufficient to describe the deterministic terms and the estimated values of d are 1.11 and 1.45 respectively, for Ghanaian and South African inflation series under white noise errors, and 1.22 and 1.18 under autocorrelated disturbances. In fact, the unit root hypothesis (i.e., $d = 1$) is rejected in favour of a higher order of integration for the Ghanaian inflation series, while this hypothesis cannot be rejected for the South African series with the model of Bloomfield (1973)³⁵.

³⁵This is based on the t -values of the coefficients in the d -differenced processes (unreported).

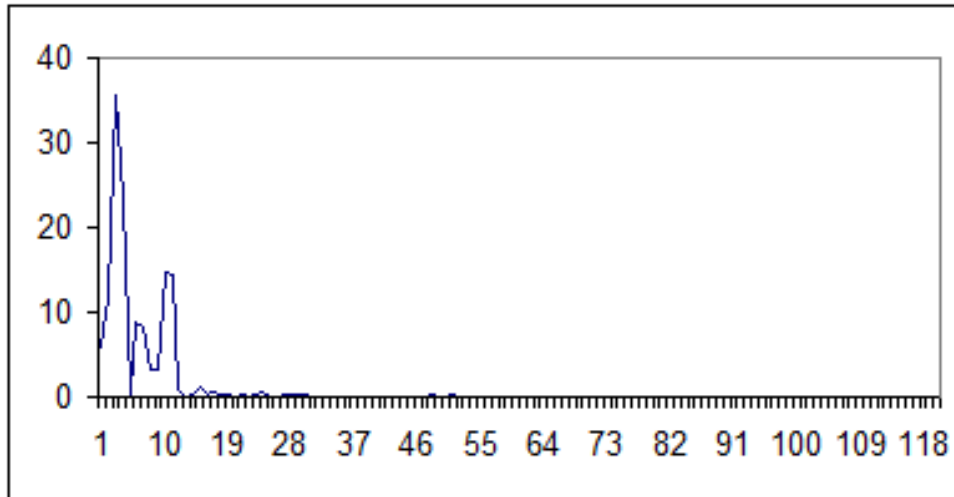


Figure 6.3: Periodogram of GHCPI inflation series for all frequencies

Note: This periodogram depicts all frequencies

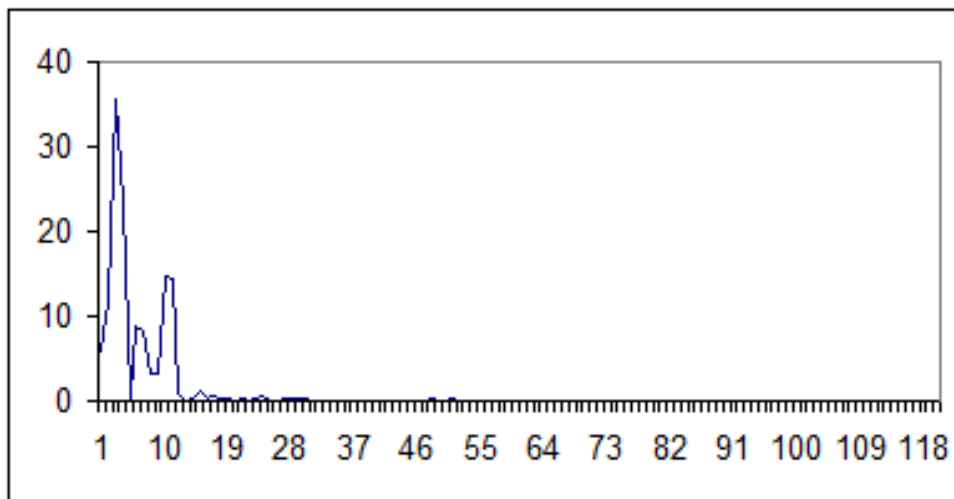


Figure 6.4: Periodogram of SACPI inflation series for all frequencies

Figures 6.3 and 6.4 display the periodograms of the two series. The periodogram is an asymptotically unbiased estimate of the spectral density function, and under the fractionally integrated specification in Equation (2.75), we expect the periodogram to display the highest value at the smallest (zero) frequency. This is precisely what we observe for the periodogram of the inflation series of Ghana in Figures 6.3. However, for the South African series, the

highest value corresponds to a non-zero frequency (Figure 6.4). Again, a close inspection of Figures 6.5 and 6.6 reveals interesting results confirming the previous results, where the third frequency corresponds to the cycle of length $\frac{T}{3}$, which is equivalent to 80 periods (months) for SACPI inflation series.

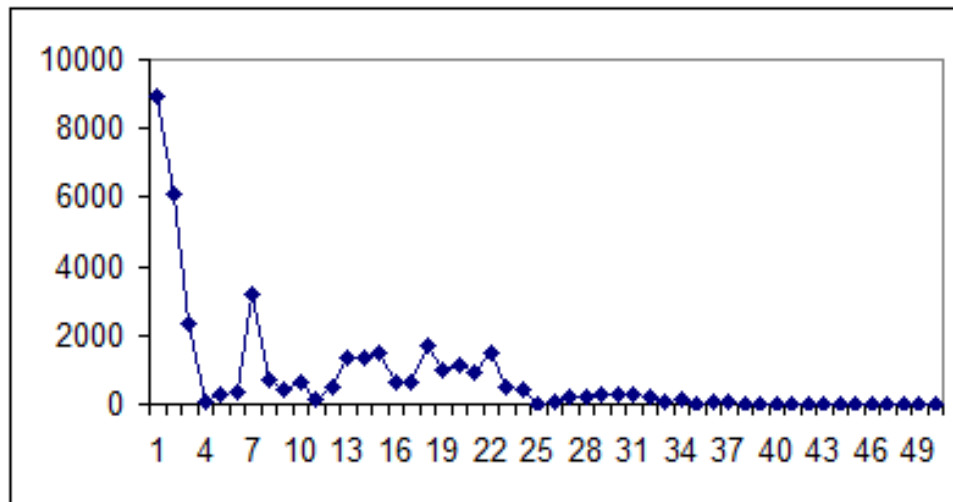


Figure 6.5: Periodogram of GH CPI inflation series for the first 50 frequencies

Note: This periodogram depicts on 50 frequencies

We now try a non-zero cyclical fractional representation for this series, using the model in Equation (2.76), but replacing the second equation by Equation (2.78). The results are reported in Table 6.3. As before, only an intercept is required to describe the deterministic part, r which is equal to 80 periods (as suggested by the periodogram in Table 6.3) and the differencing parameter, d found to be 0.71, implying non-stationarity but mean reverting behaviour in SACPI inflation series.

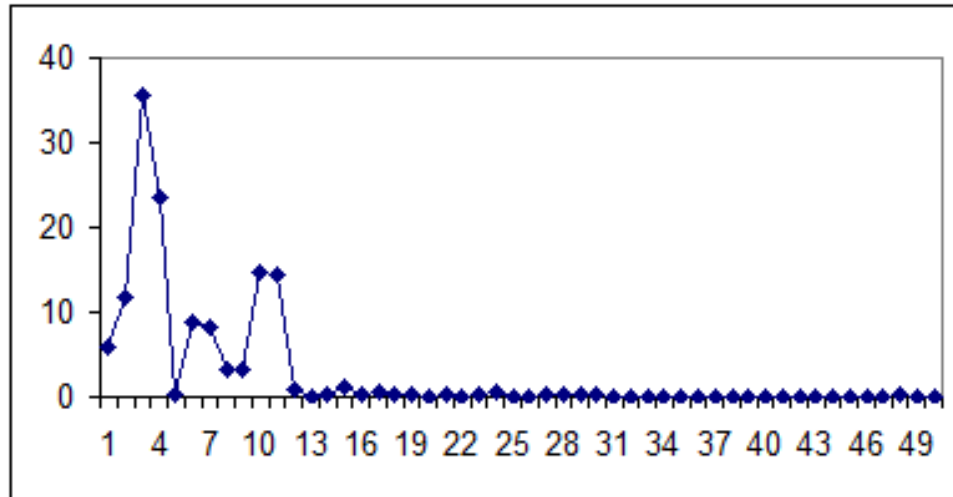


Figure 6.6: Periodogram of SACPI inflation series for the first 50 frequencies

Note: This periodogram depicts on 50 frequencies

Table 6.3: Estimate of d in the context of cyclical fractional integration for SACPI

Series	r	Constant	d
SACPI inflation series	80	6.24(17.65)	0.71 (0.64, 0.81)

Note: The value in parenthesis in the third column is a t -value, while in the fourth column is the 95% confidence band.

6.5 Summary

This Chapter investigated the Ghanaian and the South African inflation series by means of fractional integration method, combined with linear and non-linear structures. The analysis began by testing for non-linearities, after adopting the approach suggested by Cuestas and Gil-Alana (2016) that allows for non-linear deterministic terms and structural breaks in the context of fractional integration. The results indicate that the two series are highly persistent with orders of integration d , far above 1 in the two series.

Both series also appeared to be well described by means of an intercept (or

mean), after applying the linear deterministic approach, with Ghana inflation series found to be highly persistent and non-mean reverting. However, for South African inflation series, a cyclical $I(d)$ model was found to be more adequate in dealing with inflation persistence. In this case, the order of fractional integration was found to be smaller than 1, (i.e., 0.7), depicting mean reversion with the length of the cycles approximated to last for 80 periods (or months).

These discrepancies in inflation persistence, to a large extent, are attributed to the introduction of the IT monetary policy by both countries. Even though inflation persistence in Ghana seems to be on a higher side and non-mean reverting, whereas that of South Africa appears to be mean reverting and cyclical, the impact of the IT monetary policy in curtailing inflation persistence in both countries seems to have achieved some success, and therefore, must be strengthened. The next chapter combines all these properties in a state space model to estimate the threshold effect of inflation on economic growth.

Chapter 7

Threshold Effect of Inflation-Growth Nexus using State Space Model

7.1 Introduction

Many macroeconomic economic policies are often geared towards high and sustained productivity and growth together with moderate or low inflation. However, the issue of the existence and the nature of relationship between inflation and economic growth is often downplayed. Even though inflation-growth nexus is still subject to discussion, the in-depth research on this matter has uncovered some important results and a relatively wide compromise about some aspects of this relationship has been reached.

Indeed, many governments have repeatedly targeted a low, but positive rate of

inflation because they believe that persistently high inflation can have damaging economic and social consequences such as income redistribution, falling real incomes, negative real interest rates, cost of borrowing, etc. For instance, Lee and Wong (2005) investigated the existence of inflation thresholds with a threshold regression model for Taiwan and Japan using a data set for the period 1962-2002 and 1970-2001, respectively. Their results revealed a threshold level of 7.25% for Taiwan and 9.66% for Japan. Similarly, Pollin and Zhu (2006) found the existence of a non-linear relationship between inflation and economic growth for 80 countries for the period 1961-2000, using the middle-income and low-income countries. An estimated inflation threshold of 15% and 18%, above which inflation is detrimental to economic growth and below which it is beneficial to economic growth, was established for middle-income and low-income, in that order.

Kan and Omay (2010) re-examined the threshold effects of inflation and its adverse effects on economic growth using panel data from six industrialised economies, namely Japan, Canada, France, Italy, UK and USA, during the period 1972-2005. In their study, they made use of a panel smooth transition regression model (PSTR) which accounted for non-linearities. After controlling for the unobserved heterogeneity in both cross-section and time dimensions, they found a threshold level of 2.52%, above which inflation negatively and significantly affects economic growth.

Mignon and Villavicencio (2011) also conducted a similar study for 44 lower-middle and low-income countries for the period 1961-2007, using panel smooth threshold regression model, and found a threshold level of 19.6%. In another study, Vaona and Schiavo (2007) conducted research on non-parametric and semi-parametric evidence on the long-run effects of inflation on growth. In their research, they adopted both non-parametric and semi-parametric instru-

mental variable estimators to show that the relationship between inflation and output growth is non-linear, and that there exists a threshold level below which inflation has no effects on growth.

In another dimension, Eggoh and Khan (2014) conducted a study on non-linear relationship between inflation and economic growth using a large panel data set from both developed and developing economies, employing the PSTR and dynamic generalised method of moments (GMM) techniques, and their study highlighted two aspects of the inflation and economic growth relationship. First, they analysed the non-linearity of the relationship and identified several thresholds for the global sample and for various income-specific subsamples. Second, they identified some country-based macroeconomic features that influence this non-linearity. Their results, to a large extent substantiated both views and validated the fact that inflation and economic growth are non-linear and sensitive to a country's level of financial development, capital accumulation, trade openness and government expenditures.

Ibarra and Trupkin (2011) re-examined the relationship between inflation and growth and ascertained whether institutions matter in developing countries. By means of a large panel of countries during the period 1950-2009, they estimated inflation thresholds above which its association with economic growth is expected to be negative, taking into account differences in institutions across countries. They found a threshold level of 19.1% for non-industrialised countries and a high speed of transition from low to high inflation regimes. First, in line with previous literature, they found the estimated threshold level for developing countries to be substantially higher than those of developed countries.

From the literature cited so far, several observations can be pointed out. In

the first place, there appears to be a compromise of some sort on the fact that inflation-growth relationship is non-linear, which then implies the existence of threshold level below which inflation has either insignificant impact or positive impact on economic growth, and above which inflation has a negative impact on economic growth. Second, there seems to be a dissimilarity of the thresholds levels across countries depending on the stage of economic development, institutional arrangements and structural realities. Thirdly, it looks as if higher threshold levels are associated with developing countries compared to the lower thresholds for the developed or industrialised countries, and this probably may be attributed to sound macroeconomic policies being implemented in those developed countries (Seleteng et al., 2013).

Lastly, data problems, methodological issues (be it linear or non-linear) and estimation techniques among others, could be the underpinning explanations to the variations in the threshold values underscored in the literature cited so far. For instance, a number of research alluded to in literature, applied linear models based on ordinary least squares (OLS) estimation techniques and threshold models such as PSTR, GMM, semiparametric instrumental variable (SIV) estimation methods and non-parametric instrumental variable (NIV) estimation methods. For example, the use of linear models based on OLS requires strict assumptions of normality for reliable parameter estimates, since time series data are often non-normal.

This thesis, therefore, utilises a state space model, which is a more robust approach to analyse the threshold effect of inflation on economic growth. We are motivated to examine the threshold effect of inflation-growth nexus of Ghana and South Africa, using state space model with Kalman filter for the following reasons: (1) there are limited studies in the two countries on inflation-growth nexus with state space model of unobserved components with Kalman filter,

and (2) to illustrate the significance and use of unobserved components models in economics and finance and its relevance in forecasting and formulation of policy. The advantages of this model over the rest cited in the literature are that: (a) the state space model can be used to model unobserved components such as shocks and even evaluate the significance of intervention programmes together with explanatory variables, (b) this model can also identify several threshold values and assess the significance of the thresholds in a time series data, and (c) this model can also be used to solve the problem of endogeneity in a model building process.

The rest of the chapter is outlined as follows: empirical research on threshold levels of inflation is presented in Section 7.2. Section 7.3 presents data and methodology, with empirical results displayed in Section 7.4. Summary and conclusion are presented in Section 7.5.

7.2 Empirical Evidence of Thresholds effect of Inflation-growth Nexus from Ghana and South Africa

In the context of Sub-Saharan Africa, especially in the two IT countries, a number of studies have investigated theoretical and empirical aspects of inflation-growth nexus, but in terms of research meant to estimate threshold levels of inflation with state space model, is limited. For instance, Seleteng et al. (2013) undertook a study on non-linearities in inflation-growth nexus in the Southern African Development Community (SADC) region using PSTR model on a panel data for the period 1980-2008. Their findings revealed a threshold level of 18.9%, above which inflation is detrimental to economic growth in the SADC

region.

Again, there seems to be a growing concern about the influence of higher or lower inflation on economic growth. Many also believe that moderate inflation could bring about economic growth, unlike high price level which often creates uncertainty and distorts economic growth (Furuoka et al., 2009). This then raises an interesting policy issue on the level of inflation necessary to boost economic growth. Perhaps another important question to ask is whether or not formulation of monetary policies took into account the stability of the time-varying parameters of economic agents. For instance, Ghana introduced IT policy in May 2007 through the BoG with its target set between 6%-9%, but was able to meet the target during 2010- 2012. Since that time, the above target has never been met, reaching inflation rate of 18.5% as at February, 2016. This may probably be due to institutional changes, profligate government expenditure, energy crisis and global market volatility. Therefore, there is need to revisit these thresholds to reflect the current situation. For example, Frimpong and Oteng-Abayie (2010) conducted a study on inflation-growth nexus and estimated a threshold level of 11% for Ghana for the period 1960-2008, even though they failed to test for the significance at that level. They also estimated a robust 11% threshold inflation level with close coefficients after dropping growth rate of aggregate labour force and money supply growth, which were found to be insignificant in the OLS models.

South Africa, to a large extent, has been able to meet its target of 3%-6% set by the SARB since the introduction of its IT policy in February 2000, even though there were some deviations in terms of meeting the target, but by and large the target has been met. Recently, the South African economy has been facing some turbulence resulting from energy crisis, service delivery protests, political uncertainties and the consequence of China's market volatility, with

current inflation going beyond the target reaching 7% in February 2016. This has laid some fears for the SARB, Government and the general public since prices will keep on rising. Therefore, it is imperative to re-examine the issue of threshold effect taking the current situation into consideration.

Phiri (2010) estimated the threshold level that is inimical to finance-growth activity for the South African economy. He employed quarterly data for the period 2000-2010, and the results revealed an inflation threshold level of 8%. In addition, Leshoro (2012), examined inflation-growth nexus for South Africa using quarterly data for the period 1980-2010 using the threshold regression model propounded by Khan and Senhadji (2001). The researcher estimated an inflation threshold level of 4%, below which there is a positive, but statistically insignificant relationship between inflation and growth, and above which the relationship becomes negative and significant.

It is evident that there are limited studies on inflation-growth nexus with state space model. This study therefore, attempts to fill the gap and also create awareness regarding a new robust way of determining the threshold level of inflation in relation to economic growth. Indeed, state space model of unobserved components developed by Koopman et al. (2006) in the structural time series analyser modeller and predictor (STAMP) package has been under-utilised . This research seeks to contribute to extant and limited literature on this matter, especially in the context of these two African countries countries on the thresholds levels of inflation and its adverse effects on economic growth using state space models.

7.3 Data and Methodology

7.3.1 Descriptive statistics

A number of studies conducted on the issue of inflation-growth nexus have made use of cross sectional and panel data with the coverage of a large number of countries. For example, Khan and Senhadji (2001), and Ahmed and Mortaza (2010) employed cross sectional data consisting of many countries in their research. Cross sectional data is preferred by most researchers because a single country often lacks the variety of inflation experiences required to determine the relationship between inflation and growth (Judson and Orphanides, 1999). Nonetheless, Bruno and Easterly (1998) re-counted that any cross sectional relationship between inflation and growth loses importance when data from countries with 40% or more inflation are left out from the analysis.

Others researchers (Fisher, 1993; Borro, 1995; Ibarra and Trupkin, 2011; Eggoh and Khan, 2014), utilised panel data in their respective research taking the time dimension of inflation and growth into account. There are also studies such as those of Mubarik (2005), Frimpong and Oteng-Abayie (2010), and Phiri (2010), that employed annual time series data to estimate threshold levels of inflation for individual countries. Our study also uses annual data for 1981-2014 to estimate threshold levels of inflation for Ghana and South Africa using a state space model with Kalman filter algorithm. Data source on GDP per capita, CPI inflation, total investments, volume of imports of goods and services, volume of exports of goods and services and growth rate in population from the IMF database (International Monetary Fund, 2014). However, the first part of the analysis explores the stability of the time-varying components of unobserved components of inflation of Ghana and South Africa.

The second part of this chapter looks at the core objective of this study, which

is the estimation of threshold effect of inflation on economic growth using state space model with Kalman filter. Annual data have been applied in this study for the following reasons: (1) many monetary policymakers are mostly interested in an annual inflation rate in order to set targets and make adjustments compared to quarterly or monthly data, and (2) using seasonally adjusted data such as quarterly or monthly adjusted data, could conceal some information needed for the analysis. Figures 7.1 and 7.2 and Table 7.1 present the descriptive and summary statistics of all the variables used in our study for Ghana and South Africa.

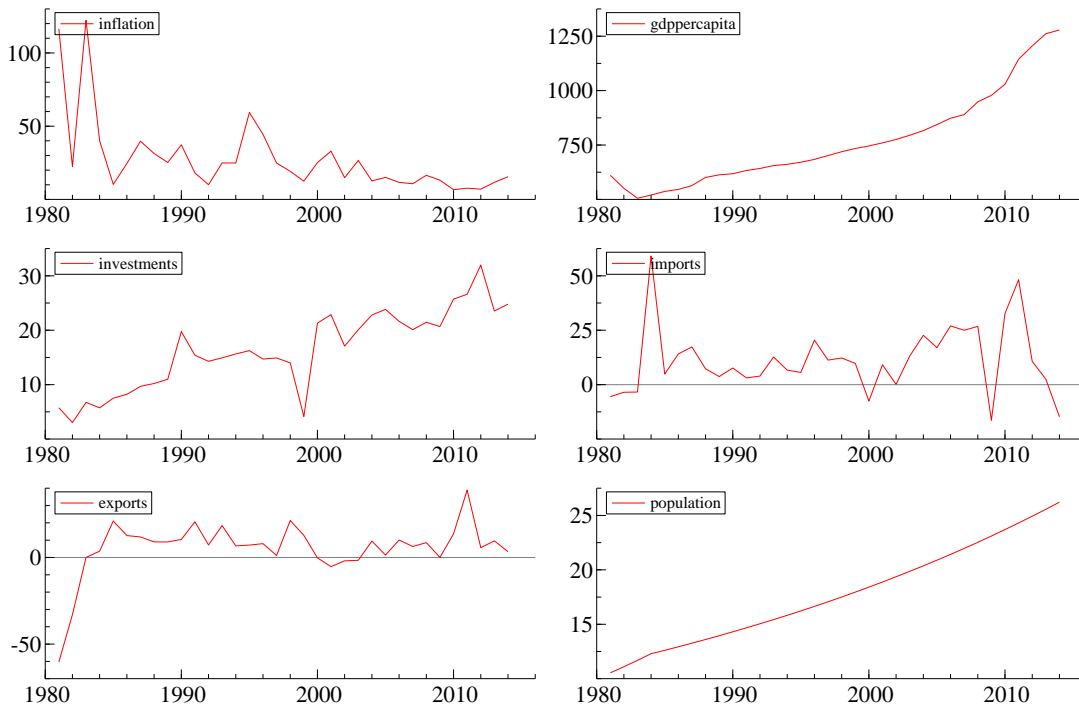


Figure 7.1: Time series plots for all the variables: Ghana (1981-2014)

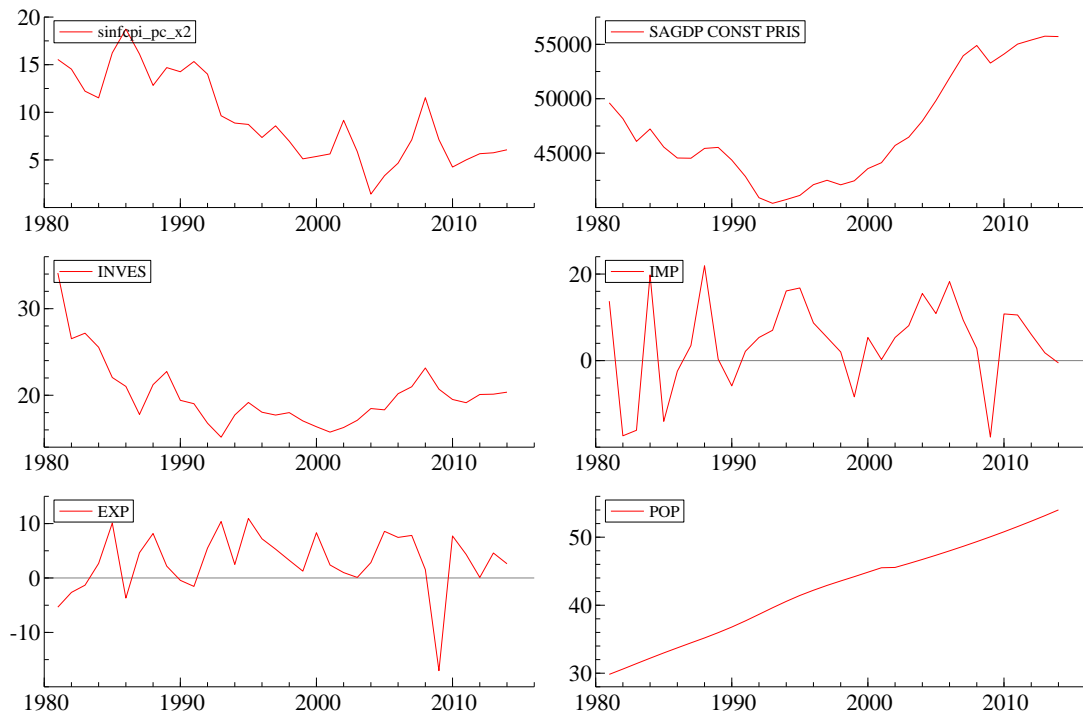


Figure 7.2: Time series plots for all the variables: South Africa (1981-2014)

7.3.2 Methodology

In this chapter, a state space model of unobserved component with Kalman filter has been applied to deal with two main issues: (1) the investigation of the stability of time-varying parameters, especially the conditional mean (i.e. mean level) of inflation, and (2) the estimation of the threshold effect of inflation on economic growth. A state space model of unobserved component consists of two main equations. These are the measurement (observation) equations, which describe the relationship between observed and unobserved variables, and the transition equation, that describes the dynamics of the unobserved variables (e.g. demand and supply shocks)³⁶. The two equations are

³⁶Details have been provided in Chapter 2.

Table 7.1: Descriptive and Summary Statistics

GHCPI Inflation						
Stats.	GDP	Exports	Imports	CPI	Invest.	Pop.
Mean	768.03	5.47	11.28	27.53	16.36	17.76
Median	710.49	7.62	9.44	20.85	15.95	17.29
Max	1278.67	38.90	59.13	122.25	31.97	26.22
Min	505.58	-60.40	-16.39	6.69	3.03	10.54
Std. Dev	214.61	16.07	15.67	26.24	7.29	4.52
Skew.	1.02	-2.17	0.97	2.60	-0.09	0.24
Kurt.	3.13	10.53	4.59	9.58	2.19	1.93
J-B test	5.87	106.86	8.93	99.66	0.98	1.96
	(0.05)	(0.00)*	(0.02)*	(0.00)*	(0.61)	(0.35)
Obs.	34	34	34	34	34	34
SACPI Inflation						
Mean	47174.20	2.98	4.28	9.39	20.08	42.29
Median	45635.94	2.75	5.34	8.66	19.29	43.24
Max	55747.86	10.94	21.91	18.74	34.12	54.00
Min	40394.35	-17.02	-17.66	1.42	15.16	29.83
Std. Dev	5035.43	5.46	10.36	4.51	3.80	7.16
Skew.	0.49	-1.36	-0.55	0.33	1.76	-0.15
Kurt	1.88	6.46	2.83	1.95	6.85	1.85
J-B test	3.09	27.52	1.78	2.18	38.49	1.99
	(0.21)	(0.00)*	(0.41)	(0.34)	(0.00)*	(0.37)
Obs.	34	34	34	34	34	34

Note: Values in parenthesis are the p-values for evaluating the significance of the Jarque-Bera test for normality; *Denotes a rejection of the null hypothesis of normality.

defined as follows:

$$\text{Observation equation : } y_t = H_t B_t + A z_t + \varepsilon_t$$

$$\text{Transition equation : } \beta_t = \mu + F \beta_{t-1} + \nu_t$$

where y_t denotes the annual CPI inflation, H_t are the coefficients, B_t unobserved state, z_t exogenous/explanatory regressors, ε_t and ν_t are the error innovations fulfilling the following assumption: $\varepsilon_t \sim iidN(0, R)$, $\nu_t \sim iidN(0, Q)$ and $E(\varepsilon_t, \nu_t) = 0$.

The second part of this methodology has to do with the estimation of threshold effect of inflation-growth nexus using the state space model of unobserved component with Kalman filter by considering the equation:

$$y_{GDP_t}^* = y_t^T + \beta_1 inf_t + \beta_2 D_t(inf_t - \delta) + z_{it} + \varepsilon_t$$

Following the definitions of Khan and Senhadji (2001), Mubarik (2005), Frimpong and Oteng-Abayie (2010), and Phiri (2010), the parameter δ denotes the threshold level of inflation-growth nexus and it is defined in terms of low inflation, (δ_1) and high inflation, $(\delta_1 + \delta_2)$. According to Khan and Senhadji (2001), high inflation connotes the significance of the inflation estimate, hence $(\delta_1 + \delta_2)$ would then be added to ascertain the impact of inflation on growth, that defines the threshold level of inflation³⁷. The value of δ is given arbitrarily for the estimation of the threshold level, with the optimal δ obtained by finding the value of the threshold that minimises the residual sum of squares (RSS) or maximises the R-squared together with AIC and the LL.

The threshold levels for the two countries were indeed selected arbitrarily based on the behaviour of the series in literature for the past two decades. State space model has been applied in this study for the following reasons: this model can decompose time series data into various components such as trend, cyclical, seasonal, autoregressive and irregular components, and this model also allows inclusion of autoregressive regressors and exogenous or explanatory variables, and it is easy to interpret. These characteristics make it more robust than previous methods cited in the literature so far (Koopman et al., 2006).

³⁷This thesis adopted the estimation approach of Khan and Senhadji (2001) and incorporated it into state space model of unobserved components with Kalman filter to estimate the threshold level of inflation for both countries.

7.4 Empirical Results

The stability of time-varying parameters such as the mean level of inflation from both countries, using the state space modelling of unobserved components, have been presented in Tables 7.2 and 7.3. A basic structural model (BSM) has been fitted to ascertain whether or not the level (i.e. mean level) of inflation of Ghana and South Africa follows a stochastic process. In Tables 7.2 and 7.3, the two BSMs together with explanatory regressors in the form of interventions, determined endogenously, are displayed³⁸.

From Table 7.3, it is observed that the mean level of GHCPI follows a stochastic process, hence time-varying. Again, a close inspection of Table 7.3 also reveals and confirms the effects of different magnitude of shocks on the mean level of the Ghanaian inflation.³⁹ For example, 1982 and 1983 marked the years where the economy of Ghana suffered an unprecedented rise in inflation.

The years 1995 and 1996 coincided with an election year which is characterised by overspending, hence a rise in inflation. In 2007, the BoG introduced the IT policy as a way of controlling the fluctuations in inflation. It was found to be insignificant and this could be attributed to the neglect of the time-varying nature of mean level of inflation. This is not surprising since the achievements of IT were felt only between 2010 and 2012.

Similarly, a visual inspection of Table 7.4 indicates the time-varying nature of the mean level of SACPI, which also confirms previous results. The consequence of the economic crisis in 2008 and its adverse effects on the South

³⁸Several interventions and their significance and shocks have been discussed in the previous Chapters, through structural breaks test and test of long range dependence and as well non-linearities. Details, confirmation and justification of this can be found in Chapters 3, 4 and 6.

³⁹This persistence of shocks was actually captured in Chapters 4, 5 and 6.

Table 7.2: State space model of unobserved components of GH CPI with Kalman filter

Variance of Disturbances (shocks)					
Component	Value	q-ratio			
Level	1.86	0.03			
Irregular	109.65	1.00			
State vector analysis at period 2014					
Level	109.65	0.00*			
Reg. effects in final state at period 2014					
Component	Coefficients	RMSE	t-value	Prob.	
OL 1983	96.38	8.37	11.52	0.00*	
LB 2007	-7.69	4.77	-1.61	0.12*	
LB 1982	-90.74	8.55	-10.62	0.00*	
OL 1995	36.66	8.16	4.49	0.00*	
OL 1996	22.14	8.16	2.71	0.01*	
Summary stats	GH CPI inflation				
T Sample size	34.00				
<i>p</i> Number of parameters	1.00				
Std. Error.	7.69				
Normality, Bowman-Shenton test	0.06				
H(9) Test of heteroscedascity	0.18				
DW Durbin-Watson test	1.64				
<i>r</i> (1) Estimated residuals at lag 1	0.03				
<i>q</i> lags of autocorrelation	5.00				
<i>r</i> (<i>q</i>) Lag <i>q</i> autocorrelation	-0.18				
<i>Q</i> (<i>q</i> , <i>q</i> - <i>p</i>) Box-Ljung stats	5.03				
<i>R</i> ² Goodness of fit	0.92				

Note: RMSE denotes root mean square error; *Denotes null hypothesis of non-stochastic process rejected against the alternative hypothesis of stochastic process, implying the time-varying nature of the mean to disturbances or shocks between 1981 and 2014 of the BSM model of the form: $y_{tinf} = level + irregular + interventions$. *OL* and *LB* denote an outlier and level breaks respectively.

African economy was also felt strongly. Currently, South Africa, missed its inflation target set between 3%-6% to about 7% in early 2016. Perhaps the reason could be due to the neglect of incorporating the time-varying nature of the mean level of inflation in the IT determination. This is confirmed in the cumsum residual plots (see Figures 7.3 and 7.4).

Table 7.3: State space model of unobserved components of SACPI with Kalman filter

Variance of Disturbances (shocks)				
Component	Value	q-ratio		
Level	4.78	1.00		
Irregular	0.00	0.00		
State vector analysis at period 2014				
Level	6.07	0.00*		
Reg. effects in final state at period 2014				
Component	Coefficients	RMSE	t-value	Prob.
OL 2008	4.41	1.55	2.85	0.01*
Summary stats		SACPI inflation		
T Sample size	34.00			
p Number of parameters	1.00			
Std. Error.	2.12			
Normality, Bowman-Shenton test	0.13			
H(9) Test of heteroscedascity	0.70			
DW Durbin-Watson test	1.82			
$r(1)$ Estimated residuals at lag 1	0.06			
q lags of autocorrelation	5.00			
$r(q)$ Lag q autocorrelation	0.33			
$Q(q, q - p)$ Box-Ljung stats	13.67			
R^2 Goodness of fit	0.79			

Note: RMSE denotes root mean square error; *Denotes null hypothesis of non-stochastic process rejected against the alternative hypothesis of stochastic process, implying the time-varying nature of the mean to disturbances or shocks between 1981 and 2014 of the BSM model of the form: $y_{tinf} = level + irregular + interventiosn$. OL denote an outlier and level breaks respectively.

Next, we incorporate the unobserved components such as structural breaks, long memory, non-linearities and the time-varying nature of mean level behaviour of inflation, together with some explanatory variables, to ascertain the threshold effect of inflation on economic growth using the state space model with Kalman filter algorithm. Figures 7.5 and 7.6 illustrate the trend in inflation and growth rate of GDP per capita of Ghana and South Africa, respectively.

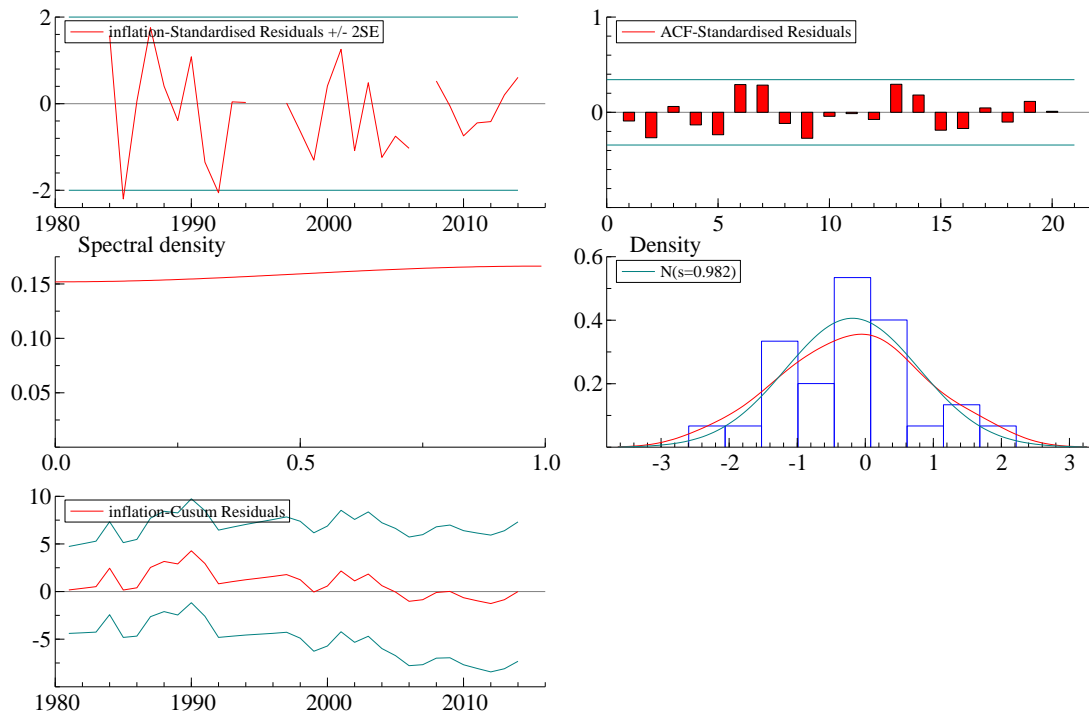


Figure 7.3: Residual analysis for GHCPI inflation with state space-BSM

Table 7.4: Pairwise Granger causality test for CPI inflation and growth rate by GDP per capita for Ghana and South Africa

GHCPI and GDP per capita				
Null hypothesis	Observations	F-stats	P-value	
GHCPI does not Granger cause GDP	32	7.09	0.00*	
GDP does not Granger cause GHCPI	32	1.08	0.35	
SACPI and GDP per capita				
SACPI does not Granger cause GDP	32	8.29	0.00*	
GDP does not Granger cause SACPI	32	0.89	0.42	

Note: *In both cases, the null hypothesis is rejected, implying the influence of inflation on economic growth.

Before estimating the threshold effect model in Equation (2.92), a Granger causality test was performed to establish the causation between inflation and economic growth for Ghana and South Africa, and the results are presented in Table 7.4. The test statistics in Table 7.4 shows a rejection of the null hy-

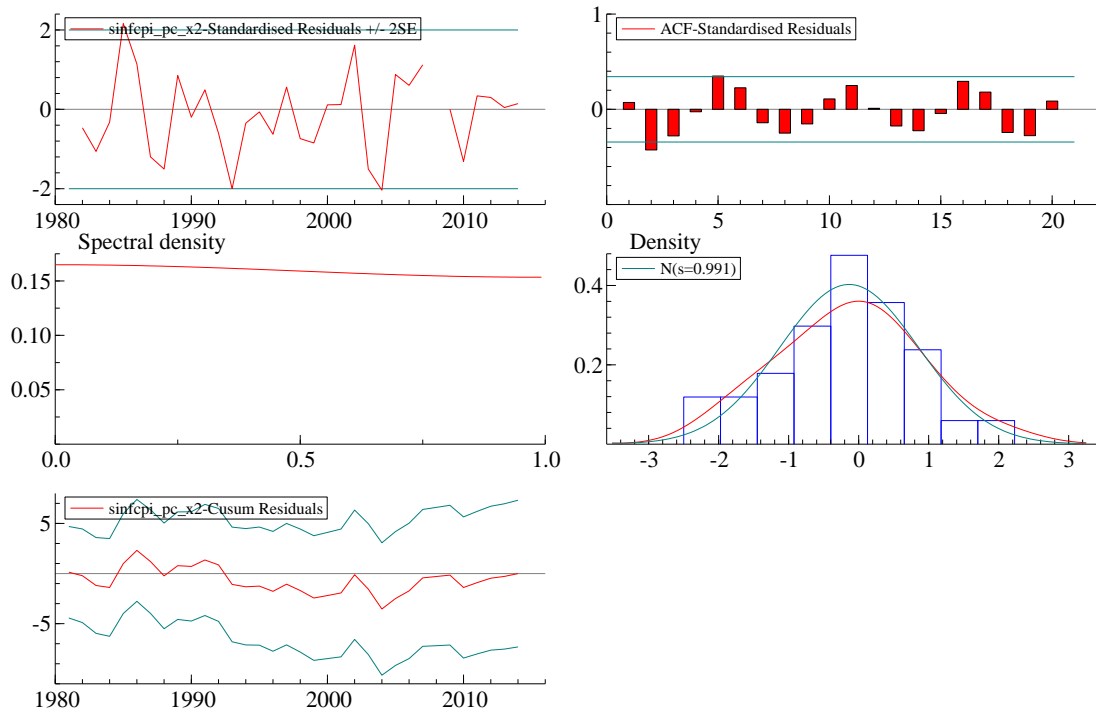


Figure 7.4: Residual analysis for SACPI inflation with state space-BSM

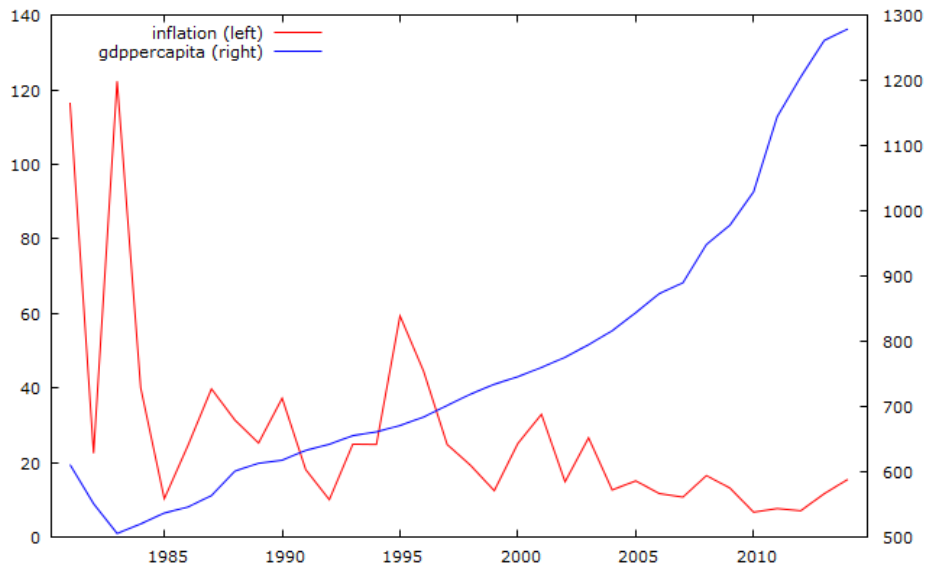


Figure 7.5: The graph of Growth rate by GDP per capita and CPI inflation series of Ghana (1981-2014)

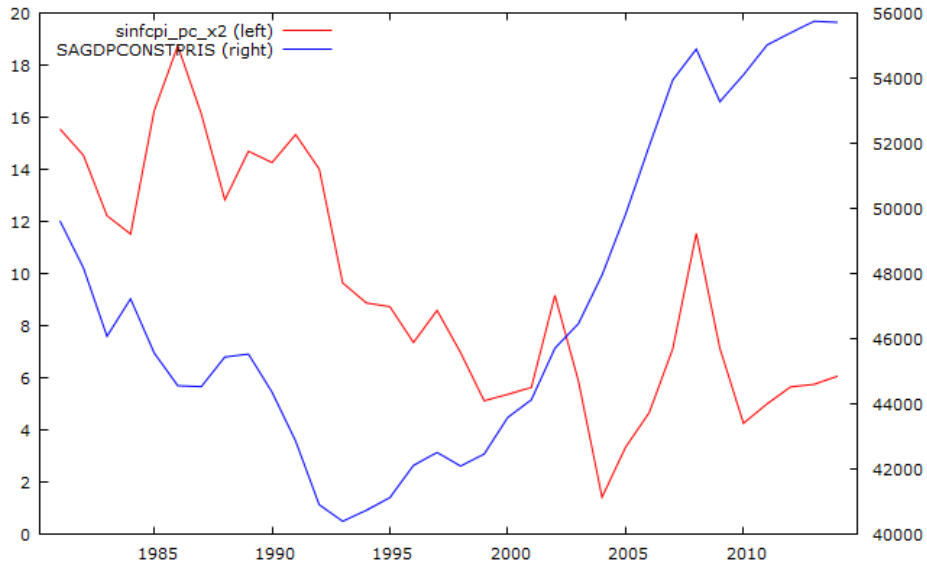


Figure 7.6: The graph of Growth rate by GDP per capita and CPI inflation series of South Africa (1981-2014)

pothesis, which means that inflation is causing GDP growth. The relationship between inflation and economic growth was also found to be unidirectional and negative (see Figures 7.5 and 7.6 and Appendices 7.1 and 7.2). This implies a rise, in inflation and will eventually have an awful consequence on economic growth.

The estimation of Equation (2.92) by EML gives a clear-cut value of threshold level and also quantifies the impact of the level on economic growth displayed in Tables 7.5 and 7.6. Also displayed in these tables are the most competing models after fitting a number of models. Indeed, threshold levels of 9%-12% and 4%-7% were respectively specified for GH CPI and SACPI inflation series based on the LL, AIC and RSS.

Estimation outputs displayed in Table 7.5 indicate that the threshold level of inflation for growth rate of GDP per capita is 9% in the Ghanaian economy. Note that this finding is dissimilar to the findings obtained by Khan and Sen-

Table 7.5: State space model of unobserved components with Kalman filter in estimating the threshold effect of inflation-growth nexus of Ghana

Variable	Coefficient	RMSE	t-value	Prob.	AIC	LL	R ²
LB 1982	-72.22	23.41	-3.08	0.01*	6.29	194.00	69.00
OL 2008	21.38	7.83	2.73	0.01*			
Inf	13.43	3.70	3.63	0.00*			
D(Inf-9)	-13.74	3.70	-3.71	0.00*			
Invest.	0.42	0.45	0.94	0.36			
Imports	0.09	0.12	0.79	0.44			
Exports	0.15	0.18	0.83	0.42			
Pop.	-79.68	73.41	-1.09	0.29			
LB 1982	-72.48	23.05	3.14	0.00*	5.58	162.00	87.000
OL 2008	20.94	7.70	2.72	0.01*			
Inf	9.25	2.68	3.46	0.00*			
D(Inf-10)	-9.55	2.67	-3.57	0.00*			
Invest.	0.43	0.45	0.95	0.35			
Imports	0.10	0.12	0.82	0.42			
Exports	0.19	0.18	1.08	0.29			
Pop.	-79.39	72.39	-1.09	0.28			
LB 1982	-77.12	23.60	-3.27	0.00*	5.56	162.00	87.000
OL 2008	20.53	7.85	2.61	0.02*			
Inf	6.43	2.01	3.20	0.00*			
D(Inf-11)	-6.76	2.02	-3.35	0.00*			
Invest.	0.39	0.45	0.87	0.39			
Imports	0.06	0.12	0.49	0.63			
Exports	0.21	0.18	1.19	0.24			
Pop.	-83.65	1.13	-74.29	0.27			
LB 1982	-80.24	24.25	-3.31	0.00*	5.540	162.00	88.00
OL 2008	17.85	8.23	2.17	0.04*			
Inf	4.19	1.54	2.73	0.01*			
D(Inf-12)	-4.54	1.55	-2.92	0.01*			
Invest.	0.31	0.47	0.66	0.51			
Imports	0.04	0.13	0.28	0.78			
Exports	0.27	0.18	1.46	0.16			
Pop.	-85.50	76.14	-1.12	0.27			

Note: Dependent variable: Growth rate by GDP per capita. The goodness of fit was selected based on the LL; *LB* and *OL* denotes level break and outlier respectively; *Denote a rejection of the null hypothesis.

Table 7.6: State space model of unobserved components with Kalman filter in estimating the threshold effect of inflation-growth nexus of South Africa

Variable	Coefficient	RMSE	t-value	Prob.	AIC	LL	R ²
Inf	155.94	210.10	0.74	0.46	13.84	382.97	64.00
D(Inf-4)	-109.97	225.35	-0.49	0.63			
Invest.	182.14	71.37	2.55	0.02*			
Imports	16.42	10.62	1.55	0.13			
Exports	29.04	17.97	1.62	0.12			
Pop.	-2381.96	880.89	-2.70	0.01*			
Inf	181.92	154.57	1.18	0.25	13.83	383.18	65.00
D(Inf-5)	-146.39	169.67	-0.86	0.39			
Invest.	184.26	70.24	2.63	0.01*			
Imports	16.27	10.47	1.55	0.13			
Exports	30.67	17.74	1.73	0.09			
Pop.	-2440.74	871.58	-2.80	0.00*			
LB 2004	2925.21	1332.31	2.19	0.04*	13.70	365.40	72.00
Inf	599.48	234.38	2.56	0.02*			
D(Inf-6)	-587.79	252.88	-2.33	0.03*			
Invest.	209.61	81.01	2.59	0.02*			
Imports	13.57	12.66	1.07	0.29			
Exports	41.08	22.15	1.86	0.08			
Pop.	-2455.58	905.86	-2.72	0.01*			
Inf	102.56	147.87	1.44	0.16	13.44	383.62	65.00
D(Inf-7)	-122.18	120.29	-1.02	0.32			
Invest.	175.74	69.48	2.53	0.02*			
Imports	16.87	10.31	1.64	0.11			
Exports	32.68	17.76	1.84	0.08			
Pop.	-2276.49	868.97	-2.62	0.01*			

Note: Dependent variable: Growth rate by GDP per capita. The goodness of fit was selected based on the AIC; *LB* and *OL* denotes level break and outlier respectively; *Denote a rejection of the null hypothesis.

hadji (2001) for developing countries. It is also lower than the results obtained by Frimpong and Oteng-Abayie (2010) for Ghana. These differences were expected since the state space model takes into account the stochastic nature of economic variables such as inflation and GDP, and also incorporated unobserved components in the estimation process.

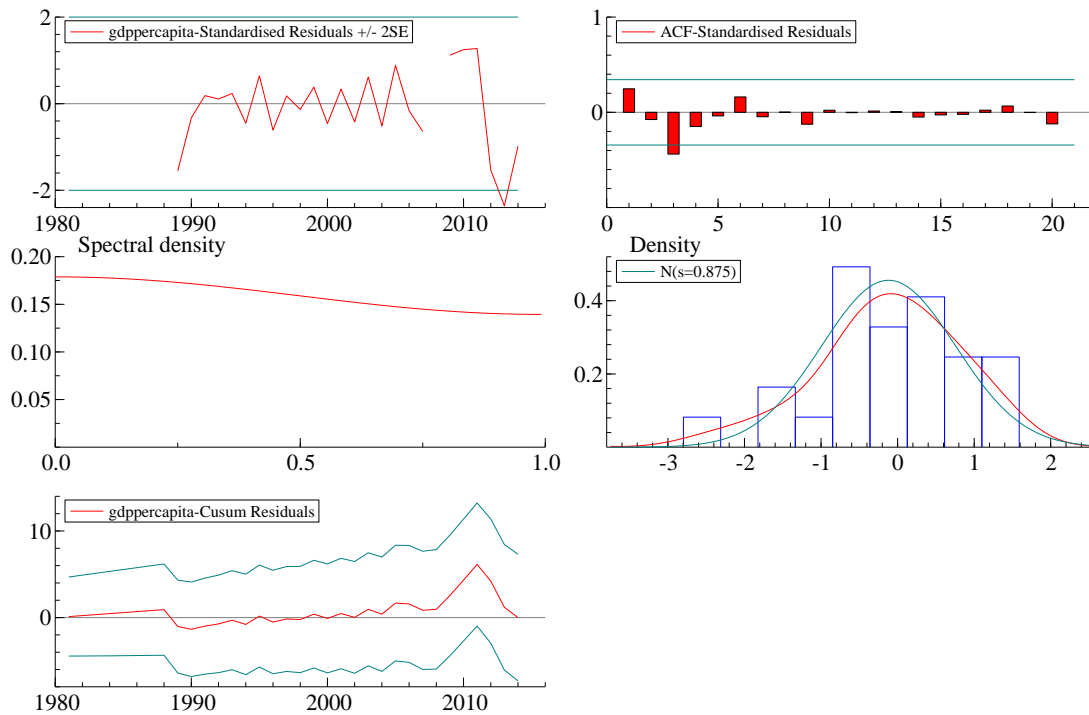


Figure 7.7: Residual analysis of the selected state space model for thresholds effect of inflation-growth nexus of Ghana

Accordingly, inflation which is lower than 9% will impact GDP growth positively, but this positive relationship will be reversed if inflation goes beyond 9%. Indeed, when inflation exceeds the 9% threshold, economic growth is expected to decline to an estimated 2.2%, i.e. $(\beta_1 + \beta_2) = 13.434 - 13.739; 0.31/13.74 = 2.2\%$. Hence, monetary policy decision makers should keep inflation under 9%. This threshold level discovered in this study is actually within the IT policy target by the BoG, but little success has been achieved so far since its introduction in May, 2007.

Similarly, the estimation results presented in Table 7.7 point to 6% threshold level of inflation for growth rate of GDP per capita in the South African economy. This result is different from the findings obtained by Phiri (2010), and Leshoro (2012) who conducted similar research on inflation thresholds neces-

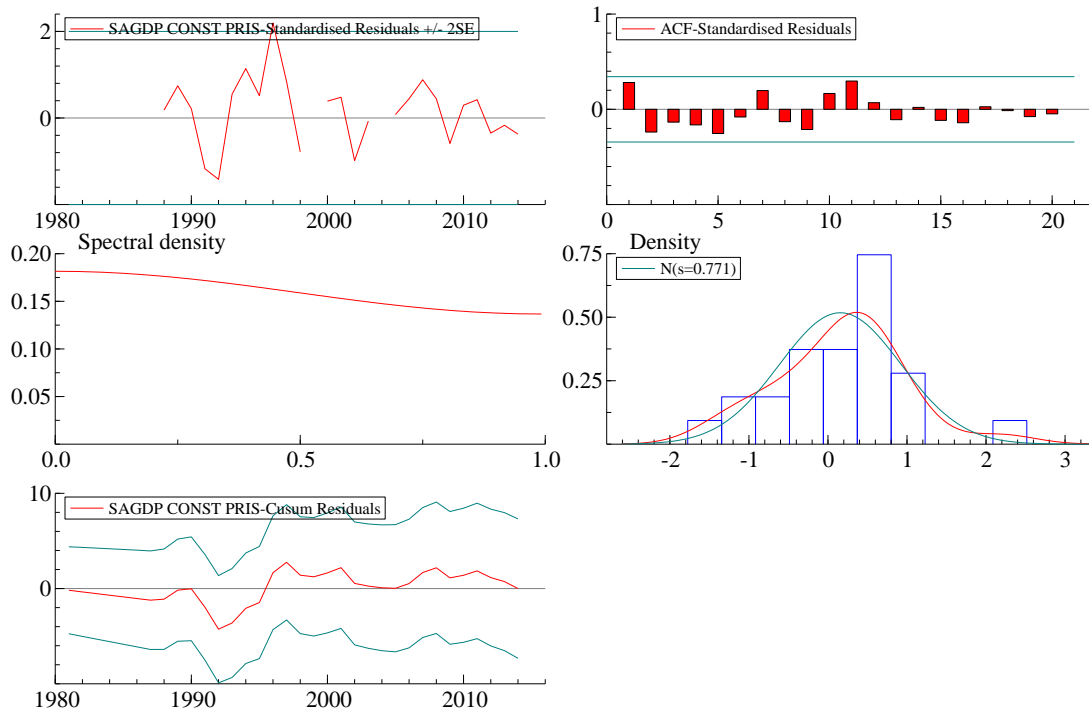


Figure 7.8: Residual analysis of the selected state space model for thresholds effect of inflation-growth nexus of South Africa

sary for economic growth in South Africa with respective thresholds levels of 4% and 8%, using the methodology developed by Khan and Senhadji (2001). These differences were anticipated since the state space model took the time varying nature of economic variables such as inflation and GDP, together with unobserved components into account in the estimation process.

Consequently, in Table 7.7, inflation which is lower than 6% will impact GDP growth positively, but this positive relationship will become negative once inflation exceeds 6%. To be precise, when inflation exceeds the 6% threshold, economic growth is expected to decline by 1.8%, i.e. $(\beta_1 + \beta_2) = 599.476 - 587.787 = 11/587.79 = 1.8\%$. As a result, monetary policy decision makers should keep inflation under 6%. Total investments and growth rate in population were also found to be significant. The residual analyses of both models are presented in

Table 7.7 indicating a good fit for both countries, together with Figure 7.7 (for Ghana) and Figure 7.8 (for South Africa)⁴⁰.

Table 7.7: Serial correlation statistics of residuals on the response variable: Growth rate by GDP per capita for Ghana and South Africa

Lag	Serial Correlation	Box-Ljung test	P-value
Residuals of GHCPI vs GDP			
1	0.26	1.91	1.00
2	-0.06	2.02	1.00
3	-0.48	9.22	0.00
Residuals of SACPI vs GDP			
1	0.28	2.19	1.00
2	-0.19	3.25	1.00
3	-0.06	3.37	0.06

Note: The null hypothesis in each case is not rejected, implying no serial correlation.

7.5 Summary

A state space model of unobserved component with Kalman filter has been applied to estimate the threshold levels of inflation-growth nexus of Ghana and South Africa. Annual data covering a period of 30 years from the two IT countries - Ghana and South Africa, were utilised. Before model estimation, a Granger causality test was applied to the two series and the results showed that CPI inflation was largely responsible for GDP growth of Ghana and South Africa. This was confirmed by the relationship between inflation and economic growth, which was found to be negative and unidirectional.

A basic structural model was fitted to the CPI inflation series of Ghana and South Africa in order to ascertain the nature of the mean level. The results

⁴⁰The estimated percentage change for the decline in growth rate was done by the author for GHCPI and SACPI inflation-growth nexus.

depicted a time-varying mean level for both countries, and this and other unobserved components such as structural breaks, long memory, volatility and non-linearities, captured by the inflation persistence parameter d , were then incorporated in the state space model in determining the threshold effect of inflation on economic growth. The empirical results strongly suggested the existence of threshold effect in the two countries beyond which inflation exerts a negative effect on economic growth. Threshold levels of 9% and 6% were effectively obtained for Ghana and South Africa, respectively. The empirical analysis suggests that inflation below the estimated levels of 9% and 6% for Ghana and South Africa, respectively, will be conducive for economic growth.

These results consolidate the exiting IT monetary policy adopted by Ghana and South Africa in reducing inflation persistence, which may have resulted from structural break shocks, volatility (or inflation uncertainty) and non-linearities (or regime shifts), among others. Although inflation persistence in Ghana appears to be high and non-mean reverting, and that of South Africa seems to be less persistent, cyclical and mean-reverting, both countries can continue to pursue and strengthen the IT monetary policy since the threshold levels necessary for economic growth are actually within the target of the BoG and the SARB.

Chapter 8

Summary, Conclusion and Recommendations

8.1 Introduction

This thesis has empirically examined persistency of inflation dynamics of Ghana and South Africa, the two countries in Sub-Saharan Africa with IT monetary policy, with fractional integration approach by Cuestas and Gil-Alana (2016), ARFIMA model extended with sGARCH and 'gjrGARCH innovations and state space model. The thesis subjected three main dissimilar but complementary properties including, but not limited to, structural breaks, long memory and non-linearities, that inform inflation persistence into a thorough investigation. Accordingly, this thesis adopted time series econometric, statistical and mathematical techniques to study the influence of structural breaks, volatility and non-linearities on inflation persistence in the conditional mean of CPI inflation series of Ghana and South Africa.

In this final chapter, Section 8.2 summarises the study and the main findings from previous chapters and outlines the specific contributions made by this study. Section 8.3 provides conclusion on policy implications from the empirical results presented with respect to the two IT countries studied. Finally, recommendations and suggestions for future studies are presented in Section 8.4.

8.2 Summary

8.2.1 Major findings

This thesis started with an overview of empirical studies on persistency of inflation dynamics by taking into consideration structural breaks, long memory and non-linearities. The main purpose of the chapter was to examine the related literature with these properties. Earlier studies employed various models and tests to examine the behaviour of the conditional mean with reference to structural breaks, long memory and non-linearities in relation to inflation persistence. Nevertheless, these models run into testing problems, hence making them inefficient and less robust. Current studies tend to rely on time series econometric techniques such as fractional integration methods, ARFIMA with GARCH extensions and state space models, among others, to investigate the behaviour of the conditional mean to recover equilibrium after a shock (also known as persistence), and its ramification on economic growth.

Chapter 3 tested the unit root hypothesis using both conventional unit root tests with and without structural breaks. The chapter began by presenting the descriptive statistics generated by the data. The results of the ADF, PP and KPSS tests imply that GHCPI and SACPI inflation series follow a ran-

dom walk hypothesis. By employing the ZA test with one-time break and BP with multiple breaks, similar results were obtained. The unit root hypothesis is an important method for testing whether shocks (e.g. demand and supply, market volatility, etc.) to a series will have a permanent or transitory effect. The presence of a unit root is indicative of the fact that a series has no tendency to revert to its equilibrium value or stable path after experiencing a shock. The most significant break occurred during various months in the periods 1971M01-2014M10 and 1995M01-2014M12, respectively for GHCPI and SACPI inflation series. After applying the ZA test, the break dates of June 1984 and October 2003 were respectively obtained for GHCPI and SACPI inflation series. All the estimated break dates coincided with major economic shocks in the two economies. Using BP test also revealed a number of estimated breaks: 1977M06, 1984M06 and 2004M05 for GHCPI, and 1999M07, 2003M06, 2006M06 and 2009M08 for SACPI inflation series. These estimated structural break shocks may have had a lasting effect on the mean, hence making it highly persistent.

Owing to the low power associated with traditional unit root tests against fractional integration alternatives (Diebold and Ruderbusch, 1991), when the true order is fractionally integrated, a more robust approach was adopted. The results from the fractional integration method indicated strong evidence of non-stationarity and persistence, hence a rejection of the unit root in favour of higher orders of integration under uncorrelated errors. However, evidence of unit roots was found under a more realistic case of autocorrelated errors for GHCPI and SACPI inflation series. Again, in the context of fractional integration, similar results were obtained, supporting the hypothesis of unit roots with multiple breaks in the two inflation series.

The long memory phenomenon in CPI inflation series is an intriguing subject in

the financial literature, since the presence of long memory contradicts the assumption of the efficient market hypothesis (EMH). Chapter 4 has investigated the presence or absence of long memory property in the GHCPI and SACPI inflation series. This chapter also modelled persistence in the conditional mean of the two inflation series after discovering the presence of long memory (which induces persistence) using the ARFIMA model, which is a follow-up to the fact that GHCPI and SACPI inflation could be fractionally integrated and highly persistent, as revealed in Chapter 3. Indeed, the distributional properties of GHCPI and SACPI inflation series are essential, not only for implementation of linear models, but also for determining the memory characteristics of an inflationary process.

Also in Chapter 4, the thesis applied two semi-parametric estimators LW and the GPH, together with EML and an R/S analysis to investigate the presence of long memory in the Ghanaian and South African CPI inflation series. The main findings are that the monthly CPI inflation series of Ghana and South Africa exhibit properties of long memory (with varied degree of persistence) and this is consistent with the results obtained by Hassler and Wolters (1994), and Doornik and Ooms (2004).

The thesis also examined the GHCPI and SACPI inflation series by modelling persistence in the conditional mean using the ARFIMA model in Chapter 4. In the first place, different ARFIMA models were specified for the two inflation series following Sowell (1992) EML method estimation. Appropriate and competing models were selected through a model selection criterion based on diagnostic tests on the residuals together with the LL and AIC. ARFIMA (3,0.38,1) and ARFIMA (3,0.43,2) were respectively specified for GHCPI and SACPI inflation series. These results, to a large extent, generally agree with the study conducted by Gil-Alana and Toro (2002) who applied a similar method in analysing

the long memory of exchange rate for five industrialised countries.

Chapter 5 extended the ARFIMA process, which has a fractionally integrated conditional mean with the sGARCH by Bollerslev and Mikkelsen (1996) and 'gjrGARCH by Glosten et al. (1993) processes to describe persistence and time-dependent heteroscedasticity under three distributional assumptions i.e. Normal, Student- t and Generalised Error distributions, following the remaining ARCH effects discovered in Chapter 4. ARFIMA(3,0.35,1)-'gjrGARCH(1,1) under GED and ARFIMA(3,0.50,2)-'gjrGARCH(1,1) under STD, provided the best fit for modelling persistence in the conditional mean of GHCPI and SACPI inflation series, with respective fractional differencing parameter, d of 0.35 and 0.50. These results show evidence of persistence and time-varying volatility in both CPI inflation series. Again, the results revealed asymmetric effects of inflationary shocks where negative shocks appear to have a greater impact on the conditional mean with time-varying volatility than positive shocks. These two models were used to predict a ten-month inflation for Ghana and South Africa, and the results depicted an increasing trend of the variable into time.

Chapter 6 investigated the Ghanaian and South African inflation series by means of fractional integration, combined with linear and non-linear structures, together with Chebyshev polynomial. The analysis began by testing for non-linearities, adopting the approach propounded by Gil-Alana (2008), that allows for non-linear deterministic terms in the context of fractional integration. The results indicated that the two series are highly persistent, with orders of integration far above 1 in each series. However, the non-linear trends were found to be statistically insignificant in the two cases (i.e., correlated and Bloomfield errors). Both series also appeared to be well described by means of an intercept (or mean) after employing the linear deterministic approach with GHCPI inflation series being highly persistent and non-mean reverting.

However, for South African inflation series, a cyclical $I(d)$ model was found to be more adequate. In this case, the order of integration was found to be smaller than 1 (i.e., 0.7), showing mean reversion, with the length of the cycles approximated to last for 80 periods (months). These results imply that the inflation structure of these two countries are different, being very persistent and non-mean reverting in the case of Ghana, and less persistent, cyclical and mean reverting in the case of South Africa. These results, obtained for Ghana and South Africa, are in agreement with those conducted by Alagidede et al. (2014) and Balcilar et al. (2016), respectively. This new evidence for South Africa, confirms the cyclical nature and mean reverting behaviour of inflation persistence, taking about 80 periods (months) to recover stability after a shock(s), compared to those by Balcilar et al. (2016) where inflation persistence was expected to take 70 periods (months) for high inflation regime and 10 periods (months) for low inflation regime to revert to equilibrium.

Finally, Chapter 7 applied a state space model of unobserved component with Kalman filter to estimate the threshold levels of inflation-growth nexus of Ghana and South Africa. Annual data covering a period of 30 years from the two IT countries, was utilised. Before model estimation, a Granger causality test was applied to the two series and the results showed that CPI inflation was largely responsible for GDP growth of Ghana and South Africa. This was confirmed by the relationship between inflation and economic growth, which was found to be negative and unidirectional. A basic structural model was fitted to the CPI inflation series for Ghana and South Africa in order to ascertain the nature of the mean level. The results depicted a time-varying mean level for both countries, and this and other unobserved components such as persistence and its related properties like structural breaks, long memory, volatility and non-linearities, were then incorporated in the state space model in determining the threshold effect of inflation on economic growth.

8.2.2 Contributions of the study

This thesis has made three significant contributions to the econometric analysis of persistency inflation dynamics of Ghana and South Africa. First, owing to the low power that often shows up in unit root testing with or without structural breaks, the thesis adopted a more robust technique developed by Gil-Alana (2004) to ascertain the true order of integration, which is the measure of persistence in GHCPI and SACPI inflation series.

Secondly, this thesis is probably one of the few studies to address the issue of persistence and its related properties such as structural breaks, long memory and non-linearities in the context of fractional integration with recently developed method by Cuestas and Gil-Alana (2016). There are, but limited number of studies that have addressed these three properties (structural breaks, long memory and non-linearities) simultaneously under correlated and Bloomfield error specifications with Chebyshev polynomial in the Ghanaian and South African context in publication.

The third significant contribution is the estimation of threshold effect of inflation on economic growth with state space model from the STAMP package developed by Koopman et al. (2006). This study incorporated the idea espoused by Chan and Tsay (1998), and Khan and Senhadji (2001) into state space model of unobserved components with Kalman filter to determine the influence of threshold effect of inflation-growth nexus of Ghana and South Africa.

The application of statistical and mathematical concepts and techniques to a traditional economic problem, is a noteworthy contribution, that could attract other scholars to undertake multidisciplinary research.

8.3 Conclusion

8.3.1 Policy implications

Some key policy implications from this study are as follows. In Chapter 3 the finding that CPI inflation of Ghana and South Africa are $I(1)$ rather than $I(0)$, even after accounting for possible endogenous structural breaks, has several implications for researchers, individual countries and investors. The knowledge of whether shocks to a series are permanent, or transitory (known as persistence) has important policy implications. This means that, economic shocks (especially negative shocks) to GHCPI and SACPI inflation series will persist for a long time and this may give the central banks of Ghana and South Africa an additional incentive to intervene by designing and enacting policies that could steer inflation back to an equilibrium path. This is crucial, since the cost of inaction on the part of monetary authorities could diverge inflation further from its long-run equilibrium value. The most significant inference of this finding is that random shocks have permanent effects on the long-run level of inflation rates and that fluctuations are not transitory, confirming the research by Nelson and Plosser (1982), and Arize and Malindrtos (2012).

In Chapter 4, the presence of long memory (which induces persistence) was discovered in the Ghanaian and South African inflation series, and this can have multiple consequences. From a theoretical perspective the presence of long memory, also called self-similar patterns, disproves the concept of efficient market hypothesis (EMH) in its weak form. It also has an important implication for critical explanation of financial time series behaviour, as it could provide an opportunity to earn speculative profits in financial markets and cast disbelief on the correctness of the EMH. The presence of long memory in inflation can

provide vital information about the likely impact of shocks (e.g. demand/supply, current market volatility in China and its adverse on South African economy and Africa as whole) on the economy with respect to time. This information can be useful for the purposes of setting up monetary policy or consolidate the IT monetary policy, in order to enhance economic growth. For instance, persistence of shocks in inflation series would require correct monetary and fiscal policies to drive the nominal inflation towards its long-run equilibrium path. Estimation of long memory in inflation series can as well serve as an evaluation tool to assess the performance of monetary policy under different dispensations. It can assist in identifying inflationary pressures in an economy.

The policy implication of the results obtained in Chapter 5, would be very useful to both countries in making good portfolio decisions and also form the basis for detecting volatility in GHCPI and SACPI inflation series. Policymakers could also design measures to control inflation due to adverse effects on market volatility. Again, the empirical differences in inflation persistence, measured by the fractional differencing parameter, d respectively, for GHCPI and SACPI inflation, raises interesting questions with respect to monetary policy guidelines and price-transmission that will be reliable with this form of behaviour. Indeed these discrepancies in the fractional integration parameter, a measure of persistence, perhaps could also be attributed to the existence of the IT monetary policy in both countries.

Although, ARFIMA(3,0.35,1)-'gjrGARCH(1,1) under GED and ARFIMA(3,0.50,2)-'gjrGARCH(1,1) under STD assumptions provided a good fit for GHCPI and SACPI inflation persistence, respectively, the implication of asymmetries discovered in the two series raises the issue of the consequence of negative shocks such as demand-supply and political uncertainties, in both countries, which could impact negatively on inflation into the future. Indeed, the impact of neg-

ative shocks were found to be three times more than positive shocks on inflation series in both countries. Another important implication of these results is that governments and central banks should be mindful of the actions and decisions they take, in the sense that unguarded decisions and unnecessary alarms could raise uncertainties in the economy, which could in turn affect the future trajectory of inflation.

The results from Chapter 6 showed a very high persistence and non-mean reversion behaviour of the Ghanaian inflation series, and the implication of this is that, monetary policy or tightening the existing IT monetary policy, will be necessary to steer inflation stability. However, a mean reversion behaviour was found in the South African inflation, but it was cyclical, taking about 80 periods (months) to revert to stability. Monetary authorities and governments should guard against negative shocks or uncertainties, since the consequences of these will affect the behaviour of inflation rate negatively.

Finally, a state space was adopted in Chapter 7 to estimate the threshold effect of inflation for the two countries with estimated threshold levels of 9% and 6%, respectively for Ghana and South Africa, above which inflation could be inimical to economic growth. These results might be useful for policymakers in providing some clue in setting an optimal inflation target or consolidating the already existing IT policy, since both threshold levels necessary for economic growth are actually within the target of the Bank of Ghana and the South African Reserve Bank. The conditional mean of both countries were also found to be time-varying, hence monetary policies must incorporate the time-varying factor in their policy formulation.

Overall, the persistence nature of inflation dynamics for both countries suggest a negative dire consequence on fixed income earners and the 'poor' are

more likely to be adversely affected, hence increasing the well-being of inflationary shocks by distorting consumption varieties/choices and the resource distribution function of markets. This ultimately impacts on poverty and income allocation with consequences for economic growth and development.

8.4 Recommendations and Suggestions for Future Research

This thesis sought to empirically analyse persistency of inflation dynamics of Ghana and South Africa. The results from this thesis, to a large extent, have revealed most of the properties underpinning the ever-changing nature of inflation including supply-side problems, mostly externally enforced, in the two IT countries (McKinley, 2008). For instance, GHCPI inflation series was found to be fractionally integrated with the integration parameter found to be 1.11, implying high persistence and non-mean reverting, whereas the SACPI inflation exhibited a mean reversion process with the integration parameter found to be 0.7, implying less persistence. SACPI inflation was also found to be cyclical with shocks persisting up to about 80 months before recovering equilibrium, whereas shocks to GHCPI inflation are expected to persist for a long period of time, and would therefore, require policy action or tightening of the IT monetary policy in order to revert inflation to stability.

Both series had similar properties such as non-linearities (or regime shifts), and asymmetric response to negatives shocks, but with varied degrees of magnitude. The conditional mean and unobserved components such as volatility (inflation uncertainty) for both countries were found to be time-varying. This thesis, therefore, recommends that the Bank of Ghana and the South African Reserve Bank, who are responsible for monetary policies, and the Finance Min-

isters of both governments, responsible for fiscal policies, take these properties into account in the formulation of monetary policies. They should also consider consolidating and tightening the already existing IT policy since these results largely confirm their targets of 6%-9% and 3%-6% respectively for Ghana and South Africa. Indeed, the persistency of inflation dynamics for both countries appears to be more of a structuralist rather than a monetarist, where a rise in inflation is largely attributed to structural maladjustments.

Generally, inflation and interest rates are related variables, and often referenced in macroeconomics. The rate at which prices for goods and services go up is referred to as inflation. In Ghana and South Africa, interest rates are respectively determined by the BoG and SARB. Indeed, more people are able to borrow more money as interest rates go down. The consequence is that consumers will have more money to spend, instigating economic growth and inflation to increase. The opposite is true for rising interest rates because consumers tend to save as result of expected increase in returns. With a less significant amount disposal income to spend as a result of the rise in savings, the economy slowdown, hence inflation drops. Even though an increase in interest rates could assist in curtailing the recent and anticipated increase in inflation rates in both countries, where targets have been missed by Ghana and South Africa, it will be prudent to legislate monetary policies around demand and supply-side since the problem of both countries is more of a structuralist than a monetarist. It is, therefore, recommended that both countries tighten the IT monetary policy in order to reduce inflation persistence. This will eventually impact on poverty and income distribution with ramifications for economic growth and/or development.

Another important implication of these results is that governments and central banks should be mindful of the actions and decisions they take, in the sense

that unguarded decisions and unnecessary alarms could raise inflation uncertainties in the economy, which could affect the future trajectory of inflation. The thesis recommends to the governments of both countries to strengthen the private sector, which is the engine of growth, for small and open economies such as Ghana and emerging markets like South Africa - as this will grow the economy through job creation and restore investor confidence.

This study also recommends the use of state space model with STAR and SETAR extensions in the context of Bayesian inference for future studies on inflation dynamics in relation to persistence.

References

- ADU, G. AND MARBUAH, G. (2011). Determinants of inflation in Ghana: An empirical investigation. *South African Journal of Economics*, **79**, 251–269.
- AGIAKLOGLOU, C., NEWBOLD, P., AND WOHAR, M. (1993). Bias in an estimator of the fractional difference parameter. *Journal of Time Series Analysis*, **14**, 235–246.
- AHMED, N. (1970). Deficit financing, inflation, and capital formation: The Ghanaian experience 1960 – 1965. *Munich Welferun Verlag*.
- AHMED, S. AND MORTAZA, M. G. (2010). Inflation and economic growth in Bangladesh: 1981 – 2005. *Policy Analysis Unit (PAU), Working Paper Series No: 3033*.
- AKAIKE, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, **19** (6), 716–723.
- ALAGIDEDE, P., COLEMAN, S., AND ADU, G. (2014). A regional analysis of inflation dynamics in Ghana. *Working Paper*, 1–32.
- AMBROSE, B. W., ANCEL, E. W., AND GRIFFITHS, M. D. (1993). Fractal structure in the capital markets revisited. *Financial Analysts Journal*, **49** (3), 73–77.
- AMUSA, K., GUPTA, R., KAROLIA, S., AND SIMO-KENGNE, B. D. (2013). The long-run impact of inflation in South Africa. *Journal of Policy Modeling*, **35**, 798–812.

- ANDERSON, T. G. AND BOLLERSLEV, T. (1997). Heterogeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns. *The Journal of Finance*, **52** (3), 975–1005.
- ANDREWS, D. W. K. (1993). Tests for parameter instability and structural change with unknown change point. *Journal of the Econometric Society*, 821–856.
- ANDREWS, D. W. K. AND PLOBERGER, W. (1994). Optimal test when a nuisance parameter is present only under the alternative. *Econometrica*, **62**, 1383–1414.
- ANIS, A. A. AND LLOYD, E. H. (1976). The expected value of the adjusted rescaled Hurst range of independent normal summands. *Biometrika*, **63** (1), 111–116.
- ARIZE, A. C. AND MALINDRTOS, J. (2012). Non-stationary and non-linearity in inflation rate. some further evidence. *International Review of Economics and Finance*, **24**, 224–234.
- ASAFU-ADJAYE, J. (2008). Analysis of inflation in Ghana 1964 – 2006. *Ghana Policy Journal*, **2**, 51–69.
- ATTA-MENSAH, J. AND BAWUMIA, M. A. (2003). A simple vector error correction forecasting model for Ghana. *Bank of Ghana, Working Paper*.
- BABA, Y., HENDRY, D. F., AND STARR, R. M. (1988). The demand for M1 in the USA 1960 – 1988. *Review of Economic Studies*, **59**, 25–61.
- BACKUS, D. K. AND ZIN, S. E. (1993). Long-memory inflation uncertainty: evidence from the structure of interest rates. *Journal of Money, Credit, and Banking*, 681–700.
- BAI, J. (1997). Testimating multiple breaks one at a time. *Econometric Theory*, **13**, 315–352.

- BAI, J. AND PERRON, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, **66**, 47–48.
- BAI, J. AND PERRON, P. (2003). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, **18**, 1–22.
- BAILLIE, R. T., CHUNG, C. F., AND TIESLAU, M. A. (1996). Analysing inflation by the fractionally integrated ARFIMA-GARCH model. *Journal of Applied Econometrics Letters*, **11**, 23–40.
- BAILLIE, R. T., HAN, Y. W., AND KWON, T. G. (2002). Further long memory properties of inflationary shocks. *Southern Economic Journal*, **68**, 496–510.
- BALCILAR, M. (2004). Persistence in inflation: Does aggregation cause long memory? *Emerging Markets Finance and Trade*, **40** (5), 25–56.
- BALCILAR, M., GUPTA, R., AND JOOSTE, C. (2016). Analysing South Africa's inflation persistence using an ARFIMA model with Markov-switching differencing parameter. *Journal of Developing Areas*, **50** (1), 47–57.
- BALL, L. AND CECCHETTI, S. G. (1990). Inflation and uncertainty at short and long horizons. *Brookings Papers on Economic Activity*, 215–254.
- BALL, L. AND SHERIDAN, N. (2003). Does inflation targeting matter? *NBER Working Paper, March*.
- BANERJEE, A. AND URGA, G. (2005). Modelling structural breaks, long memory and stock market volatility: An overview. *Journal of Econometrics*, **129**, 1–34.
- BANK OF GHANA (2014). Cpi inflation data.
URL: *www.bog.gov.gh: Accessed 23-04-2014*.
- BARKOULAS, J. T., BAUM, C. F., AND TRAVLOS, N. (2000). Long memory in the Greek stock market. *Applied Financial Economics*, **10**, 177–184.

- BARNES, M. L. (1998). Aggregation of short-memory processes, the volatility of stock market return indices and long memory. *Working paper No: 1998-10, Adelaide University*.
- BARSKY, R. B. (1987). The fisher hypothesis and the forecastability and persistence of inflation. *Journal of Monetary Economics*, **19** (1), 3–24.
- BAUM, C. F., BARKOULAS, J. T., AND CAGLAYAN, M. (1999). Persistence in international inflation rates. *Southern Economic Journal*, **65**, 900–914.
- BELKHOUSA, M. AND MOOTAMRI, I. (2016). Long memory and structural change in the g7 inflation dynamics. *Economic Modelling*, **54**, 450–462.
- BEN, A. M. AND JOUINI, J. (2003). Structural breaks in the US inflation process. *Applied Economic Letters*, **10**, 633–636.
- BEN, A. M. S., BOUTAHAR, M., AND JOUINI, J. (2004). Bai and Perron's and spectral density methods for structural change detection in the US inflation process. *Applied Economic Letters*, **11**, 109–115.
- BENATI, L. (2008). Investigating inflation persistence across monetary regimes. *Quarterly Journal of Economics*, **123**, 1005–1060.
- BERAN, J. (1994). *Statistics for Long-Memory Processes*. New York, Chapman and Hall.
- BEVILAQUA, A. S., MESQUITA, M., AND MINELLA, M. (2008). Brazil: Taming inflation expectations. In De Mello, L. ed., *Monetary Policies and Inflation Targeting in Emerging Economies*, 43–67.
- BLIGNAUT, Z., GREG, F., VICTOR, M., AND RANGASSAMY, L. (2009). A note on the trimmed mean measure of core inflation in South Africa. *South African Journal of Economics*, **77** (4), 538–552.

- BLOOMFIELD, P. (1973). An exponential model in the spectrum of a scalar time series. *Biometrika*, **60**, 217–226.
- BOLLERSLEV, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, **31** (3), 307–327.
- BOLLERSLEV, T. AND MIKKELSEN, H. O. (1996). Modeling and pricing long memory in stock market volatility. *Journal of Econometrics*, **73** (1), 151–184.
- BORRO, R. J. (1995). Inflation and economic growth. *NBER Working Paper No: 5326*.
- BOS, C. S., FRANSES, P. H., AND OOMS, M. (2002). Inflation, forecast intervals and long memory regression models. *International Journal of Forecasting*, **18** (2), 243–264.
- BRITO, R. D. (2010). Inflation targeting does not matter: Another look at oecd economies' output sacrifice ration. *Journal of Money, Credit, and Banking*, **42**, 1679–1688.
- BRITO, R. D. AND BYSTEDT, B. (2010). Inflation targeting in emerging economies: Panel evidence. *Journal of Development Economics*, **91**, 198–210.
- BROCKWELL, P. AND DAVIS, R. A. (1995). *Time Series: Theory and Methods*. Willey.
- BRUNNER, A. D. AND HESS, G. D. (1993). Are higher levels of inflation less predicatable? a state-dependent conditional heteroscedasticity approach. *Journal of Business and Economic Statistics*, **11**, 187–197.
- BRUNO, M. AND EASTERLY, W. (1998). Inflation crises and long-run growth. *Journal of Monetary Economics*, **41** (2), 3–26.

- BURGER, P. AND MARINKOV, M. (2008). *Inflation targeting and inflation performance in South Africa*. Annual Forum.
- CAMPBELL, J. Y., LO, A. W., AND MACKINLAY, A. C. (1997). *The Economics of Financial Markets*. Princeton University Press, Princeton.
- CANER, M. AND KILLIAN, L. (2001). Size distortions of tests of the null hypothesis of stationarity: Evidence and implications for the PPP debate. *Journal of International Money and Finance*, **20**, 639–657.
- CAPORALE, B. AND CAPORALE, T. (1998). Asymmetric effects of inflation shocks on inflation uncertainty. *Atlantic Economic Journal*, **30** (4), 385–388.
- CAPORALE, G. M., CARCEL, H., AND GIL-ALANA, L. A. (2015). Modelling inflation rates: Non-linear deterministic terms and long-range dependence. *Applied Economics Letters*, **22** (2), 421–424.
- CECCHETTI, S. G. AND DEBELLE, G. (2006). Has the inflation process changed? *Economic Policy*, **21**, 311–352.
- CHAMBERS, M. J. (1998). Long memory and aggregation in macroeconomic time series. *International Economic Review*, 1053–1072.
- CHAN, K. S. AND TSAY, R. S. (1998). Limiting properties of the least squares estimator of a continuous threshold autoregressive model. *Biometrika*, **85** (2), 413–426.
- CHEN, M. Y. (2002). Testing stationarity against unit roots and structural changes. *Applied Economics Letters*, **9**, 459–464.
- CHEUNG, Y. W. AND DIEBOLD, F. X. (1994). On maximum likelihood estimation of the differencing parameter of fractionally-integrated noise with unknown mean. *Journal of Econometrics*, **63** (2), 301–316.

- CHEUNG, Y. W. AND LAI, K. S. (1995). A search for long memory in international stock market returns. *Journal of International Money and Finance*, **14**, 597–615.
- CHHIBBER, A. AND SHAFIK, N. (1990). Exchange reform, parallel markets, and inflation in africa: The case of Ghana. *World Bank Working Papers*, *WPS 427*.
- CHIANG, T. C. AND DOONG, S. C. (2001). Empirical analysis of stock returns and volatilities: Evidence from seven asian stock markets based on TAR-GARCH model. *Review of Quantitative Finance and Accounting*, **17**, 301–318.
- CHIMOBI, O. P. (2010). Inflation and economic growth in Nigeria. *Journal of Sustainable Development*, **3** (2), 1–59.
- CHOI, S. M. AND WOHR, M. E. (1992). The performance of the GPH estimator of the fractional difference parameter: Simulation results. *Review of Quantitative Finance and Accounting*, **2**, 409–417.
- CHOWDHURY, A. (2014). Inflation and inflation-uncertainty in India: The policy implications of the relationship. *Journal of Economic Studies*, **41**, 71–86.
- CHRISTIANO, L. J. (1992). Searching for a break in GNP. *Journal of Business and Economic Statistics*, **10**, 237–249.
- CIOCZEK-GEORGES, R. AND MANDELBROT, B. B. (1995). A class of micropulses and anti-persistent fractional Brownian motion. *Stochastic Process and their Applications*, **60**, 1–18.
- CLARK, P. K. (1987). The cyclical component of US economic activity. *The Quarterly Journal of Economics*, 797–814.

- CROWDER, W. AND HOFFMAN, D. (1996). The long-run relationship between interest rates and inflation: The Fisher equation revisited. *Journal of Money, Credit and Banking*, **28**, 102–118.
- CUESTAS, J. C. AND GIL-ALANA, L. A. (2016). Testing for long memory in the presence of non-linear deterministic trends with Chebyshev polynomials. *Studies in Non-linear Dynamics and Econometrics*, **20** (1), 57–74.
- CULVER, S. E. AND PAPELL, D. H. (1997). Is there a unit root in the inflation rate. evidence from sequential break and panel data models? *Journal of Applied Econometrics*, **12** (4), 435–444.
- DAHLHAUS, R. (1989). Efficient parameter estimation for self-similar processes. *The Annals of Statistics*, 1749–1766.
- DELGADO, M. AND ROBINSON, P. M. (1994). New methods for the analysis of long memory time series: Application to Spanish inflation. *Journal of Forecasting*, **13**, 97–107.
- DER MERWE, E. J. V. (2004). *Inflation Targeting in South Africa*. Pretoria, South Africa: South African Reserve Bank.
- DIAS, D. A. AND MARQUES, C. R. (2010). Using mean reversion as a measure of persistence. *Economic Modeling*, **27** (1), 262–273.
- DICKEY, D. A. AND FULLER, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association*, **74** (366), 427–481.
- DIEBOLD, F. X. AND INOUE, A. (2001). Long memory and regime switching. *Journal of Econometrics*, **105**, 131–159.
- DIEBOLD, F. X. AND RUDEBUSCH, G. D. (1989). Long memory and persistence in aggregate output. *Journal of Monetary Economics*, **24** (2), 189–209.

- DIEBOLD, F. X. AND RUDERBUSCH, G. D. (1991). On the power of the Dickey-Fuller tests against fractional alternatives. *Economic Letters*, **35**, 155–160.
- DING, Z., GRANGER, C. W. J., AND ENGLE, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, **1**, 83–106.
- DONALD, W., ANDREWS, K., AND GUGGENBERGER, P. (2003). A bias reduced log-periodogram regression estimator for long memory parameter. *Econometrica*, **71** (2), 675–712.
- DOORNIK, J. A. AND OOMS, M. I. (2004). Inference and forecasting for ARFIMA models with an application to US and UK inflation. *Studies in Non-linear Dynamics and Econometrics Letters*, **8**, 1–23.
- DORDUNOO, C. (1994). The structure and policy implications of a macroeconomic model of Ghana. *World Development*, **22** (8), 1243–1251.
- EGGOH, J. C. AND KHAN, M. (2014). On the nonlinear relationship between inflation and economic growth. *Research in Economics*, **68** (2), 133–143.
- ENET, C., CHAKRABARTI, G., AND SARKA, A. (2010). Long memory in foreign exchange market: Myth or reality? a study of selected currencies around the globe in terms of Indian Rupee. *Ioannina Meeting on Applied Economics and Finance IMAEF 2010, Ioannina, Greece*.
- ENGLE, C. (2000). Long-run PPP may not hold after all. *Journal of International Economics*, **57**, 243–273.
- ENGLE, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, **50**, 987–1007.
- ENGLE, R. F. AND NG, V. (1993). Measuring and testing the impact of news on volatility. *Journal of Finance*, **45** (5), 1749–1778.

- ENGLE, R. F. AND SMITH, A. D. (1999). Stochastic permanent breaks. *Review of Economics and Statistics*, **81**, 553–574.
- ERICSSON, N. R., HENDRY, D. F., AND MIZON, G. M. (1998). Exogeneity, cointegration and economic policy analysis. *Journal of Business and Economic Statistics*, **16** (4), 370–387.
- ERICSSON, N. R. AND IRONS, S. I. (1994). *Testing Exogeneity (ed.)*. London University Press.
- EVANS, M. D. AND LEWIS, K. K. (1995). Do expected shifts in inflation affect estimates long-run fisher relation? *Journal of Finance*, **50**, 225–253.
- EWUSI, K. (1977). The determinants of price fluctuations in Ghana. *ISSER Discussion Paper, University of Ghana, Legon*.
- FAMA, E. F. AND GIBBONS, M. R. (1982). Inflation, real returns and capital investment. *Journal of Monetary Economics*, **9** (3), 297–323.
- FILARDO, A. AND GENBERG, H. (2010). Targeting inflation in asia and the pacific: Lessons from the recent past. *The International Financial Crisis and Policy Challenges in Asia and the Pacific BIS Papers No.52, Bank of International Settlement*.
- FISHER, S. (1993). The role of macroeconomic factors in growth. *Journal of Monetary Economics*, **32**, 485–512.
- FOX, R. AND TAQQU, M. S. (1986). Large-sample properties of parameter estimates for strongly dependent stationary gaussian time series. *Annals of Statistics*, **14**, 517–523.
- FRIEDMAN, M. (1970). The counter-revolution in monetary theory. *IEA Occasional Paper No. 33, Wincott Memorial Lecture, Institute of Economic Affairs, First published by the Institute of Economic Affairs, London, 1970, 1–14*.

- FRIMPONG, J. M. AND OTENG-ABAYIE, E. F. (2010). When is inflation harmful? estimating the threshold effect for Ghana. *American Journal of Economics and Business Administration*, **2** (3), 232–239.
- FURUOKA, F., MANSUR, K., AND MUNIR, Q. (2009). Inflation and economic growth in Malaysia: A threshold regression approach. *ASEAN Economic Bulletin*, **26** (2), 180–193.
- GADEA, M. D., SABATE, M. A., AND SERRANO, J. M. (2004). Structural breaks and their trace in the memory inflation rate series in the long-run. *Journal of International Financial Market Institutions and Money*, **14**, 117–134.
- GEWEKE, J. AND PORTER-HUDAK, S. (1983). The estimation and application of long-memory times series models. *Journal of Time Series Analysis*, **4**, 221–238.
- GIL-ALANA, L. A. (2004). The use of bloomfield (1973) model as an approximation to ARMA process in the context of fractional integration. *Math.Computer model*, **39**, 429–436.
- GIL-ALANA, L. A. (2008). Fractional integration and structural breaks at unknown periods of time. *Journal of Time Series Analysis*, **29**, 163–185.
- GIL-ALANA, L. A. (2011). Inflation in South Africa: A long memory approach. *Economics Letters*, **111** (3), 207–209.
- GIL-ALANA, L. A. AND ROBINSON, P. M. (1997). Testing of unit root and other nonstationary hypotheses in macroeconomic time series. *Journal of Econometrics*, **80** (2), 241–268.
- GIL-ALANA, L. A. AND TORO, J. (2002). Estimation and testing of ARFIMA models in the real exchange rate. *International Journal of Finance and Econometrics*, **7**, 279–292.

- GIRAITIS, L. AND SURGAILIS, D. (1990). A central limit theorem for quadratic forms in strongly dependent linear variables and its application to asymptotic normality of Whittle's estimate. *Probability Theory and Related Fields*, **86**, 87–104.
- GLOSTEN, L. R., JAGANNATHAN, R., AND RUNKLE, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, **48** (5), 1779–1801.
- GOLAKNATH, N. C. AND REDDY, Y. V. (2002). Long memory in Rupee-Dollar exchange rate: An empirical study. *UTIICM Conference Paper*.
- GONCALVES, C. E. S. AND SALLES, J. M. (2008). Inflation targeting in emerging economics; what do the data say? *Journal of Developing Economics*, **85**, 312–318.
- GRANGER, C. W. AND TERASVIRTA, T. (1999). A simple nonlinear time series model with misleading linear properties. *Economics Letters*, **62**, 161–165.
- GRANGER, C. W. J. AND HYUNG, N. (2004). Occasional structural breaks and long memory with an application to the SP 500 absolute stock returns. *Journal of Empirical Finance*, **11**, 399–421.
- GRANGER, C. W. J. AND JOYEUX, R. (1980). An introduction to long memory series. *Journal of Time Series Analysis*, **1**, 15–30.
- GRANGER, C. W. J. A. (1980). Long-memory relationships and the aggregation of dynamic models. *Journal of Econometrics*, **14**, 227–238.
- GREENE, W. H. (2003). *Econometric Analysis*. Fifth Ed, New Jersey, Prentice Hall, Upper Saddle River.
- GREGORIOU, A. AND KONTONIKAS, A. (2006). Inflation targeting and the stationarity of inflation: New results from an ESTAR unit root test. *Buletin of Economic Research*, **54** (4), 309–322.

- GUPTA, R. AND JOSINE, U. (2012). Comparing South African inflation volatility across monetary policy regimes: An application of Saphe cracking. *The Journal of Developing Areas*, **46** (1), 45–54.
- GUPTA, R., KABUNDI, A., AND MODISE, M. P. (2010). Has the sarb become more effective post inflation targeting? *Economic Change and Restructuring*, **43** (3), 187–2014.
- GUPTA, R. AND UWILINGIYE, J. (2007). Comparing South African inflation volatility across monetary policy regimes: An application of saphe cracking. *Working Paper Series, 200906, University of Pretoria, Department of Economics*, **77**, 3–32.
- HAMILTON, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, **57**, 357–384.
- HAMMING, R. W. (1973). *Numerical methods for scientists and engineers*. McGraw Hill, New York.
- HANSEN, B. (1999). Threshold effects in non-dynamic panels: Estimation, testing and inference. *Journal of Econometrics*, **93** (2), 345–368.
- HANSEN, B. E. (2001). The new econometrics of structural change: Dating breaks in the US labour productivity. *Journal of Econometrics*, **15**, 117–128.
- HARVIE, C. AND PAHLAVANI, M. (2006). Testing for structural breaks in the Korean economy 1980-2005: An application of the innovational outlier and additive outlier models. *Journal of the Korean Economy*, **7** (2), 179–212.
- HASLETT, J. AND RAFTERY, A. E. (1989). Space-time modelling with long-memory dependence: Assessing Ireland's wind power resource. *Applied Statistics*, **46**, 1–50.
- HASSLER, U. (1993). Regression of spectral estimators with fractionally integrated time series. *Journal of Time Series Analysis*, **14**, 369–380.

- HASSLER, U. AND WOLTERS, J. (1994). On the power of unit root tests against fractional alternatives. *Economic Letters*, **45**, 1–5.
- HEINTZ, J. AND NDIKUMANA, L. (2011). Is there a case for formal inflation targeting in sub-saharan Africa. *Journal of African Economics*, **20** (2), 67–103.
- HENDRY, D. F. (2001). Modelling UK inflation. *Hydrology and Earth System Sciences*, **16**, 255–275.
- HODGE, D. (2006). Inflation and growth in South Africa. *Cambridge Journal of Economics*, **30**, 163–180.
- HOSKING, J. R. M. (1981). Fractional differencing. *Biometrika*, **68**, 165–176.
- HOSOYA, Y. (1997). A limit theory with long range dependence and statistical inference on related models. *Annals of Statistics*, **25**, 105–137.
- HSIEH, S. F. AND SHYU, S. D. (2009). Long-term dependence in Asian foreign exchange markets. *Journal of Asia Business Studies*, **4** (1), 49–55.
- HSU, C. C. (2005). Long memory or structural changes: An empirical examination on inflation rates. *Economics Letters*, **88**, 289–294.
- HUANG, B. N. AND YANG, C. W. (1995). The fractal structure in multinational stock returns. *Applied Economics Letters*, **2** (3), 67–71.
- HURST, H. E. (1951). Long-term storage capacity of reservoirs. *Transaction of the American Society of Civil Engineers*, **116**, 770–799.
- HYUNG, N., FRANSES, P. H. A., AND PENM, J. (2006). Structural breaks and long memory in US inflation rates: Do they matter for forecasting? *Research in International Business and Finance*, **20**, 95–110.
- IBARRA, R. AND TRUPKIN, D. (2011). The relationship between inflation and growth: A panel smooth transition regression approach. *Research Network*

and Research Centers Program of Banco Central del Uruguay, Working Paper No: 6.

INTERNATIONAL MONETARY FUND (2014). Data.

URL: *www.imf.org/en/data: Accessed 23-02-2015.*

JACOBSEN, B. (1996). Long term dependence in stock returns. *Journal of Empirical Finance*, **3** (4), 393–417.

JAHANSEN, S. (1992). Testing weak exogeneity and order of cointegration in UK money demand data. *Journal of Policy Modelling*, **14** (3), 313–334.

JOUINI, J. AND BOUTAHAR, M. (2005). Evidence of structural changes in US time series. *Economic Modelling*, **22**, 391–422.

JUDSON, R. AND ORPHANIDES, A. (1999). Inflation, volatility and growth. *International Finance*, **1** (1), 117–138.

KABUNDI, A., SCHALING, E., AND MODESTE, S. (2014). Monetary policy and heterogeneous inflation expectation in South Africa. *A paper presented at a conference titled: Fourteen years of inflation Targeting in South Africa and the challenge of a changing mandate Organised by SARB on October 30-31.*

KALMAN, R. E. (1960). A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, **82** (1), 35–45.

KAN, E. AND OMAI, T. (2010). Re-examining the threshold effects in the inflation–growth nexus with cross-sectionally dependent non-linear panel: evidence from six industrialised economies. *Economic Modelling*, **27**, 996–1005.

KASEERAM, I. (2012). *Essays on the impact of inflation targeting in South Africa*. Doctoral Thesis on DCom, University of Zululand, South Africa.

KHAN, M. S. AND SENHADJI, A. S. (2001). Threshold effects in the relationship between inflation and growth. *IMF Staff Papers: 1*, **48**.

- KIM, J. C. (1993). Unobserved-component time series models with Markov-switching heteroscedasticity: Changes in regime and the link between inflation rates and inflation uncertainty. *Journal of Business and Economic Statistics*, **11** (3), 341–349.
- KING, R. G., PLOSSER, C., STOCK, H. J., AND WATSON, N. W. (1991). Stochastic trends and economic fluctuations. *American Economic Review*, **81**, 819–840.
- KOCK, G. D. (1998). Inflation under scrutiny. *South African Reserve Bank Quarterly Bulletin*, No 168, 21–25.
- KOIRALA, T. P. (2013). Time-varying parameters of inflation model in Nepal. *State Space Modeling*, 67–77.
- KONTONIKAS, A. (2004). Inflation and inflation uncertainty in the United Kingdom: Evidence from garch modelling. *Economic Modelling*, **21**, 525–543.
- KOOPMAN, S. J., HARVEY, A. C., DOORNIK, J. A., AND SHEPHARD, J. (2006). *STAMP: Structural Time Series Analyser, Modeller and Predictor*. Timberlake Consultants.
- KUAN, C. M. AND HORNIK, K. (1995). The generalized fluctuation test: A unifying view. *Econometric Reviews*, **14**, 135–161.
- KWIATKOWSKI, D., PHILLIPS, P. C., SCHMIDT, P., AND SHIN, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, **54** (1), 159–178.
- LAMOREAUX, C. G. AND LASTRAPES, W. D. (1990). Persistence in variance, structural change and the GARCH model. *Journal of Business and Economic Statistics*, **8** (2), 225–234.

- LAWSON, R. M. (1966). Inflation in the consumer market in Ghana. *Economic Bulletin of Ghana*, **X** (1).
- LEE, C. AND WONG, S. Y. (2005). Inflationary threshold effects in the relationship between financial development and economic growth: evidence from Taiwan and Japan. *Journal of Economic Development*, **30** (1), 1–49.
- LEE, J. (2005). Estimating memory parameter in the US inflation rate. *Economics Letters*, 207–210.
- LESHORO, T. (2012). Estimating the inflation threshold for South Africa. *University of South Africa Working Paper No: 285*.
- LEVIN, A. T., NATALUCCI, F. M., AND PIGER, J. M. (2004). The macroeconomic effects of inflation targeting. *Federal Reserve Bank of St. Louis Review*, **86** (4), 51–80.
- LEYBOURNE, S. J. A. AND NEWBOLD, P. (2003). Spurious rejections by cointegration test induced by structural breaks. *Applied Econometrics*, **35**, 1117–1121.
- LIN, S. AND YE, H. (2009). Does inflation targeting make a difference in developing countries? *Journal of Development Economics*, **89**, 118–123.
- LO, A. W. (1991). Long-term memory in stock market prices. *Econometrica*, **59**, 1279–1313.
- LO, A. W. AND MACKINLAY, C. (1988). Stock markets prices do not follow random walks: Evidence from a simple specification test. *Review of Financial Studies*, **1**, 41–66.
- LO, A. W. AND MACKINLAY, C. (1990). An econometric analysis of non-synchronous trading. *Journal of Econometrics*, **45**, 181–211.

- LUCAS, R. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, **1**, 19–46.
- LUI, M. (2000). Modeling long memory in stock market volatility. *Journal of Econometrics*, **99**, 139–171.
- LUMSDAINE, R. L. A. AND PAPELL, D. H. T. (1997). Multiple trend breaks and the unit root hypothesis. *Review of Economics and Statistics*, **79**, 212–218.
- MACDONALD, R. AND MURPHY, P. D. (1989). Testing for long-run relationship between interest rates and inflation using cointegration technique. *Applied Economics*, **21**, 439–447.
- MACKAY, A. AND SOWAH, N. K. (2014). Does the inflation hit the poor harder? **URL:** <http://www.org/inflation/ghana/poor.pdf> accessed: Accessed 01-03-2015.
- MALLIK, G. AND CHOWDHURY, A. (2001). Inflation and economic growth: Evidence from South Asian countries. *Asian Pacific Development Journal*, **8** (1), 123–135.
- MAMO, F. (2012). *Economic growth and inflation: A panel data analysis*. Master of Social Sciences: Economics Thesis, University of Sodertorns.
- MANDELBROT, B. A. AND WALLIS, J. (1968). Noah, Joseph and operational hydrology. *Water Resources Research*, **4**, 909–918.
- MANDELBROT, B. B. (1963). The variation of certain speculative prices. *Journal of Business*, **36**, 394–419.
- MANDELBROT, B. B. (1971). When can price be arbitrated efficiently? a limit to the validity of the random walk and martingale models. *Review of Economics and Statistics*, **53**, 225–236.

- MANDELBROT, B. B. (1972). Statistical methodology for non-periodic cycles: From the covariance to R/S analysis. *Annals of Economic and Social Measurement*, 259–290.
- MANDELBROT, B. B. AND NESS, J. V. (1968). Fractional Brownian motions, fractional noises and applications. *SIAM Review*, **10** (4), 422–437.
- MANDELBROT, B. B. AND WALLIS, J. R. (1969a). Computer experiments with fractional gaussian noises, part 1, averages and variances. *Water Resource Research*, **5**, 228–241.
- MANDELBROT, B. B. AND WALLIS, J. R. (1969b). Computer experiments with fractional gaussian noises, part 2, averages and spectra. *Water Resource Research*, **5**, 242–259.
- MANDELBROT, B. B. AND WALLIS, J. R. (1969c). Computer experiments with fractional gaussian noises, part 3, mathematical appendix. *Water Resource Research*, **5**, 260–267.
- MANKIW, N. G., ROMER, D., AND WEIL, D. N. (2004). A contribution to the empirics of economic growth. *International Library of Critical Economics*, **179**, 163–193.
- MBOWENI, T. T. (2003). *Inflation targeting in South Africa*. Pretoria: BIS/South African Reserve Bank, Management Seminar Dinner, 2nd September 2003.
- MCKENZIE, M. D. (2001). Non-periodic Australian stock market cycles: Evidence from rescaled range analysis. *Economic Record*, **77** (239), 393–406.
- MCKINLEY, T. (2008). Inflation targeting in sub-saharan africa: Why now? why at all? *Centre for Development Policy and Research No 1, School of Oriental and African Studies*.

- MCLEOD, A. I. AND HIPEL, K. W. P. (1978). Preservation of the rescaled adjusted range: A reassessment of the Hurst phenomenon. *Water Resources Research*, **14**, 491–508.
- MIGNON, V. AND VILLAVICENCIO, A. (2011). On the impact of inflation on output growth: Does the level of inflation matter? *Journal of Macroeconomics*, **33**, 455–464.
- MILES, W. (2008). Inflation targeting and monetary policy in Canada: What is the impact on inflation uncertainty? *North American Journal of Economics and Finance*, **19**, 235–248.
- MONTGOMERY, A. L., ZARNOWITZ, V., TSAY, R. S., AND TIAO, G. (1998). Forecasting the US unemployment rate. *Journal of American Statistical Association*, **93**, 478–493.
- MOOLMAN, E. AND DU TOIT, C. (2004). Modelling price determination in South Africa. *Department of Economics, University of Pretoria, SAJEMS NS No 1:151*, **7**.
- MORANA, C. AND BELTRATTI, A. (2004). Structural change and long-range dependence in volatility of exchange rate: Either, neither or both. *Journal of Empirical Finance*, **11**, 629–658.
- MOTLEY, B. (1998). Growth and inflation: A cross-country study. *Economic Review-Federal Reserve Bank of San Francisco*, **1**, 1–15.
- MUBARIK, Y. A. (2005). Inflation and growth: An estimate of the threshold level of inflation in Pakistan. *SBP-Research Bulletin*, **1**, 35–43.
- NELSON, C. R. AND PLOSSER, P. (1982). Trends and random walks in macroeconomic time series: Some evidence and implications. *Journal of Monetary Economics*, **10**, 139–162.

- NELSON, C. R. AND SCHWERT, G. J. (1997). Short-term interest rates as predictors of inflation: On testing the hypothesis that the real rate of interest is constant. *American Economic Review*, **67**, 478–486.
- NG, S. AND PERRON, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, **66** (6), 1519–1554.
- NICOLETTA, B. AND LAXTON, D. (2007). Under what conditions can inflation targeting be adopted. *Monetary Policy under Inflation Targeting*, edited by Klaus Schmidt-Hebbel and Frederic Mishkin. Santiago: Central Bank of Chile.
- NORIEGA, A. E., CAPISTRAN, C., AND RAMOS-FRANCIA, M. (2014). On the dynamics of inflation persistence around the world. *Empirical Economics*, **44** (3), 1243–1265.
- OCRAN, M. K. (2007). A modelling of Ghana's inflation experience: 1960 – 2003. *African Economic Research Consortium Research Paper No: 169, Nairobi*.
- ODHIAMBO, N. (2012). Inflation dynamics and economic growth in Tanzania: A multivariate time series model. *The Journal of Applied Business Research*, **28** (3), 317–324.
- OMANE-AJEPONG, M., ODURO, F. T., AND ODURO, S. D. (2013). Determining the better approach for short-term forecasting of Ghana's inflation: Seasonal ARIMA vs Holt-Winters. *International Journal of Business, Humanities and Technology*, **3** (1), 69–79.
- O'REILLY, G. AND WHELAN, K. (2005). Has euro-area inflation persistence changed over time? *The Review of Economics and Statistics*, **87**, 709–720.
- ORPHANIDES, A. AND WIELAND, V. (2000). Inflation zone targeting. *European Economic Review*, **44** (7), 1351–1387.

- OTERO, J. AND SMITH, J. (2005). The KPSS test with outliers. *Computational Economics*, **26**, 241–249.
- PAHLAVANI, M., VALADKHANI, A., AND WORTHINGTON, A. (2005). The impact of financial deregulation on monetary aggregates and interest rates in Australia. *Applied Financial Economics Letters*, **1** (3), 157–163.
- PAPELL, D. H. AND PRODAN, R. (2004). The uncertain unit root in US real GDP: evidence with restricted and unrestricted structural change. *Journal of Money, Credit, and Banking*, **36**, 423–427.
- PARKE, W. R. (1999). What is fractional integration? *The Review of Economics and Statistics*, **81**, 632–638.
- PARKER, R. E. AND ROTHMAN, P. (1997). The current death-of-recession and unemployment-rate forecasts. *Studies in Non-linear Dynamics and Econometrics*, **2**, 151–158.
- PERRON, P. (1989). The great crash, the oil price shock and the unit root hypothesis. *Journal of Econometrics*, **57**, 1361–1401.
- PERRON, P. (1997). Further evidence on breaking trend functions in macroeconomic variables. *Journal of Econometrics*, **80** (2), 355–385.
- PERRON, P. A. AND VOGELSSANG, T. J. (1992). Nonstationarity and level shifts with an application to purchasing power parity. *Journal of Business and Economic Statistics*, **10**, 301–320.
- PETERS, E. E. (1994). *Fractal market analysis*. New York, John Wiley and Sons.
- PFAFF, B. (2008). *Analysis of integrated and cointegrated time series with R*. 2nd edition. Springer-Verlag, New York.

- PHILLIPS, P. C. B. AND PERRON, P. (1988). Testing for unit root in time series regression. *Biometrika*, **75**, 335–346.
- PHIRI, A. (2010). At what level is inflation least detrimental towards finance-growth activity in South Africa? *Journal of Sustainable Development in Africa*, **12** (6), 354–364.
- POLLIN, R. AND ZHU, A. (2006). Inflation and economic growth: A cross-country non-linear analysis. *Journal of Post Keynesian Economics*, **28** (4), 593–614.
- POTTER, S. M. (1995). A nonlinear approach to US GNP. *Journal of Applied Econometrics*, **10** (2), 109–125.
- QUARTEY, P. (2010). Price stability and the growth maximizing rate of inflation for Ghana. *Modern Economy*, **1**, 180–194.
- RANGASAMY, L. (2009). Inflation persistence and core inflation: The case of South Africa. *South African Journal of Economics*, **77**, 430–444.
- RAPACH, D. E. AND WEBER, C. E. (2004). Are real interest rates really non-stationary: New evidence from tests with good size and power. *Journal of Macroeconomics*, **26** (3), 409–430.
- REISEN, V. A., CRIBARI, F., AND JENSEN, M. J. (2003). Long memory inflationary dynamics: The case of Brazil. *Studies in Non-linear Dynamics and Econometrics*, **7** (1-3.).
- ROBBINS, M., GALLAGHER, C., LUND, R., AND AUE, A. (2011). Mean shift testing in correlated data. *Journal of Time Series Analysis*, **32**, 498–511.
- ROBINSON, P. M. (1978). Statistical inference for a random coefficient autoregressive model. *Scandinavian Journal of Statistics*, **5**, 163–168.

- ROBINSON, P. M. (1994). Efficient tests of non-stationary hypotheses. *Journal of American Statistical Association*, **80**, 1420–1471.
- ROBINSON, P. M. (1995). Gaussian semi-parametric estimation of long range dependence. *Annals of Statistics*, **23**, 1630–1661.
- ROSE, A. K. (1988). Is the real interest rate stable? *Journal of Finance*, **43**, 1095–1112.
- ROTHMAN, P. (1998). Forecasting asymmetric unemployment rates over business cycle. *Journal of Macroeconomics*, **13**, 164–168.
- RUCH, F. AND DIRK, B. (2013). Towards a measure of core inflation using singular spectrum analysis. *South African Journal of Economics*, **81** (3), 307–329.
- RUSSELL, B. AND BANERJEE, A. (2008). The long-run Phillips curve and non-stationary inflation. *Journal of Macroeconomics*, **30**, 1792–1815.
- SAMUELSON, P. A. AND SOLOW, R. M. (1956). Analytical aspects of anti-inflationary policy. *The American Economic Review*, **50** (2), 177–194.
- SASIKUMAR, A. (2011). Testing for weak market efficiency in the Indian foreign exchange market. *The IUP Journal of Monetary Economics*, **9**, 7–19.
- SELETENG, M., BITTENCOURT, M., AND EYDEN, R. V. (2013). Non-linearities in inflation–growth nexus in the SADC region: A panel smooth transition regression approach. *Economic Modelling*, **30**, 149–156.
- SHUMWAY, R. H. AND STOFFER, D. S. (1982). An approach to time series smoothing and forecasting using the EM algorithm. *Journal of Time Series Analysis*, **3** (4), 253–264.
- SHYH, W. C. (2010). Regime non-stationarity and non-linearity in inflation rates: Evidence from OECD countries. *International Journal of Finance and Economics*, **463**, 47–57.

- SIDRAUSKI, M. (1967). Rational choice and patterns of growth in a monetary economy. *American Economic Review*, **57**, 534–544.
- SIKLOS, P. L. (2008). Inflation targeting around th world. *Emerging Markets Finance and Trade*, **44**, 17–37.
- SMYTH, G. K. (1998). *Polynomial Aproximation*. John Wiley and Sons, Ltd, Chichester.
- SO, M. K. P. (2000). Long-term memory in stock market volatility. *Applied Financial Economics Letters*, **10**, 519–524.
- SOOFI, A. S. AND ZHANG, Y. (2006). Testing for long memory in the Asian foreign exchange rates. *Journal for Systematic Science and Complexity*, **19**, 182–190.
- SOWA, N. K. AND ABRADU-OTOO, P. (2007). Inflation management in ghana - the output factor. *Paper presented at the Bank of England, Cornell University Workshop on New Developments in Monetary Policy in Emerging Economies, London*, 17–18.
- SOWAH, N. K. (1994). Fiscal deficit, output growth and inflation targets in Ghana. *World Development*, **22**, 1105–1117.
- SOWAH, N. K. (1996). Policy consistency and inflation of Ghana. *AERC Research Paper No: 43, African Economic Research Consortium, Nairobi, Kenya*.
- SOWAH, N. K. AND KWAKYE, J. K. (1993). Inflationary trends and control in Ghana. *AERC, Nairobi Research Paper No: 22*.
- SOWELL, F. (1992). Maximum likelihood estimation of stationary univariate fractionally integrated time series model. *Journal of Econometrics*, **55**, 165–188.

- STATISTICS SOUTH AFRICA (2014). Does the inflation hit the poor harder?
URL: *statasa.gov.za: Accessed 11-09-2014.*
- STEEL, F. (1979). Hyperinflation in Ghana. *Legon Observer*, 308–312.
- STOCK, J. H. AND WATSON, N. W. (1993). Introduction to business cycles, indicators and forecasting. *University of Chicago Press*, **9**, 1–10.
- TAQQU, M. S., WILLINGER, W., AND SHERMAN, R. (1997). Proof of a fundamental result in self-similar traffic modeling. *Computer Communications*, **27**, 5–23.
- TEMPEL, J. (2000). Inflation and growth: Short stories and tall. *Journal of Economic Surveys*, **14** (4), 996–1005.
- TILLMANN, P. (2012). Has inflation persistence changed under emu? *German Economic Review*, **13**, 86–102.
- TKACZ, G. (2001). Estimating the fractional order of integration of interest rates using a wavelet OLS estimator. *Studies in Non-linear Dynamics and Econometrics*, **5** (1), 19–32.
- TRADING ECONOMICS REPORT (2016a). Inflation rate in Ghana.
URL: Available: *http://www.tradingeconomics.com/ghana/inflation-cpi: Accessed 25-06-2015.*
- TRADING ECONOMICS REPORT (2016b). South Africa inflation rate.
URL: Available: *http://www.tradingeconomics.com/south-africa/inflation-cpi: Accessed 25-06-2015.*
- TWENEBOAH, G., ANOKYE, M. A., DADZIE, E. A., AND AMOAKO, G. K. (2015). Long memory behaviour of real interest rates in Ghana. *Journal of Business and Enterprise Development*, 1–7.

- VAN RUTH, F. (2014). Inflation dynamics in the Netherlands: a linear and non-linear analysis and the influence of economic conditions. *Statistics Netherlands Discussion Paper: 1 August 2014*, **22**, 1–35.
- VAONA, A. AND SCHIAVO, S. (2007). Non-parametric and semi-parametric evidence on the long run effects of inflation on growth. *Economics Letters*, **94**, 452–458.
- VELASCO, C. AND ROBINSON, P. M. (2000). Whittle pseudo-maximum likelihood estimation for non-stationary time series. *Journal of the American Statistical Association*, **95**, 1229–1243.
- WHITTLE, P. (1951). *Hypothesis Testing in Time Series Analysis*. Uppsala, Almqvist, Wiksells Boktryckeri AB.
- WORLD BANK REPORT (2016a). The economy of Ghana: Overview.
URL: Available: <http://www.worldbank.org/en/country/ghana/overview>:
Accessed 19-01-2016.
- WORLD BANK REPORT (2016b). The economy of South Africa: Overview.
URL: Available: <http://www.worldbank.org/en/country/southafrica/overview>:
Accessed 19-01-2016.
- YILDIRIM, E. (2004). Counting and constructing boolean functions with particular difference distributions vectors. *Master Thesis, The Graduate School of Applied Mathematics, Middle East Technical University, Turkey*.
- ZAFFARONI, P. (2004). Contemporaneous aggregation of linear dynamic models in large economics. *Journal of Econometrics*.
- ZIVOT, A. A. AND ANDREWS, D. W. K. (1992). Further evidence on the great crash, the oil price shock, and unit root hypothesis. *Journal of Business and Economic Statistics*, **10**, 251–270.

-
- ZIVOT, E. AND WANG, J. (1994). *Modeling financial time series with S-Plus*.
New York, Springer.

Appendices

APPENDIX 4.1: PROPERTIES OF THE FRACTIONAL INTEGRATION PARAMETER, d

Memory	FI-parameter	Mean-reverting	Variance	Characteristics
No	$d = 0$	Yes	Finite	Covariance stationary
Short	$0 < d < 0.5$	Yes	Finite	Covariance stationary
Long	$0.5 \leq d < 1$	Yes	Infinite	Covariance non-stationary
Infinite	$d \geq 1$	No	Infinite	Covariance non-stationary

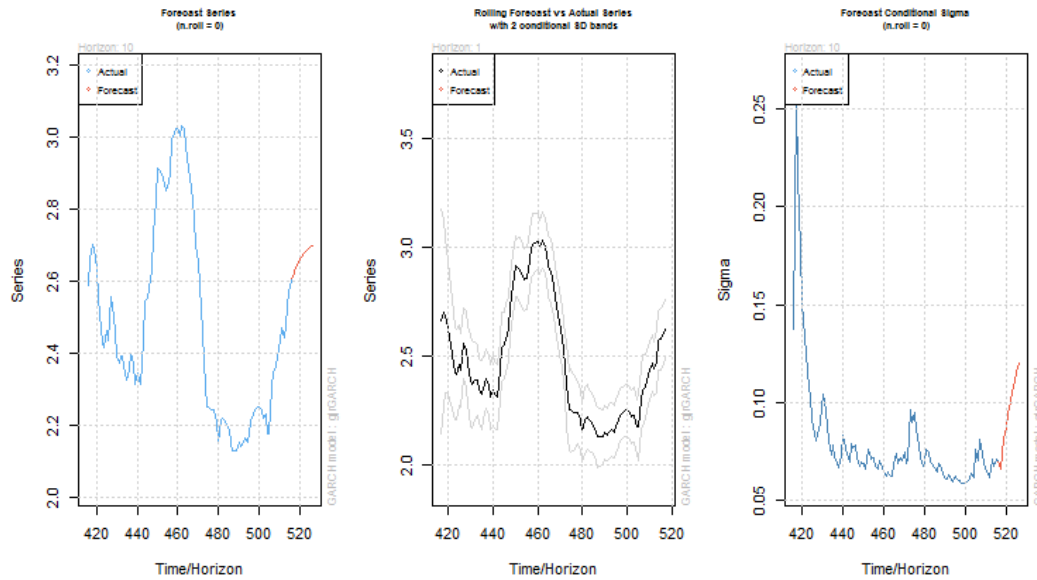
Note: FI denotes fractional integration parameter (or fractional differencing parameter). Source: Granger (1980), Granger and Joyeux (1980), Hosking (1981), and Tkacz (2001).

APPENDIX 5.1: PREDICTED VALUES FOR GHCPI INFLATION SERIES USING ARFIMA(3,0.35,1)-'GJR'GARCH(1,1) UNDER GED

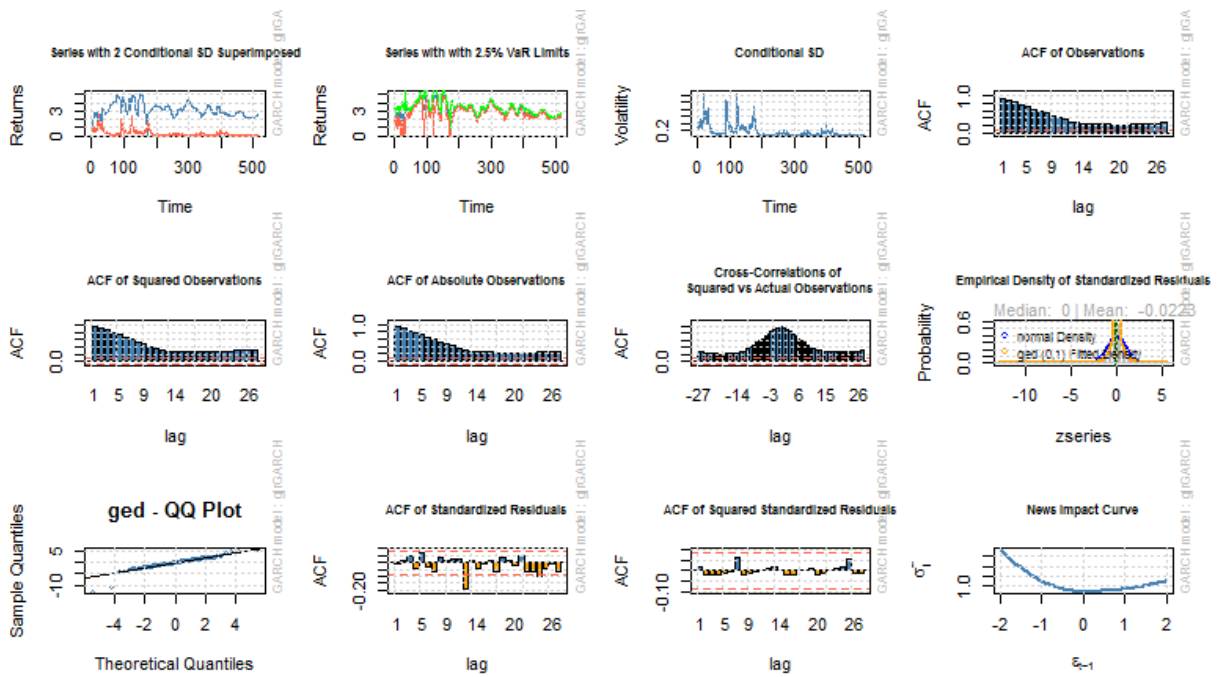
Observations	σ	Predicted conditional mean inflation
2014M01	0.07	2.62
2014M02	0.07	2.64
2014M03	0.08	2.65
2014M04	0.08	2.66
2014M05	0.09	2.67
2014M06	0.10	2.68
2014M07	0.10	2.69
2014M08	0.11	2.69
2014M09	0.12	2.69
2014M10	0.12	2.70
Forecast Accuracy	Roll-0	Roll-1
MSE	0.00	0.00
MAE	0.04	0.04
DAC	1.00	1.00
N	10.00	9.00

Note: MSE denotes the Mean Square Errors; MAE is the Mean Absolute Error; DAC denotes Directional Accuracy test, N depicts the number of in-sample forecast observations and σ denotes the conditional standard deviation. All these forecast performance measures point to the closeness of the predicted mean to actual conditional mean of 3.12, respectively for GHCPI inflation series.

APPENDIX 5.2: RESIDUALS PLOTS FOR THE TEN-MONTH FORECASTS OF ARFIMA(3,0.35,1)-'GJR'GARCH(1,1) FOR GHCCI INFLATION SERIES UNDER GED



APPENDIX 5.3: RESIDUALS ANALYSIS OF ALL PLOTS OF ARFIMA(3,0.35,1)-'GJR'GARCH(1,1) FOR GHCPPI INFLATION SERIES UNDER GED

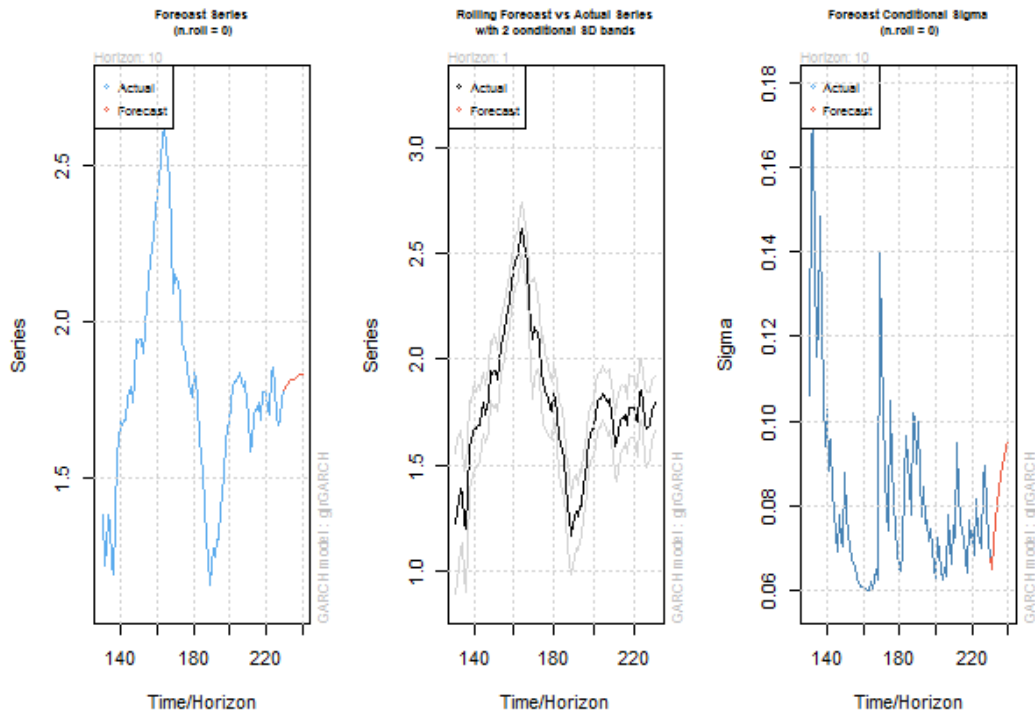


APPENDIX 5.4: PREDICTED VALUES FOR SACPI INFLATION SERIES USING ARFIMA(3,0.50,2)-'GJRGARCH(1,1) UNDER STD

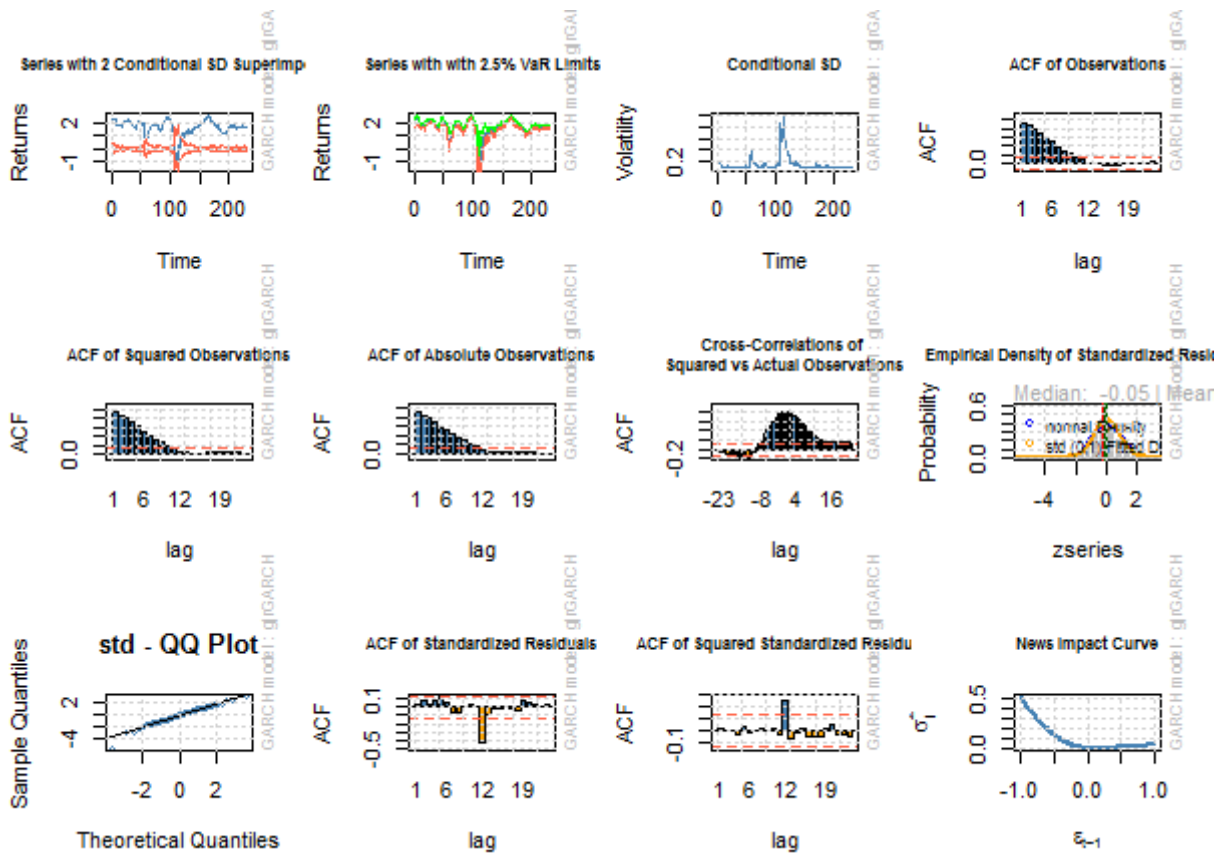
Observations	σ	Predicted conditional mean inflation
2014M03	0.07	1.78
2014M04	0.07	1.79
2014M05	0.07	1.80
2014M06	0.08	1.81
2014M07	0.08	1.81
2014M08	0.08	1.82
2014M09	0.08	1.83
2014M10	0.09	1.83
2014M11	0.09	1.83
2014M12	0.09	1.83
Forecast Accuracy	Roll-0	Roll-1
MSE	0.00	0.01
MAE	0.05	0.00
DAC	1.00	1.00
N	10.00	9.00

Note: MSE denotes the Mean Square Errors; MAE is the Mean Absolute Error; DAC denotes Directional Accuracy test, N depicts the number of in-sample forecast observations and σ denotes the conditional standard deviation. All these forecast performance measures point to the closeness of the predicted mean to actual conditional mean of 1.68, respectively for SACPI inflation series.

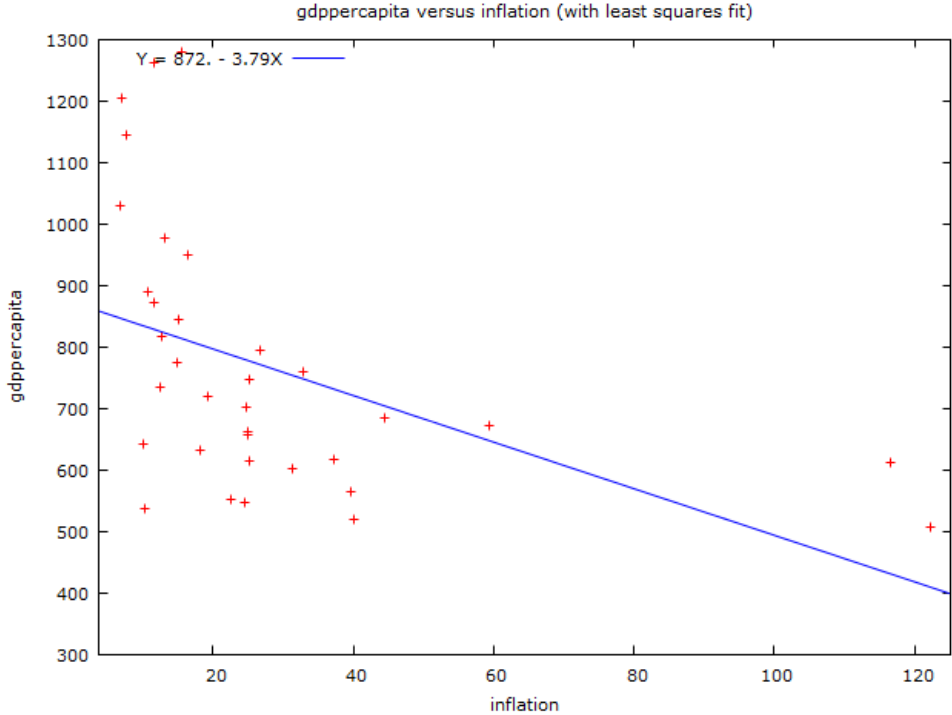
APPENDIX 5.5: RESIDUALS PLOTS FOR THE TEN-MONTH FORECASTS OF ARFIMA(3,0.50,2)-'GJR'GARCH(1,1) FOR SACPI INFLATION SERIES UNDER STD



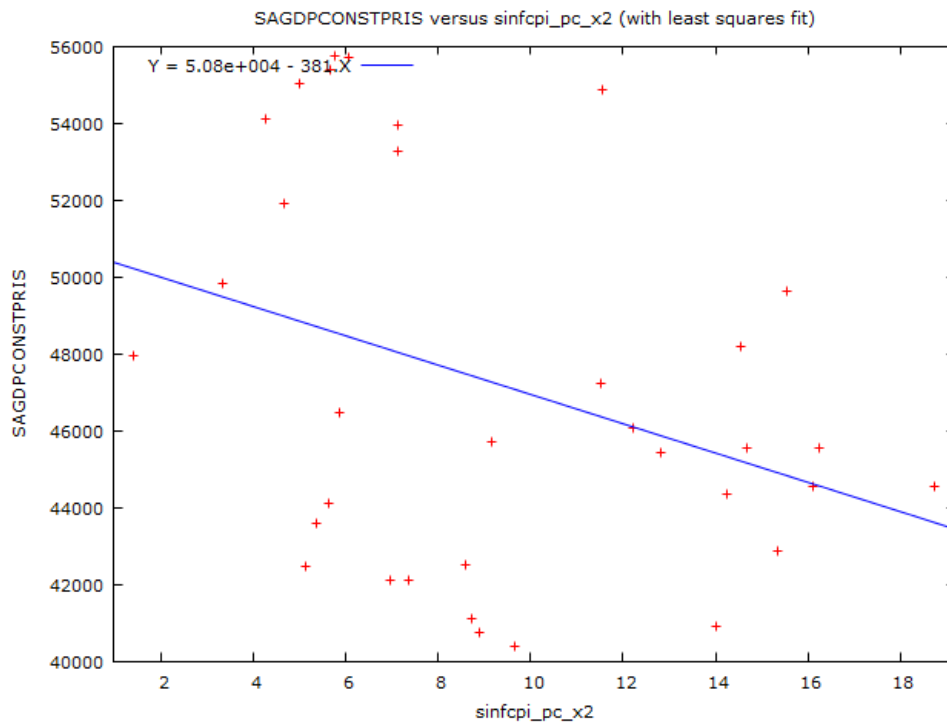
APPENDIX 5.6: RESIDUALS ANALYSIS OF ALL PLOTS OF ARFIMA(3,0.50,2)-'GJR'GARCH(1,1) FOR SACPI INFLATION SERIES UNDER STD



APPENDIX 7.1: RELATIONSHIP BETWEEN INFLATION AND GROWTH RATE BY GDP PER CAPITA FOR GHANA



APPENDIX 7.2: RELATIONSHIP BETWEEN INFLATION AND GROWTH RATE BY GDP PER CAPITA FOR SOUTH AFRICA



Note: The main statistical packages used throughout this thesis are Oxmetrics, R, Eviews, Gretl and STATA.