ENACTMENT OF MATHEMATICAL AGENCY: A NARRATIVE ANALYSIS OF CLASSROOM INTERACTIONS

by

LEKWA LAZARUS MOKWANA

RESEARCH DISSERTATION

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DECLARATION

I, L. L. Mokwana, declare that the research dissertation hereby submitted to the University of Limpopo, for the degree of Master of Education in Mathematics Education has not previously been submitted by me for any degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.

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Mokwana L.L. (Mr) Date

DEDICATION

This dissertation is dedicated to my late mother; Matsemela Betty, Mokwana.

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I am thankful to God, for without him I wouldn't have completed this work.

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ABSTRACT

The qualitative study reported here was aimed at documenting and describing how agency is enacted through students' interactions in a mathematics classroom. A case study design was adopted and focused on a grade 11 mathematics class with all the students being participants. These participants were purposefully selected as they formed the class which was allocated to me for day-to-day mathematics teaching. The research question which the study sought to address was: how is agency enacted through students' interactions in a mathematics classroom? The classroom in which data was generated adopted a sociocultural perspective as a referent for its practice. Due to this perspective, agency was thus employed as conceptualised by Pickering (1995).

Data was generated through interviews and participant observation. However, the interviews were not employed in their 'tradition' view, but were mostly like focus-group interviews in nature. Data also emerged from classroom discussions, when students in their groups, worked through learning activities. These interactions together with the interviews were audio recorded. Meanwhile, observation data was recorded in a researcher journal in which entries were made after each lesson. Data was analysed following Polkinghorne's (1995) narrative analysis of eventful data. During the analysis the researcher listened to the audio records a number of times, and then transcribed all the audio into text. This was followed by reading through the textual data which led to a selection of excerpts used in data analysis.

It was found that agency was enacted during student-material interactions, as students engaged in the 'dance of agency' when deciding on learning a new approach or using an old one to respond to questions. Furthermore, agency was enacted during student-student interactions when students initiated either group or whole class discussion and they were able to sustain the discussions to completion

without the teacher's intrusion. Finally, during teacher-student interactions, students accounted for their actions and shared their experience and decision making process.

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CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

There is a proliferation of research on agency aimed at explaining how students' learning endeavours and experiences are improved or supported. This results from the paucity of students' participation in learning environments, which underpinned such research. Consequently, this proliferation is recently seen at all education levels. For example, in Foundation Phase (Norén, 2015; Shaik & Ebrahim, 2015), Intermediate and Senior Phase (Arnold, 2012; Soini, Pietarinen, Toom, & Pyhältö, 2015; Vaughn, 2014), Further Education and Training (FET) Phase (Deed et al., 2014; Kangas et al., 2014) and in higher education and training both undergraduate (Lindgren & McDaniel, 2012; Ragpot, 2014) and postgraduate (Braathe & Solomon, 2015) level.

In these studies, focus was on student's agency (Braathe & Solomon, 2015; Norén, 2015; Shaik & Ebrahim, 2015), teacher's agency (Soini et al., 2015) and for some on both (Deed et al., 2014). Although all based on agency, the difference is on its conceptualisation, due to the varied fields and areas of specialisation in which the studies are based. For instance, studies were conducted in the field of teacher education (Soini et al., 2015; Shaik & Ebrahim, 2015; Deed et al., 2014), psychology education (Ragpot, 2014), language education (Vaughn, 2014), science education (Doyle, 2015; Kangas et al., 2014) and mathematics education (Braathe & Solomon, 2015; Norén, 2015). Therefore it should be noted that the study reported here is in the field of mathematics education conducted at the FET phase. This results from being an FET mathematics teacher with growing interest on teaching, learning and research in the classroom.

In mathematics education research, a person's agency is understood as initiating ideas, agreeing with others, to elaborate and critique, to question or disagree with others (Gresalfi, Martin, Hand, & Greeno, 2009). In the same way Biesta and Tedder (2006) suggest that agency should be understood as in an ecological way that is strongly connected to context. In other words, they imply that agency should not be seen as an individual's capacity, rather as an achievement. Furthermore, they assert that the agentic dimension "lies in the ways in which we have control over ways in which we respond to the situation" (pp. 20-21). As a result, in a classroom context agency is thus not how the students act in particular situations, instead it is how they have control over ways in which they act in the classroom.

Reported studies on mathematical agency in the classroom (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Wagner, 2007), have a common aspect in the examination of how students' agency change through specific programmes. Hence, they report on the change as an "instance" and as such do not detail the process(es) of how students became mathematical agents. Typically, they report on students' "pre" and "post" agency. For that reason, I refer to their narratives as the "back" and "front" of mathematical agency. On the contrary, how students actually enact agency in these mathematics classrooms is what is missing. In the same way Doyle (2015) supports this view and as a result argues for a need to study "how it [agency] can and does function in educative environments" (p. 278). Correspondingly, the purpose of the study reported here is to document and describe how agency is enacted through student interaction in a mathematics classroom.

In studying how agency is enacted, Braathe and Solomon (2015) concluded that agency is enacted through the appropriation of cultural resources and self-authoring. Even though they reported on enactment of agency, their study was not classroom based as the one reported. On the other hand, Norén's (2015)

work stands alone in the exploration of how agency is enacted in the classroom. Her study aimed at exploring discursive practices to understand how students act, and are positioned by enacted discourses in a multilingual mathematics classroom. Unlike in the reported study, she used agency as an analytical tool for both locating forms of interactions in the classroom, and students' intentions in those interactions. Her study focused on a Grade 1 mathematics classroom, and she drew conclusions based on how agency and positions were made available [by the teacher] and implications on reform-oriented pedagogy. The reported study on the other hand focused on enactment of agency in a Grade 11 mathematics classroom where students are not positioned by the teacher.

Research on student's agency in mathematics education draws on Pickering's (1995) metaphor of the "dance of agency" (Boaler & Greeno, 2000; Cobb, Gresalfi & Hodge, 2009; Wagner, 2007). This dance is a metaphor for the tension between individual's initiatives and structures or conventions in relation to mathematics per se (Andersson, 2011). Similarly, the reported study also adopted Pickering's (1995) conceptual and disciplinary agency to examine students' enactment of agency through interactions in the classroom.

In presenting the case of the "dance of agency", Boaler (2003), argued for the "need to study classrooms practices in order to understand relationships between teaching and learning" (p.3). She contends that the practices of classrooms need to be captured in order to cross a divide between research and practice. "The capturing of some of the practices of teaching and converting them into a set of carefully documented records of practice assist researchers in producing artefacts that encourage a special kind of analysis-grounded in practice" (p.15).

Andersson and Valero's (2011) work stands alone in an attempt to respond to Boaler's (2003) call. They examined stories of context and students' mathematical agency. The context they refer to is a "real-life" situation, which is not necessarily within the parameters of the mathematics curriculum expected to be taught. This is in contrast to the prescribed mathematics curriculum in the reported study. Nevertheless they detailed the students' process through which students achieved agency. For the reported study, the enactment of agency is detailed as it unfolds in a natural setting of a mathematics classroom. The curriculum taught is prescribed and major deviations may disadvantage students as they are expected to write common external examinations.

Furthermore, Andersson (2011) through her work on achievement of agency states that "it became apparent that account also had to be taken for the relationship between students' identities and their achievement of agency as they impact, together on students' decisions to participate in mathematics education and hence in their learning of mathematics" (p.159). The latter implies that mathematical agency should be studied together with mathematical identities, however that is not the case with the study reported here. Without doubt, it is notable that most studies forefront mathematical identities over mathematical agency, when the two are studied together. While that is the case, for this study I deliberately fore fronted mathematical agency.

1.2 PURPOSE OF THE STUDY

The purpose of this qualitative case study was to document and describe how agency is enacted through students' interactions in a mathematics classroom, for a Grade 11 mathematics class allocated to me for day-to-day mathematics teaching (Creswell, 2009). This was the case as Doyle (2015) argues for the need to study "how it [agency] can and does function in educative environments" (p. 278). Apart from that, Boaler (2003) also makes a call for the "need to study

classrooms practices in order to understand relationships between teaching and learning" (p.3).

1.3 RESEARCH QUESTION

The research question is: How is agency enacted through students interactions in a mathematics classroom?

1.4 OVERVIEW OF THE DISSERTATION

The structure of this dissertation is framed through the following headings: literature review, methodology, analysis and interpretation, and conclusion and recommendations. These headings are only the order in which the chapters of this dissertation are arranged, and should not be interpreted as the order in which they were written.

In chapter 2, I start by presenting my perception of what it means to learn mathematics using Vygotsky's socio-cultural learning theory as a referent for learning. The latter is followed by an outline of the classroom setting of the class in which data was constructed. Then I present an analysis and interpretation of the literature which assisted me in making sense of agency and mathematical agency. Lastly, I present the conceptual framework which was adopted as lens for data interpretation.

In chapter 3, I start by presenting the research approach which was adopted by this study. This is followed by a detailed outlined of the design of the study, its participants, data gathering techniques, and data analysis method. Finally, I present a discussion about how quality was insured in the study and the ethical issues which were considered.

In chapter 4, I present data collected through various methods. This is immediately followed by analysis and interpretation done in an integrated approach. Throughout this chapter I raise important issues which respond to the research question and address the purpose of the study.

In chapter 5, I present the conclusion of this dissertation by responding to the research question. Then I outline the limitations of the study, and offer some recommendations.

1.5 SUMMARY

In this chapter I outlined the background of the study, by offering the different fields in which agency was researched, delimiting my study to agency in the mathematics classroom. Then I outlined the purpose of the study together with the research question which the study sought to address. Lastly, I outlined the structure of this dissertation by offering the synopsis of all the chapters.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter firstly, I start by discussing what it means to learn mathematics. Secondly, I give the setting of the mathematics classroom in which data for the study was constructed, focusing particularly on classroom interactions. Thirdly, I discuss what agency means as conceptualised by Bandura and Pickering. Fourthly, I discuss what mathematical agency means by highlighting studies which focused on this phenomenon. Lastly, I present the conceptual framework which was used as a lens through which data was interpreted.

2.2 CONCEPTUALISING MATHEMATICS LEARNING

My engagement with the work of Lev Vygotsky was an attempt to answer the question: what does it mean to learn mathematics? This was the case because enactment of agency in the reported study was documented during students' mathematics learning experience. Therefore an understanding of what it means to learn mathematics needed to be clarified.

According to Vygotsky (1978), learning is acquisition of abilities to think about a variety of things. Furthermore, he argues that environment should be the starting point of learning, and that student-centred learning should be designed within their zone of proximal development. On the other hand, he rejects assumptions and views that learning is purely external, and that having learned one thing, leads to being able to do others which are entirely unrelated. As a result, he asserts that "human learning presupposes a specific social nature". Consequently, this social nature which learning depends on, leads to the sociocultural learning theory.

From a sociocultural perspective, individual learning takes place as a result of social interactions and culture (Cobb & Yackel, 1996). Culture refers to both the classroom culture and the culture of the student within and outside the classroom. Cobb and Yackel (1996) state that culture contributes to learning, while the student contributes to the culture in the classroom. The latter means that the student and the culture of the classroom have an influence on each other. Social interactions, on the other hand, refer to the conversations (in this case mathematical conversations) that the student engages in with other students or the teacher. It is this view of learning that was adopted in the reported study.

Furthermore, Cobb and Yackel (1996) define learning as "a constructive process that occurs while participating in and contributing to the practices of the local community" (p. 185). They however make it clear in their work that the construction of an idea will be different for each student even within the same environment. For me it means that mental processes (cognition) that students go through will be different. While agreeing with them, it is interesting to note that their view of learning is recognised by Vygotsky. Then again, Vygotsky clarifies that the mind does not and cannot exist outside the social practices (Packer & Goicoechea, 2000). As such, students' mental processes are not the only aspects which are significant, but also the social interaction which triggers such processes.

Vygotsky's sociocultural theory has three major attributes about learning. First, cognition exists between and among people in social learning settings, and that from these social settings the student moves ideas into his or own psychological realm (Forman, 2003). Second, the way in which information is learned depends on whether it was within a student's zone of proximal development (ZPD), which is the difference between student's assisted and unassisted performance on a task (Vygotsky, 1978). Third, the way in which information moves from the social plane to the individual plan and this is referred to as semiotic mediation. Semiotic mediation according to Forman (2003) refers to

interaction through language, diagrams, pictures and actions. In view of this, Cobb and Bowers (1999) view of learning as “a process in which students actively reorganise their ways of participating in classroom practices” (p. 9) was more meaningful. Therefore, in summary, learning involves students, social interactions in the classroom and the culture within and beyond the classroom. And this is the view on learning which the present study adopted.

2.3 CLASSROOM INTERACTIONS

The classroom interactions which took place in the mathematics classroom in which the study was conducted are appropriately summarised in Figure 1 below in the next page.

2.3.1 Student-Material Interaction Stage

The material given to students is designed in such a way that concept formation is driven by means of questions. A series of questions ranging from those easily accessible to the advanced ones are raised. In the process students are asked a lot of reflective questions that require them to generate theories on what they are learning. Subsequently, lots of questions are also raised that require them to put into practice their emerging theories. At this initial stage, it is important that each student becomes aware of his or her own thoughts, ideas, position, challenges, etc. This serves as the point of departure in engaging with student-student interaction stage or student-teacher interaction stage.

2.3.2 Student-Student Interaction Stage

At this stage students have opportunities to put to test emerging ideas. Whilst always ready to engage in a robust debate, the students are also ready to reformulate or adapt to new ideas that come as a result of this interaction. It is at this stage that communication skills are put to a robust test.

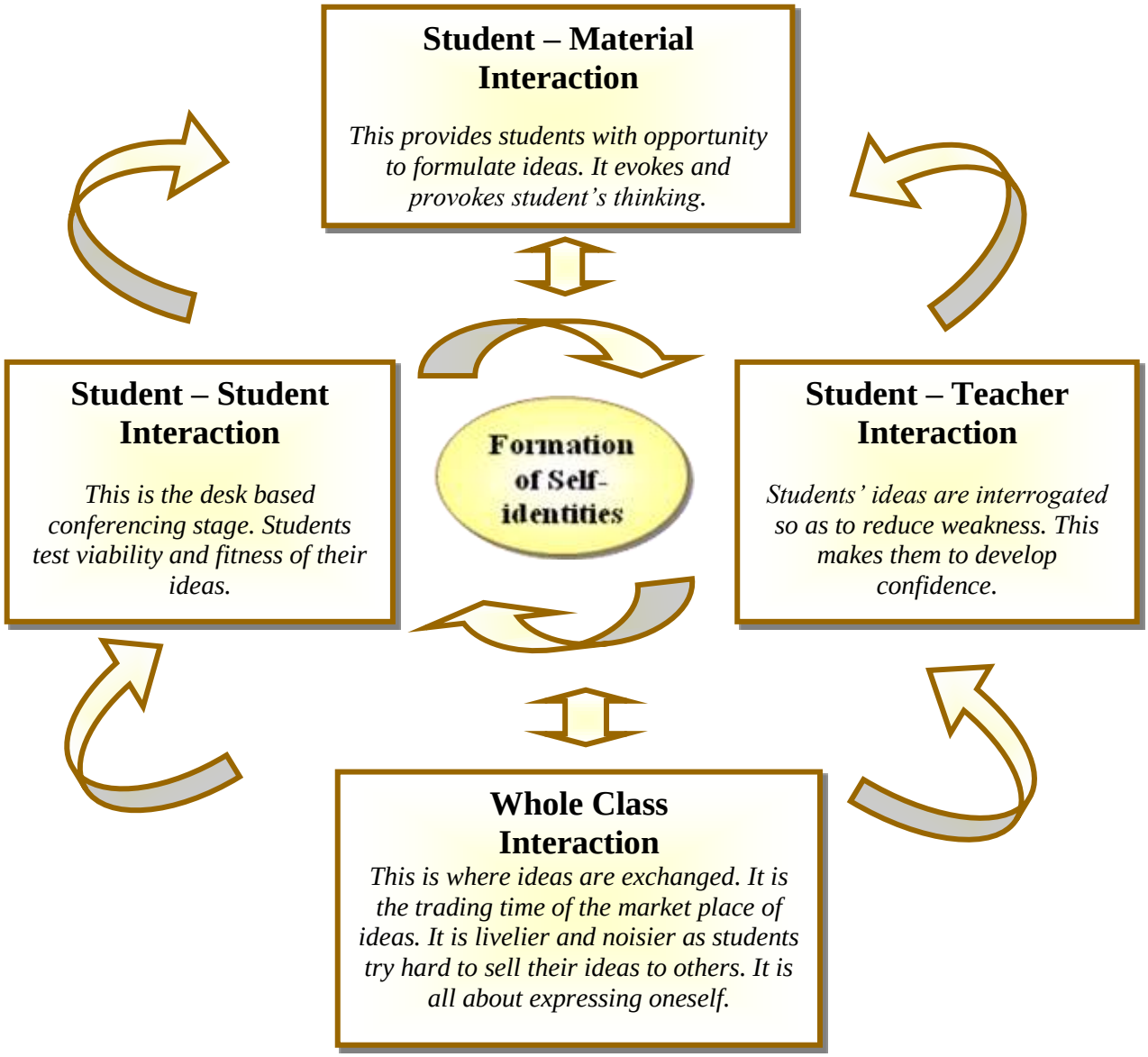

Figure 1: Classroom interactions (Masha, 2004)

2.3.3 Student-Teacher Interaction Stage

Through students' responses to the questions raised, the teacher has initial ideas of individual student's current knowledge on the issues raised. The responses act as a base for engaging students in further questions that are aimed at exploring the students' responses further. In some cases, opportunities for contradicting situations are provided. The core of this stage is to make students to

become more aware of their thoughts and how robust and viable these are. The idea is to inculcate reflective process in thought formation.

2.3.4 Whole-Class Interaction Stage

Issues that have been identified as common in student-teacher interaction stage normally are addressed at the class level. At this stage the classroom environment becomes livelier as individual students or groups try hard to defend their ideas. The responses become spontaneous as weaknesses are explored whilst strong points are reiterated. In many instances, these discussions are settled through consensus. I must indicate that whilst in the early days in the year it is normally the teacher who initiates the whole class interactive stage, as time goes by, more and more it becomes students who initiate the discussions. It is for this reason that the model has students' identities at its core. That is, individual students become aware of their own positions in these interactions. They develop individual identities that become critical in the sustenance of the interactions. But even more important, this stage is aimed at providing students with experiences to gauge how knowledge is constructed, especially its human aspect of it.

As it can be perceived from the model, teaching and learning were mainly driven from students' interpretations of content. The classes were highly interactive, allowing continuous reflection on students' learning and adequacy of learning material. This was the case of how learning was conceptualised as a socially constructed activity that required an agent, a committed human being who makes the decision of engaging herself in the activity of learning (Stentoft & Valero, 2010).

2.4 CONCEPTUALISING AGENCY

2.4.1 Bandura's Agency

The idea of human agency has two philosophical underpinnings. Firstly, it is embedded in psychology where the formation of agency is in social cognitive theory which is modelled on emergent interactive agency (Bandura, 1999). Here Bandura asserts that individuals are "neither autonomous agents nor simply mechanical conveyers of animating environmental influences" (p. 22) because individuals' learn through observing behaviour and its related consequences. It is this view of learning that informed how Bandura conceptualised human agency. Bandura conceptualised human agency as having four core properties. One is intentionality; people either as individuals or in groups form intentions which include action plans of how to achieve them. However there is no absolute agency from this property of human agency, as most human pursuits involve other participating agents. The second is forethought, people set goals and anticipate likely outcomes of their actions in order to motivate their efforts. This property of human agency leads to temporal extension of agency. The third is self-reactiveness, people construct courses of action and regulate their execution. Bundura views this as a self-regulatory process which bridges the gap between thought and action. The fourth is self-reflectiveness, people continuously reflect on their thoughts, actions and meaning of the pursuits.

This view by Bandura is not adopted for use in this study because the classroom from which data is constructed is not teacher-centred for students to imitate fully developed knowledge in mathematics. Instead this classroom adopted sociocultural theory as a referent for their practice. As explained earlier in this chapter, sociocultural theory in summary involves students, social interactions in the classroom and the culture within and beyond the classroom. And again, this is the view on learning which the reported study adopted.

2.4.2 Pickering's Agency

Pickering (1995) and his ideas on "the dance of agency" is a proponent of human agency in sociocultural theory. While reading through the work by Pickering and listening to his videos on his book, there seems to be a thin line between how Pickering and Bandura view human agency. They both view components of human agency as intentionality. They both talk of coevolution in human agency. But as indicated earlier, Bandura's view of human agency centres on the notion that the social exists to be imitated, to inform plans, and is predictive on potential outcomes based on existing knowledge. On the other hand, Pickering deviates from this and asserts that the social is not an independent course of scientific beliefs. As a result, he argues against the predictive nature of the social because knowledge is viewed as being in 'the dance of agency'.

In the dance of agency' Pickering claims not to use science as a body through which knowledge is explained but through which things are done. He thus talks of performative/agency idiom which is achieved through a process that he conceived as the Mangle. The Mangle is interpreted as dialectical resistance and accommodation where resistance is experienced when something either does not work or goes wrong. Furthermore, resistance is clarified as a block between current position and intentionality – as destination. On the other hand, accommodation happens when adjusting performance because of resistance. Thus, in the Mangle there is exploration of what individuals might become without knowing where they are going; the new sense of self come into being in the Mangle. It is this explanation of the Mangle that justifies Pickering's view of the extension of culture as open-ended in the sense of not being prescriptive. This notion of agency is the one that is adopted in this study where data was constructed in an environment where participants were allowed to develop mathematical knowledge.

On account of his interest in how action/agency/performance is used to think of the world, and his interest in the material component of things, Pickering (1995) does not only talk of human agency; he also talks of material agency. The difference between human agency and material agency is in the former, as it has an intentional and social structure, and intentionality. People maneuvering in the field of material agency "capture, seduce, download, recruit, enroll, or materialise that agency, taming and domesticating it, putting it at service, often in the accomplishment of tasks" (p.6). Human agency is itself emergently reconfigured in its engagement with material agency. This relationship between human agency and material agency is also emphasised by Malafouris (2008). He asserted that "There is no way that human and material agency can be disentangled" (p. 22).

In concluding on the concept of human agency as perceived by Pickering, it should be emphasised that because of the process of the mangle in "the dance of agency", theory cannot be standardised to be the centre of any social construct; theory grows. Given his views of knowledge as knowledge as mangled, it evolves. This he observed to be inclusive of concepts in Physics and Mathematics.

2.5 AGENCY AND MATHEMATICS LEARNING

Reported research on human agency in mathematics education draws on Pickering's (1995) "dance of agency" metaphor in studying teaching and learning of mathematics (Boaler, 2003; Grootenboer & Jorgensen, 2009; Grootenboer & Zevenbergen, 2008; Wagner, 2007). Boaler (2003) uses the metaphor when illustrating the importance for mathematics students to have an empowering identity in relation to school mathematics. She reports on students' engagement in the "dance of agency", in one of the "reform" classes she observed. Her report does not detail the process through which students achieved mathematical agency; it seems this was the case due to two reasons. First, her study was aimed at "understanding the ways students are positioned in classes and the beliefs and

identities they develop as students" (p. 5). Second, students are given agency by the teachers according to her. Her work focused on the opportunities teachers afforded students to engage in the dance and how the students identified (positioned) themselves.

Grootenboer and Zevenbergen (2008) noted that Mathematics teachers have to engage in a "dance of agency" when they decide to encourage students' own agency as mathematicians. On the other hand, Grootenboer and Jorgensen (2009) argue that teachers' sense of agency allowed them to expand their sense of learning and achievement. In both studies, focus was on teachers and how their agency influenced that of the students, and this was the case with Boaler (2003). For them, students' learning experience is used to reflect on the teaching approaches adopted by the teachers. Hence, it seems they were more concerned about how teaching (or teachers) orchestrates students' agency.

Unlike in the studies highlighted above, the one reported here focused particularly on students and their learning experience. Students' learning experience is used to showcase how agency is enacted through interaction in the classroom, and how engagement in the dance of agency is possible. Studies with a similar focus (students and their learning experience) are also seen in the work of Wegner (2007) as he investigates students' voices in utterances. However, his focus is not on achievement of agency through mathematics learning. Instead, he discusses issues of language in Mathematics with students who had agency in the discourse and control in the classroom communication.

Similarly, Boaler and Greeno (2000) and Lange (2010) interviewed secondary and primary school students, respectively about their learning experience and how they felt being in control of learning. Both studies focused on students and their sense of agency in their learning experience. Even though this was the case, in the present study agency is accounted for as in action. That is, students' actions and utterances were used to account for their sense of agency.

As noted before, Boaler, Greeno and Lange also went back to re-interpret students' responses in relation to the different teaching approaches adopted in the (mathematics) classrooms and subsequently the learning environment they were exposed to. In the reported study, students' agency as they engage in mathematics learning was observed without talking to or asking the students about agency in particular.

Additionally, Fernandes (2013) conducted a study which aimed at highlighting the emergence of students' agency in the interplay with roots and discussing the role of agency in participation and in the learning of mathematics. As a consequence, he concluded that the introduction of robots made agency to emerge on students who usually did not participate. Furthermore, he explained that the students acted on the field of material agency, brought by the robots by capturing this agency and placing it in service of the task they had to carry out. Though his work draws on Pickering's human agency and is in mathematics learning, however he does not refer to the "dance of agency" which Pickering regards as the mangle.

Furthermore, Andersson and Valero (2011) examined achievement of agency when introducing new critical pedagogical discourse to a group of social sciences students. Thus, they concluded that this pedagogy which introduced projects made it easier for students to achieve personal agency. Then again, they explained that students got the opportunity to decide individually on personal learning objectives to reach within each project. Though their study is related to the one reported here, the studies differ as follows:

- First, their work does not draw on Pickering's "dance of agency";
- Students who participated in the reported study were doing mathematics as a subject of their choice (over mathematical

literacy).While those in their study did mathematics as a compulsory subject; and

- Students in the reported study could not personally decide on individual objectives during mathematics learning due to policy implementation.

Then, Andersson and Valero's (2011) work drew on Biesta and Tedder's (2006) achievement of agency; this was the case as the main focus of their study was on students' mathematical identities. Hence, Biesta and Tedder's (2006) work on achievement of agency afforded them the opportunity to illustrate the interrelatedness between student's formation of identities and achievement of agency.

In understanding the concept, agency, Biesta and Tedder (2006) put forward two key ideas. First, they suggested that agency should be understood in an ecological way. That is, strongly connected to context and second, they imply that agency should be seen as achieved and not as an individual's capacity:

> …agency should not be understood as a possession of the individual, but rather as something that is *achieved* in and through the engagement with a particular temporal relational situation. The idea of *achieving agency* makes it possible to understand why individuals can be agentic in one situation but not in another. It moves the explanation away, in other words, from the individual and locates it firmly in the transaction (which also implies that the achievement in one situation does not mean that it will necessarily be achieved in other situations as well). (Biesta & Tedder, 2006, p.18-19)

Agency is thus not how we act in particular situations; rather, the agentic dimension lies in the ways in which we have control over the ways in which we respond to the situation. Within a socio-cultural theoretical framework, regarding learning as a social activity, a definition of agency in line with Biesta and Tedder's ecological understanding fits particularly well (Andersson, 2011).

Furthermore, Biesta and Tedder argue that a dimension of agency can be understood as the way in which actors bring their past experiences and future orientations to bear on the present situation, resonating with Skovsmose's (2005) understanding of students' background and fore grounds as reasons for students' intentions for engaging in mathematics learning. In addition, Biesta and Tedder (2006) address "the extent to which people are able to distance themselves from their agentic orientations, i.e. make such orientations the object of reflection and imagination" (p. 21). This way of reasoning resonates with the definition of identity as the reified, endorsed and significant narratives told about a person as suggested by Sfard and Prusak (2005) and thus firmly connecting the concept of achieving agency with changes of identity. Since the focus of the reported study as aforementioned was on agency, this view of agency as explained by Biesta and Tedder is neither adopted by the study nor rejected as an explanation of what agency is. Instead it was highlighted here in order to understand why Anderson and Valero did not adopt Pickering's view of agency in their study.

2.6 CONCEPTUAL FRAMEWORK

The conceptual framework which guided the study reported here is grounded mainly within a socio-cultural perspective. Concerns about agency in education research relate, from the socio-cultural perspective, to the empirical conditions of agency as when and in what ways agency is possible (Biesta & Tedder, 2006). Mathematics learning is thus viewed as a social activity (Lerman, 2000; Radford, 2008). I put it that, a socio-cultural perspective implies comprehending mathematics classrooms as social systems that are organised as shared practices. These social systems shape in different ways how individuals are expected or allowed to act and participate (Andersson, 2011). In short, from a socio-cultural perspective, learning is dependent on other students, the social interactions in the classroom, and the culture within and beyond the classroom.

Thus, within the socio-cultural perspective the study adopted Mueller, Maher and Yankelewitz's (2012) framework which highlights three modes of student collaborations, the discursive nature of each, and the significance and interplay of agency and neighbour interactions that take place during collaborative instances. The first form of collaboration occurs when students engage in the *co-construction* of ideas. This is a form of collaboration in which the dialogue occurs in a back and forth nature until the argument is built. The second form of collaboration is that of *integration*. This form of collaboration is identified when a student's argument is strengthened using ideas from their peers. In other words, the ideas, explanations, or representations of others are assimilated into their original argument. The third form of collaboration is that of *modification*. This occurs as students attempt to correct a peer or assist him/her in making sense of a model or argument that was originally expressed in an unclear or incorrect way.

Figure 2 below illustrates the nature of the discourse and of the agency that is typical of each form of collaboration. First, co-construction is typified by negotiated discourse, and all participants equally share agency in the discussion. Second, integration makes use of both informative and interpretive discourse. As a result, original argument is interpreted by the second participant, who then enhances the argument in a way that informs the originator and allows the first participant to assimilate the information in a meaningful way. In addition, the originator of the argument is the principal agent, whereas the second participant influences the mathematical outcome and thus has minimal agency in the discussion. Lastly, modification is characterised by interpretive discourse, as one student attempts to makes sense of another's flawed argument. Moreover, this sense-making student also has the primary agency in the discourse, since he/she has the ultimate control in the mathematical outcome of the discussion (Mueller, Maher, & Yankelewitz's, 2012). These modes of collaboration do not occur in a linear way as discussed here, rather in an integrated way, and therefore their occurrence should be viewed as such.

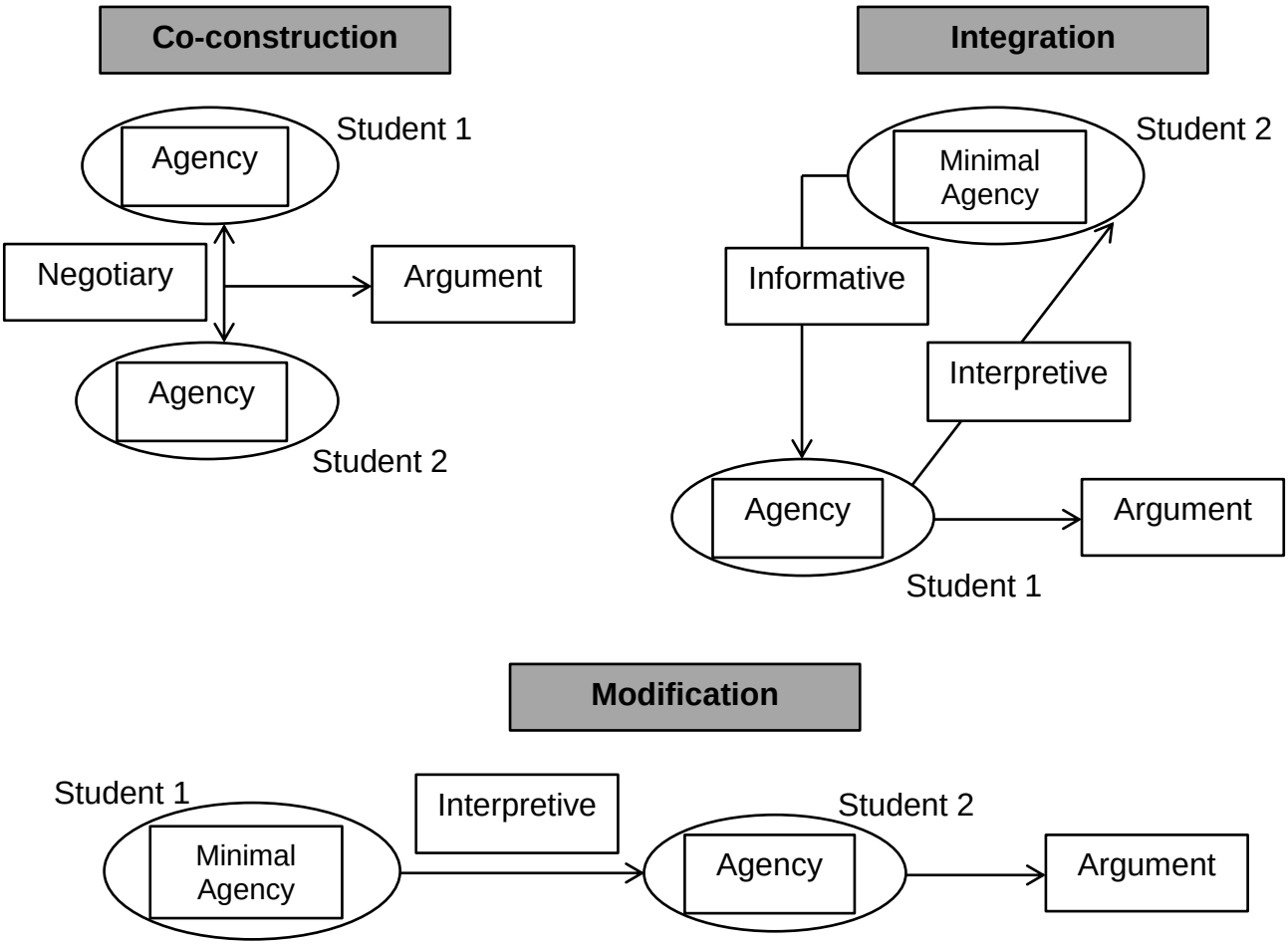

Figure 2: Three modes of collaboration
(Mueller, Maher, & Yankelewitz, 2012)

2.7 SUMMARY

In this chapter I discussed the meaning of mathematics learning from a socio-cultural perspective. Then I discussed agency and mathematical agency through analysis and interpretation of literature. Lastly, I discussed the framework which the study adopted, as a lens for data interpretation.

CHAPTER 3: METHODOLOGY

3.1 INTRODUCTION

In this chapter first, I outline the research approach which the study adopted; my choice of approach is defended through the socio-cultural perspective which the study employed for classroom practise. Second, I present the design of the study. Third, I outline the participants of the study. Fourth, I discuss the data gathering techniques which were employed by the study. Furthermore, I account for the manner in which I used these techniques, as it differs with how they are usually used in "traditional" research. Fifth, I discuss the data analysis method and the actual data analysis process as it unfolded. With this section I found audio data transcription to be a very slow and tedious exercise; nevertheless I transcribed the data and did the analysis. Sixth, I outline and discuss how I accorded various quality criterions throughout the study and lastly, I outline the ethical issues which I considered throughout the study.

3.2 RESEARCH APPROACH

It was made clear in Chapter 2 that the study uses sociocultural perspective as a referent for classroom practice and data interpretation. Owing to the philosophical assumptions made by this theory in terms of epistemology and ontology, a qualitative research approach was more suitable. Denzin and Lincoln's (2005) definition of qualitative research is as follows:

> Qualitative research is a situated activity that locates the observer in the world. It consists of set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings,

> attempting to make sense of, or interpret, phenomena in terms of meanings people bring to them (p. 3)

This definition is relevant as it captures a number of methodological issues which are evident in this report. Amongst others are: the strategy of inquiry which was employed in the study, methods which were used for data collection and the setting in which data was collected. These aspects will be discussed in more detail later in this chapter. Nevertheless, a definition which outlines the process of qualitative research from philosophical assumptions to procedures involved throughout the study was also relevant, because it indicates of how the research unfolded. One such definition is that of Creswell (2007) captured below:

> Qualitative research begins with assumptions, a worldview, the possible theoretical lens, and the study of research problems inquiring into the meaning individuals or groups ascribe to a social or human problem. To study this problem, qualitative researchers use an emerging qualitative approach to inquiry, the collection of data in a natural setting sensitive the people and places understudy, and data analysis that is inductive and establishes patterns or themes. (p. 37)

Due to the philosophical assumptions made by this study and the descriptive nature of this study a qualitative approach was adopted. The study intended to offer a thick description of how agency is enacted in the classroom; as a matter of fact the latter could not be achieved using a quantitative approach.

3.3 STUDY DESIGN

Enactment of agency through students' interactions in a mathematics classroom is not a predetermined and a predictable phenomenon. Rather, it is a contemporary phenomenon deeply rooted in actions taken by students during a mathematics learning experience. Therefore studying this phenomenon should be done in its real-life context, and in this case the context is a mathematics classroom. Studies conducted in their real life contexts are best done through case studies (Merriam, 1998; Stake, 1995; Yin, 2003). Case study as a research design

has been thoroughly explored by Merriam, Stake and Yin (Brown, 2008; Njie & Asimiran, 2014; Yazan, 2015) in particular. Hence, I will discuss what case studies are as conceptualised by the three distinguished authors.

Yin (2003) defined case studies as an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident. On the other hand, Stake (1995) defines it as a form of research defined by interest in individual case, not by the method of inquiry used. Unlike Yin and Stake, Meriam (1998) does not offer a precise definition of a case study. Instead she clearly defines a "case" and creatively discusses characteristics of case study (Yazan, 2015). From her definition of a case and discussion of characteristics of case study, I then offer a definition of case study to represent her point of view. Merriam vied a case study as an intensive, holistic, descriptive and analysis of a single instance, phenomenon or social unit.

It seems all these definitions of a case study fit well with my study, as they all emphasise on a phenomenon studied within its real life context. However that is not the case; the three authors' views of case study differ. Firstly, they differ in their epistemological commitments. Secondly, they differ in their conceptualisation of the "bounded nature" of the case.

In terms of stance on reality or meaning, Yin does not explicitly articulate his orientation. However, Yazan (2015) argues that his approach to case study is towards a positivist philosophical stance. This worldview cannot be relevant since agency is a social phenomenon. Mathematical agency, on the other hand, is a social phenomenon during (re-) construction of mathematical knowledge. It is therefore needless for me to say that reality or meaning is socially constructed. Merriam and Stake both subscribe to the constructivist paradigm (Njie & Asimiran,

2014; Yazan, 2015). I then find myself in harmony with a view of case study through their lens.

In terms of their views of the bounded nature of the case study, I will discuss that of Stake and Merriam only. To this point their views of case study epistemologically seem relevant to my study. Stake views a case study as an integrated system (Yazan, 2015), which means multiple cases and various contexts. In my study the case is enactment of mathematical agency and this is bounded in a sociocultural mathematics classroom. It is therefore clear that the case study described in this study is the one viewed as a bounded system (Merriam, 1995).

Merriam (1998) further explains a case study according to three distinctive attributes: Particularistic, descriptive and heuristic. She explains particularistic as a focus on particular situation, event, programme or phenomenon. This, according to Brown (2008) it suggests what to do in a similar situation. Descriptive case study is explained as one which yield rich and thick description of the phenomenon studied. Lastly, heuristic case study is one which clarifies the phenomenon under study. Brown and Yazan (2015) contend that it allows for a broader understanding on the phenomenon.

These traits are interpreted as types of case studies (Brown, 2008) and often as characteristics of a case study (Yazan, 2015). For the reported study, I did not either choose one of the attributes or subscribe to all of them. These resulted from the purpose of the study which was to document and describe how students enact agency through interactions in a mathematics classroom; making the study to have descriptive elements. Documenting and describing enactment agency will broaden the readers' understanding of agency in a mathematics classroom.

The use of case study as a research design in studying mathematical agency is not foreign to research in mathematics education. Andersson (2011) in her study on student's achievement of agency also reported a case study from upper secondary critical mathematics innovation. Fernandes (2013) reported on a case study about the emergence of students' agency in the interplay with robots. The use of case study in research on mathematical agency is also seen in the work of Biesta and Tedder (2006), Goodall and Johnston-Wilder (2015), and Tan et al. (2012).

3.4 CHOOSING PARTICIPANTS

Sampling is defined by Merriam (1998) as the selection of a research site, time, people and events in a field research. Purposive sampling which is a non-random method of sampling where the researcher selects information rich case for conducting an in-depth study was employed (Cohen, Manion, & Morrison, 2000). This was the case as Merriam indicates that a non-probability sample is effective when, as in this study, the researcher is exploring and describing what is occurring. She suggested that such a purposive sample has a logic and power and provides rich information.

Merriam (1998) further explains that purposive sampling emphasizes on a criterion based selection of information rich cases from which a researcher can discover, understand and gain more insight into issues crucial for the study. The latter is an indication that purposive sampling is consonant with case study design. Participants in the study comprised 49 Grade 11 Mathematics students from a rural school, located in the Sekhukhune district of Limpopo Province, South Africa. This cohort was selected as it was the only mathematics class allocated to me to teach, hence it was conveniently accessible.

Though the sample consists of 49 students, the data and the findings were based on the sample's grain-sizes. The grain-sizes were made up of either groups or pairs of students working together (with the teacher) on teaching and learning activities in a natural setting of my classroom. Instances analysed did not represent the experiences of each group nor all individual students. Since the study's interest was describing how mathematical agency is achieved in the classroom, their strength was in showcasing when and in what ways was agency enacted. Therefore external validity was traded off for transferability (Guba & Lincoln, 1989).

3.5 DATA GATHERING TECHNIQUES

There is a variety of data collection methods used in qualitative research documented by several authors, for example Creswell (2012), Maree (2011), McMillan and Schumacher (2010), and Struwig and Stead (2013). The common ones which made it in the list of all the authors cited here are interviews and observations. These methods, as in this study, are prominent data collection methods in case studies as noted by Merriam (1998).

3.5.1 Interviews

Interviews are described as the process in which the researcher asks one or more open-ended questions (Struwig & Stead, 2013) to the participants and record their answers (Creswell, 2012). Merriam (1998) noted that interviews are the most common sources of data in a case study research. Her elaboration on interviews include the types, asking good questions, questions to avoid, probes, the interview guide, beginning the interview, the interaction between interviewee and respondent, recording and evaluating interview data.

Interviews in this study were not employed in their "traditional" view as described above, where the researcher asks the participants questions. They were like focus-group interviews in nature. Creswell (2012) defines a focus group as "the

process of collecting data through interviews with a group of people, typically four to six" (p. 218). The focus-group interview occurred during instances where the teacher (researcher) discussed with students their learning experience and engagement in the learning process. The questions were not predetermined; instead they emerged from the immediate context and were asked in the natural setting of the events. This allowed the participants to express their views or opinions freely (Struwig & Stead, 2013).

Data also emerged from classroom discussions, when students in their groups worked through activities given to them as part of their day-to-day mathematics learning. These interactions were also like focus-group interviews, which were audio recorded while the teacher was not monitoring the groups but present in the class. These are not "chain-rule" interviews (Silverman, 1993, p. 116) but a form of group interview where data emerged as members of the group interacted on a topic that was supplied (Cohen et al., 2000). The topic under discussion was guided by the activities supplied to the students by the teacher. These activities were aligned to subject content to be taught in the Grade as prescribed by the Curriculum Assessment and Policy Statement (CAPS) (Department of Basic Education, 2011). It is therefore needless to say that the research agenda did not shape the data which emerged.

3.5.2 Observations

Observation is defined as the process of gathering information by observing people or places (Creswell, 2012) in their real life context (Struwig & Stead, 2013). Its use in case studies is supported by Cohen and colleagues (2000) when they assert that "whatever the problem or the approach, at the heart of every case study lies a method of observation" (p. 185). On the same note, Merriam (1998) also mentions that observations are important in case studies. However,

she cautions that they are subjective data sources; hence she recommends that they be carefully considered.

For Merriam (1998) this subjectivity may affect the credibility of the data collected which is a threat to the findings. She argues that the researcher mostly enters the situation without observation schedule and engages in constructing an understanding of the research environment through self-interpretation of what happens. For that reason qualitative research often produces results which are “an interpretation by the researcher of others’ views filtered through his or her own” (p. 23). Qualitative research however acknowledges the researcher’s subjectivity (Guba & Lincoln, 1989) and it is therefore not a threat as such. Later in this chapter I will discuss how credibility of the collected data was accorded for in this study.

Observations can be conducted with or without the researcher being a participant (Creswell, 2013; Gatley, 2001). Participant observation which Creswell (2013) describes as the role adopted by the researcher when they take part in the situation they are observing, was employed in this study. Therefore as a researcher, through a close interaction with the actors, I became a “passionate participant” (Guba & Lincoln, 1994, p.115). My participation emanated from the role that I played as the participants’ mathematics teacher.

During observations as students engaged in activities there were instances when I was a silent participant observer. The latter means I was a member of the group who did not participate in conversation. Though my presence in the group did not affect the data produced, it kept students on task. Entries were made in the researcher’s journal during group conversations; this was however mostly done later while reflecting after the lesson. The reason was that, as the teacher, in most cases I was immersed (by listening) in students’ discussions, that I forgot to immediately make entries in the researcher journal.

3.6 DATA ANALYSIS

Data analysis according to Merriam (1998) is “the process of making sense out of the data. And making sense out of the data involves consolidating, reducing, and interpreting what people have said and what the researcher has seen and read – it is the process of making meaning.” (p. 178). This view of data analysis is consonant with systematic data analysis in qualitative research (Creswell, 2013; Kim, 2015; Struwig & Stead, 2013). The systematic qualitative data analysis involves identifying concepts from raw data through coding, linking similar or related codes to form a category, identifying repeated patterns from categories and creating themes that represent similar patterns (Kim, 2015). However, in this study, narrative analysis was adopted instead of the “traditional” qualitative data analysis.

Narrative analysis is “an act of finding narrative meaning” (Kim, 2015, p. 190). Subsequently, narrative meaning is defined by Polkinghorne (1995) as a cognitive process that organises human experiences into temporally meaningful episodes. Then again, narrative analysis concerns different aspects of experience that involve human actions or events (Kim, 2015). Additionally, Kim posits that narrative analysis is a meaning finding act used to attempt to elicit implications for better understanding of human actions. Therefore, the reported study adopted narrative data as conceptualised though the work of Polkinghorne.

Polkinghorne (1995) in his work divides narrative data analysis into two categories: analysis of narratives and narrative analysis. Analysis of narratives follows after the systematic qualitative data analysis discussed earlier as the final step. Furthermore, Polkinghorne posits that with this analysis common themes are examined in storied data, and organised under several categories using stories as data. However, this method was not used in this study since no storied data was collected. Since the data collected during this study was actions, events and text,

Polkinghorne's narrative analysis of eventful data was adopted for use in making meaning of the data.

In narrative analysis the search is for data that will reveal uniqueness of the individual or bounded system and provide an understanding of its idiosyncrasy and particular complexity (Polkinghorne, 1995). With this analysis instead of separating data, data is synthesised into a coherent developmental account by linking the themes which emerge during data sorting when interpreting data. Hence, the purpose of this analysis is, according to Kim (2015), to help understand why and how things happened and participants acted, in the way they did.

During the analysis I listened to the audio records a number of times until I reached a stage where I accepted the picture they portrayed (Chuene, 2011). Then I transcribed all the audio into text (Creswell, 2013), followed by reading through the textual data several times, which led to selection of excerpts used in data analysis. The choice of the excerpts was guided by their strength in fulfilling the purpose of the study and responding to the research question. In the same way, data collected through observation was also read through several times. Finally, the findings were presented using vignettes from related excerpts of discussions that were found substantive to the narrative.

3.7 QUALITY CRITERIA

The quality of a qualitative research is measured through its trustworthiness. A study is said to be trustworthy if and only if the reader of the research report judges it to be so (Rolfe, 2006). Trustworthiness of qualitative research is generally questioned by positivists, as their concepts of validity and reliability cannot be addressed in the same way in naturalistic studies (Shenton, 2004). Qualitative investigator like Guba and Lincoln (1985, 1989) address these concepts, however they prefer to use different terminology to distance themselves

from the positivist paradigm. They divide trustworthiness into credibility, dependability, transferability and confirmability. In the discussion which follows I offer a brief explanation of each and explain how it was addressed in this study.

3.7.1 Credibility

Credibility is defined as the extent to which data, data analysis and conclusions are believable and trustworthy (McMillan, 2008). In the case of positivist researchers, they refer to internal validity, which ensures that their studies measure or test what is actually intended (Shenton, 2004). Shenton further suggests that Merriam (1998) refers to credibility by saying the qualitative researcher's equivalent concept deals with the question, "How congruent are the findings with reality?" Even though both internal validity and credibility deals with issues of reality in research, the reality they refer to differs due to epistemological assumptions. The assumption in qualitative research is that reality is multidimensional and ever-changing; not a single, fixed and measured phenomenon as in quantitative research (Merriam & Tisdell, 2015).

Guba and Lincoln (1989) elaborated on seven techniques which qualitative researchers can employ in order to address credibility. The techniques included: prolonged engagement, persistent observation, peer debriefing, negative case analysis, progressive subjectivity, member check and triangulation. For the study reported here, three techniques to enhance credibility were employed; prolonged engagement, persistent observation and triangulation.

3.7.1.1 Prolonged engagement

Credibility of data collected was accorded through prolonged engagement with the students I taught, and through which rapport was developed (Guba & Lincoln, 1989). The latter was possible because the data providers were students I taught mathematics daily a year before the study was conducted until the study

was conducted. I therefore did not expect students to deliberately mislead, distort or misconstrue their discussions or actions (Chuene, 2011).

3.7.1.2 Persistent observation

Persistent observation is a technique which ensures depth of experience and understanding during prolonged engagement (Williams, 2015). Its purpose is "to identify those characteristics and elements in the situation that are the most relevant to the problem or issue being pursued and focusing on them in detail" (Guba & Lincoln, 1985, p. 304). It gives an understanding of the participants' views and minimises the effects of the researcher's presence (Anney, 2014). The prolonged engagement allowed persistent observation to take place and result in an in-depth study (Bitsch, 2005).

3.7.1.3 Data triangulation

(Data) triangulation refers to the use of a variety of methods in collecting data (Bitsch, 2005; Merriam, 1998). It helps the researcher to reduce bias and it cross examines the integrity of the participants' responses (Anney, 2014). This is where most qualitative researchers employ several data collection techniques in a study but, usually, select one as the central methods (McMillan & Schumacher, 2010). Even though both observations and interviews were used for data collection, observation was the central method. These methods permitted triangulation of data collected, resulting in increased credibility of findings. This process conformed to case study as a research design (Cohen *et al.,* 2000).

3.7.2 Dependability

Dependability is defined as "the stability of findings over time" (Bitsch, 2005, p. 86). According to Merriam (1998), it refers to the extent to which research findings with similar subjects and in a similar context. In the case of positivist researchers they refer to reliability, which they regard as "a technique to show that, if the work were repeated, in the same context, with the same methods and with

the same participants, similar results would be obtained" (Shenton, 2004, p. 71). Shenton acknowledges that such provision is problematic to qualitative researchers due to the changing nature of the phenomena under study. This is because it is unlikely for participants in a later similar study to provide identical responses, as their understanding have changed or developed due to their reflection on the initial research (Carcary, 2009). Guba and Lincoln (1985) put forward inquiry audit as a technique for establishing dependability.

Inquiry audit is a process where the researcher who was not involved in the research process examines both the process and product of the research study for consistency (Guba & Lincoln, 1985). In this case my supervisor was the one who examined the research process and product. Guba and Lincoln, further assert that "since there can be no validity without reliability (and thus no credibility without dependability), a demonstration of the former is sufficient to establish the latter" (p. 316). This means if a researcher has demonstrated credibility of the data collected, then that is enough to regard the data as dependable.

3.7.3 Transferability

Transferability is defined as the extent to which the findings of one qualitative study can be applied to other situations (Guba & Lincoln, 1985; Merriam, 1998). In naturalistic paradigm transferability of findings cannot be specified by the researcher; but only the reader can determine whether the findings are applicable to a new situation (Guba & Lincoln). The latter can be achieved when the researcher provides "thick description" of the study (Guba & Lincoln, 1985; Merriam, 1998; Stake, 1995).

Thick description refers to the detailed account of field experiences in which the researcher makes explicit the patterns of cultural and social relationships and puts them in context (Holloway, 1997). Thick description enables judgement of

how well a research context fits in others (Anney, 2014); without it it becomes difficult for the reader to determine the extent to which the findings are true (Shenton, 2004).

3.7.4 Confirmability

Confirmability refers to the extent to which the researcher can demonstrate the neutrality of the research interpretations (Guba & Lincoln, 1985). This is according to Anney (2014), the extent to which findings of a study can be corroborated by other researchers. Guba and Lincoln (1985) recommend confirmability audit and triangulation as techniques for establishing confirmability. The interpretation of triangulation is still that of using multiple data sources during data collection as discussed earlier in this section. In the same vein, confirmability audit is still the process where a reviewer examines the process and product of the research as discussed earlier in this section.

3.8 ETHICAL CONSIDERATIONS

A letter requesting for permission to collect data from my classroom, during normal teaching hours was submitted to the principal and the Department of Education. In the letter I informed them about the study, its purpose and the envisaged participants. After permission had been granted, I informed the students that I would be recording our lesson, including their interactions during our mathematics periods. The following ethical principles which were shared with both the management and the participants were employed: informed consent, confidentiality and dual role of the teacher researcher (Orb, Eisenhauer, & Wynaden, 2001; Leedy & Ormrod, 2005).

3.8.1 Informed Consent

The participants exercised their rights as autonomous persons to voluntarily accept or refuse to participate in the study. They were informed of the nature of the study and what was expected from them, so as to allow them to make an informed consent. They were informed of their right to freely participate and withdraw from the study at any time without penalty. Given that data was collected during normal teaching hours, the participants were informed that if they did not want their group interactions recorded they should say so without fear. In the case of whole class discussions they were informed that their identity would be held confidential when reporting. For this study all students in the Grade 11 mathematics class accepted to take part, as this study was integrated with their day-by-day teaching.

3.8.2 Confidentiality

Information collected from the participants was to be treated as confidential by using pseudonyms when reporting. However, as I explained the nature of the study and how it will be reported, the students wanted to be part of it through the use of their real names, except for one student. I have to acknowledge that confidentiality in case study is often a challenge due to the thick descriptions which are given. At times these descriptions may reveal identity of the participants or even the research site. Consequently, the decision that was taken by the students made my reporting easy.

3.8.3 Dual Role of the Teacher Researcher

Entering the classroom as both the teacher and researcher presented some challenges. However, I took an oath that I would always take the lead and as such I did not allow the research agenda to interfere with the normal teaching of the students.

3.9 SUMMARY

In this chapter I presented the research approach which guided the study and the study design which was adopted. Then I described the participants of the study and discussed the data gathering techniques which the study employed. Furthermore, I discussed the qualitative data analysis method which the study employed, outlined the quality criteria and lastly the ethical issues which were considered throughout the study.

CHAPTER 4: FINDINGS, ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

In this section I analyse data and throughout the analysis I show case in what ways students' enacted mathematical agency. The analysis is focused according to various classroom interactions which took place in the classroom as described earlier in chapter 2. Data is presented through excerpts from various moments of the classroom discussions, these excerpts were selected because they are substantial to the narrative.

4.2 ANALYSIS

4.2.1 Student – Material and Student - Student Interactions

In this section I analyse related excerpts from the first lesson I facilitated with the intention to engender mathematical agency. It was a revision lesson on factorising quadratics with a view to preparing the students to solve quadratic equations. However, the approach to factorisation that was used in the material was different from how the students had done before. As a result, the students were learning through revision. This approach to revision was also different. Normally a set of questions would be given to the students to answer. However, this time their revision material included notes and worked examples for students to go through.

I requested the students to get into groups and go through number 3 in the material then respond to an exercise. There were eight groups with members ranging between four and six, all groups were of mixed gender and mixed ability. The number 3 which students were requested to work on was as captured below:

Factorising quadratics

To learn how to factorise, let us study again the removal of brackets from $(x+3)(x+2)$.

$$(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

Clearly the number 6 in the final answer comes from *multiplying* the numbers 3 and 2 in the brackets. This is an important observation. The term $5x$ comes from *adding* the terms $2x$ and $3x$.

So, if we were to begin with $x^2 + 5x + 6$ and we were going to reverse the process we need to look for two numbers which multiply to give 6 and add to give 5.

$$?\times? = 6$$
$$?+? = 5$$

What are these numbers? Well, we know that they are 3 and 2, and you will learn with practise to find this by inspection.

Using these two numbers which add to give 5, we split the $5x$ into $3x$ and $2x$. We can set the calculation out as follows.

$x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$= x(x+3) + 2x + 6$ by factorising the first two terms

$= x(x+3) + 2(x+3)$ by factorising the last two terms

$= (x+3)(x+2)$ by noting the common factor of $x+3$

The quadratic has been factorised. Note that you should never get this wrong, because the answer can always be checked by multiplying-out the brackets again!

Example

Suppose you want to factorise the quadratic expression $x^2 - 7x + 12$.

Starting as before we look for two numbers which multiply to give 12 and add together to give – 7. Think about this for a minute and you will realise that the two numbers we seek are – 3 and – 4 because

$$-4 \times -3 = 12, \quad \text{and} \quad -4 + -3 = -7$$

So, using the two numbers which add to give – 7 we split the $-7x$ term into $-4x$ and $-3x$. We set the calculation out like this:

$x^2 - 7x + 12 = x^2 - 4x - 3x - 12$

$= x(x-4) + 3x + 12$ by factorising the first two terms

$= x(x-4) - 3(x-4)$ by factorising the last two terms extracting a factor of – 3 in order to leave $x-4$

$= (x-4)(x-3)$ by noting the common factor of $x-4$

Once again, note that the answer can be checked by multiplying out the brackets again.

Example

Suppose we wish to factorise the quadratic expression $x^2 - 5x - 14$.

Starting as before we look for two numbers which multiply together to give – 14 and add together to give – 5. Think about this for a minute and you will realise that the two numbers we seek are – 7 and 2 because

$$-7 \times 2 = -14, \quad \text{and} \quad -7 + 2 = -5$$

So, using the two numbers which add to give – 5 we split the $-5x$ term into $-7x$ and $+2x$. We set the calculation out like this:

$$\begin{aligned} x^2 - 5x - 14 &= x^2 - 7x + 2x - 14 \\ &= x(x-7) + 2x - 14 && \text{by factorising the first two terms} \\ &= x(x-7) + 2(x-7) && \text{by factorising the last two terms} \\ &= (x-7)(x+2) && \text{by noting common factor of } x-7 \end{aligned}$$

So the factorisation of $x^2 - 5x - 14$ is $(x-7)(x+2)$.

After the students had got into groups and settled, there was total silence. The latter meant students were interacting with the material, through reading. In the midst of this silence, one student spoke loudly, and that initiated a conversation in her group. Consequently, I went closer to the group to observe. The excerpt which follows captures the conversation which took place as I observed the group. I use it to showcase instances in which students were (not) agentic.

Excerpt 1

1.1	Fridah	Why do they have to complicate this? Akere (isn't) it can just be done by inspection.
Pause		
1.2	Patrick	I think it's because if you factorise like this over and over again eventually you will be able to do it by inspection.
Pause		
1.3	Moganedi	Guys look at this example (reffering to the factorisation of $x^2 - 7x + 12$) they say we are looking for two numbers which we multiply to get 12 and then add up to – 7. They decided the numbers are – 3 and – 4 I don't understand.
1.4	Fridah	Which one? Wait let me see, what is confusing you actually?

1.5	Moganedi	I don't understand why they say they are adding since a positive times a negative is a negative (referring to $-3+-4$). I think they must say they subtract.
1.6	Fridah	Moganedi, actually it does make sense, the idea here is to come up with sort of a general way in which factorisation can be done. You see in the first example they spoke of addition, even now they still stick to addition.
1.7	Patrick	Fridah is right we are always going to add but you see our numbers are integers. Actually they were supposed to write $(-3)+(-4)$, to show the addition of two negative integers.
1.8	Moganedi	Ok
1.9	Fridah	Is it ok now?
1.10	Moganedi	Mmm
Pause		
1.11	Fridah	So what are you doing now?
1.12	Moganedi	I am factorising another one to see if I understand the explanation.
1.13	Fridah	Ok
1.14	Patrick	Explain it to us as you go so we can also see that you really understand.

Fridah's first utterance (1.1) was an indication that she did not appreciate the new approach to factorisation. In this case, being apprehensive about changing from the method of factorisation she was used to. On the surface, this can be seen as refusing to participate and make things happen and hence her attitude can be interpreted as not being agentic. However, the question is why did she resist the new approach aloud? Nevertheless, this showed she had agency in the discourse (Wagner, 2007), as she could have resisted silently. Another essential point is that she made a choice and intentionally resisted aloud, though her intentions remain unknown, her action denoted human agency (Bandura, 2001; Johnson, 2000). Additionally, her loud utterance seems to have been an invitation to others to participate in a conversation.

Similarly, Patrick's response (1.2) to Fridah also denoted human agency (Bandura, 2006; Pickering, 1995) as he intentionally influenced her thinking by inviting her to read the given information and making her aware that it was

necessary. Therefore, both students acted on the field of material agency (Malafouris, 2008), their collaboration about reading (or participating) can be seen as co-construction (Mueller et al., 2012).

The picture portrayed above can be viewed as an instance where human agency precedes mathematical agency. The use of the two concepts: human agency and mathematical agency might raise the question: how do they differ? While human agency refers to the capacity of individuals to make choices (Malafouris, 2008), mathematical agency emphasises a positive self-concept of individual's ability to make sense of mathematical tasks (Meyer, 2012). With this distinction in mind, it now becomes clear that Fridah displayed elements of mathematical agency as well. Moreover, she also contributed towards understanding the text used in the task at hand (1.6).

Fridah's engagement in the "dance of agency" was twofold: Firstly, she resisted learning the new approach because it did not fit her old approach. The latter can be interpreted that Friday saw this as a forced move, while she preferred a free move. Secondly, she accommodated the explanation that they got from the learning material. According to Pickering (1995, p. 22), this is explained as "dialectic of resistance and accommodation".

In conclusion, students in mathematical classrooms, engage in the "dance of agency" when they decide either to act on mathematical tasks as outlined in mathematical text or not. Not acting on mathematical tasks, in this case does not mean not doing the task but rather doing the task differently. Hence, in Fridah's case the issue was not about factorising quadratics per se, but factorising in a "particular" way. Furthermore, it can be concluded that the picture painted above also shows how students interact with learning materials. Though there was no instruction which specified that students factorise the given mathematical

expressions using the approach in the text, Fridah saw this massage as implying that it can be ignored.

Moganedi initiated another conversation (1.3) in the same group, his interaction with the other members showed that all the participants enacted agency during the discussion. Notably, he engaged in integration and modification collaborations (Mueller et al., 2012) with Fridah and Patrick respectively. Therefore, it is needless to say his sense of agency varied in the two collaborations. For instance, Moganedi was the principal agent when he interpreted the example, while Fridah had minimal agency. However, Moganedi had minimal agency when Patrick assisted him in making sense of the addition. Within the same conversation Moganedi's levels of agency (or achievement thereof) do vary. In the same way Fridah was a principal agent in the first conversation but she later had minimal agency. This observation is not entirely surprising as Biesta and Tedder (2006) allude that it is difficult to understand why achievement of agency changes from one situation to another. As a result they conclude that agency should not be seen as possession of the individual. Their conclusion is complement to the students' changing sense of agency discussed here.

Excerpt 1 does not only portray human agency that the students portrayed. It also portrays how the participants in the reported study are mathematically enculturated. Furthermore, it portrays how the participants interact with the given material and with each other as in the model captured in Figure 1 of Chapter 2. Through Fridah's interaction with Moganedi, the imaginary outcome was for Moganedi to make sense of the information from the learning materials. On the other hand, Patrick's explanation (1.7) to Moganedi confirms Vygotsky's assertion that the mind does not and cannot exist out of the social realm. Moganedi can be seen as engaged in semiotic mediation (Forman, 2003) as he moves ideas on factorisation that he got from social plane into his individual plane. These interactions are meaningful within a sociocultural classroom.

One of the striking features of Excerpt 1 is that, on the one hand, my contribution to the interactions can be perceived as absent. Such is the nature of the classroom from which data was constructed. This is also aligned with the interaction model that I presented in Chapter 2. On the other hand, in the case of this excerpt, I observed but held back from active participation. While this is allowed in participant observation, holding back throughout the observed interactions was a struggle. My frustration came when students started questioning the explanation given in the learning material because I was unable to defend it and convince them to play along while deep inside I wanted to. At this stage of holding back, I overlooked that though I removed the physical teacher from the group's interactions I was still part of the interactions in my ideas that were captured on the learning material. Therefore, in a covert way, I was part of their interaction through student-material interaction of Model 1.

This way of letting go of my role of encouraging the students to participate and giving directions to their discussion should be considered highly and encouraged in classrooms where the "dance of agency" is at play. Care should be taken that when students discuss in order to understand related information required for working out an activity, and not rushing through to bring the activity to an end, it is not a waste of time. By doing this as a teacher, I provided the students with "waiting time" as recommended for classrooms that engage in questioning and answering (Kaur, Singh, & Hashim, 2014). The worry should not have been, whether they were going to complete what I wanted them to or not. I should have known that with "waiting time", someone like Moganedi, would lead the group in working out the next problem and by so doing lessening my concerns.

Another essential point is that, as the lesson progressed I came to realise that both Patrick and Fridah left their group and joined new groups. Whereas Moganedi seemed to have been busy explaining to the other (three) members of his group who remained quiet during the conversation captured in excerpt 1. As a

result I was then interested in knowing their roles in these new groups and their reasons for joining them. Subsequently, I visited the group which Patrick joined; the excerpt below is a caption of the conversation observed. I use it to illustrate how agency enabled students to assist each other.

Excerpt 2

2.1	Patrick	You cannot just say you don't know how to factorise and you don't even understand the examples given here. We did factorisation last year, can you explain how you understood it then?
2.2	Potego	I told you I didn't understand it then and even now I don't understand.
2.3	Thomas	You (referring to Patrick) explain to us how factorisation can be done easily.
2.4	Patrick	I am going to use the approach in the handout, even though it is long but it is simple to understand. Ok let us look at how $x^2 - 7x + 12$ was factorised…
2.5	Teacher	Patrick, before you carry on. I am interested in knowing how you ended up being part of this group.
2.6	Patrick	Eish, Thomas asked me to come and assist them, is there a problem, sir?
2.7	Teacher	No, I was just curious. You may carry on.

Thomas took an initiative to ask for assistance (2.6), unfortunately I am in no position to account as to why that was the case because I was not part their group earlier. However, I can justify his choice of Patrick as influenced by his knowledge of Patrick's active participation during whole class discussions. Here Thomas's choice is influenced by the culture of Patrick in culture of the classroom (Cobb & Yackel, 1996). Apart from that, his decision portrays elements of human agency as he shows "forethought" which is, according to Bundura (2006), a property of human agency. Unlike in excerpt 1, here agency is enacted through seeking assistance. Furthermore, Thomas was mathematically agentic, he saw himself having the potential of making sense of the mathematical task through assistance. This positive self-concept he had about his ability to make sense of a mathematical task is what Meyer (2012) refers to as mathematical agency. Then

again this resonates with the practice of the classroom in which data was constructed, as this class accepts that learning occurs within the ZPD, and that is through assisted efforts. In brief, in classrooms where agency is not engendered the teacher should trade off authority. As a result, that allows achievement of agency to become a tool for mathematics learning through student-student interactions. On top of that, student's forethought of their ability to carry out a mathematical task through assisted efforts is an indication of the student's achievement of mathematical agency.

Unlike Thomas above, Patrick on the other hand is offering assistance to this group. In doing that, firstly he asked them to explain how they understood factorisation (2.1). This was an invitation to other members of the group to participate. In other words, he wanted to engage in integration collaboration with them. However, his invitation was rejected (2.2) immediately. As a result, this led to the group engaging in modification collaboration. While agency can be enacted in the latter, it is the former that would have allowed the group members to be principal agents. For this group, Thomas then had minimal agency and as for Potego she remained not agentic up to that point. As such, Potego's lack of agency appears to disable her from even sharing her confusion. Meanwhile, Patrick's achievement of mathematical agency made him to take up the role of teacher in a "traditional" mathematics classroom. This was the result of other members refusing to actively participate, but one can argue that as the discussion proceeds other students would eventually participate. Complementary to this, is that Patrick has shown that he was conscious of the practice of his classroom, by first inviting others to participate. Nevertheless, he was unable to influence them in engaging as he had wished; his failure was due to his lack of probing experience which could have led Thomas and Potego into talking about their experiences of factorising.

The concern then was on Potego, who refused to participate actively and opted to only listen. One wonders if this benefited her. Within the sociocultural

perspective it is acknowledged that whatever happens in the individual's cognitive plane emanates from the social interactions. Therefore, one can argue that she benefited. Correspondingly, Masha (2004) in support of this argument, asserts that students who remain quite during discussions observe for gaps and instances where they can make meaningful contributions. As a result, it means whatever cognition (if any), she engaged in, was due to the interactions between Patrick and Thomas. Therefore, it can be concluded that their achievement of agency in this case afforded her the opportunity to learn. In short, students' achievement of agency in a sociocultural classroom is an opportunity for learning to other students, even those who are not agentic.

To summarise, it seems in a classroom where agency is engendered the cared for becomes the care givers. Notwithstanding that I was not giving direction and instruction, students were able to request assistance and correspondingly others offered assistance. Again, this shows that students were enculturated in the classroom practice, where through the interactions modelled in chapter 2, the whole becomes one. For that reason, the students did not see themselves as individuals but rather as one team and supporting each other was the norm. At this stage of letting go, I had accepted equal, if not even having less power during facilitation as agency allowed the students to invite help if and when needed.

Finally, the period drew to an end. At that time I learned that some students were almost done with the exercise, others were only starting and on the other hand others wrote nothing all together. Reflecting on this lesson raised a number of aspects a teacher should take care of, for students to be mathematically agentic. First, allow the students to interact without the teacher's intrusion. Second, facilitate learning in way that will allow the students to make their own decisions about engaging in the tasks at hand. Lastly, use teaching and learning support material to initiate interactions.

4.2.2 Student – Teacher Interactions

The excerpt presented below, captures a conversation during a lesson which took place the following day after the preceding excerpts. On my arrival in the class, I requested the students to get into groups again. Then, I requested them to go through the factorisation of quadratics where the coefficient of x^2 was not 1. Unlike the previous day, this time the students' movement continued for a while as some changed groups. Amongst others it was Thato, who went to almost all the groups before he settled. Therefore, I was concerned about his movement because over the time he was not only playful but destructive as well. As a result I went to observe what he was up to, and to establish the reasons for his many movements. I use excerpt 3 to show how his sense of agency enabled him to find a group which operated within the same zone of proximal development as him.

Except 3

3.1	Teacher	Thato, why were you moving around all the groups?
3.2	Thato	My group members are done with this section and are busy with the special cases. So I was looking for a group which was just starting like me.
3.3	Teacher	Why have you not completed this section with your group?
3.4	Thato	Ah, Sir, I was waiting for corrections of the previous exercise.
3.5	Teacher	Let me see what you struggled with in the previous exercise (reaching for his book)
3.6	Thato	Well, Sir, I got them all but you never said we should complete the next sections, that is why I didn't do it just like these ones.
3.7	Teacher	I am going to come back a little later and ask you to share your understanding with me.
3.8	Thato	Eh, okay Sir

Eventually

3.9	Thato	Sir, you want me to explain how this factorisation is done?
3.10	Teacher	No, I want you to share with me what you learned as you revised.

3.11	Thato	Yesterday I found this method too long *nje* (for no reason), but today I find it helpful. You see *nna* (I) I always cheated when the coefficient was not one. I will use the quadratic formula to solve for x, then work in reverse to factorise.
3.12	Teacher	How did moving from your group and joining this one benefit you?
3.13	Thato	Here we listen to each other and help one another, there they were just talking about things I did not understand. But you see now I am also going to go on to other sections in advance.
3.14	Teacher	Ok

Thato's movements were in search of a group where his colleagues were doing a similar section as him (3.2). Notwithstanding that he could have either let his group members explain to him or just copy to cover what they had completed already; he did neither the latter nor the former. Instead, he took responsibility for his learning as development; again, this view of learning is within a sociocultural perspective adopted for classroom practice in this study. Resulting from his decision, Thato engaged in co-construction collaboration (3.13), in which negotiations were back and forth and all the students got engaged and shared agency equally. Thus, it is satisfactory that Thato was not only enacting agency, but he was also engaged in learning as he actively reorganised his participation in the classroom practice (Cobb & Bowers, 1999).

One of the striking features of Excerpt 3 is that, unlike in other lessons, Thato was not troublesome. It can be concluded that when the teacher allows for learning decisions to be taken by students, they tend to take those which will benefit them. However, I am not arguing for teachers to leave classes unattended to because their presence on its own is a tool for classroom management. In brief, Thato's sense of agency allowed him to seek for learning opportunities in which he would continue being agentic as they work through a mathematical task. Therefore, it is needless to say that had he stayed in his old group, he would not have developed as in the current group. The reason is that, he would have engaged in a modification form of collaboration, as a result, he would have had minimal agency.

The excerpt below captures a whole class conversation I had with the students, a day after I had been absent from school. Since we had completed linear number patterns in previous lessons, I informed the students that we were to start with quadratic number patterns. Considering my absence the previous day, I decided to start the lesson by inquiring from the students what they had done during the period. Therefore, I use the excerpt to illustrate how students enact agency while working through mathematical tasks unsupervised.

Excerpt 4

4.1	Teacher	What did you do during our period yesterday?
4.2	Students	Quadratic patterns (some saying no in the background)
4.3	Teacher	Who is saying no?
4.4	Students	Billy.
4.5	Mahlako	It means he was not listening.
4.6	Students	(Making further comments in a chorus)
4.7	Teacher	Can we have perhaps one person raising up a hand to tell us what you did yesterday? What did you do yesterday during my period (pointing at Thato)?
4.8	Thato	Eh, eh. They were doing the quadratic…
4.9	Teacher	What did you do?
4.10	Thato	I did nothing.
4.11	Teacher	You did nothing. What did you do yesterday? (referring to another student)?
4.12	L1	We were solving quadratic and linear sequences.
4.13	Teacher	Quadratic and?
4.14	L1	Linear sequences.
4.15	Teacher	Linear sequences. Thwala! What were you doing?
4.16	Thwala	Eh, I remember we were doing linear…(inaudible). (*students in the background saying quadratic with low tone*)
4.17	Teacher	Moganedi!
4.18	Moganedi	*Meneer* (Sir).
4.19	Teacher	What did you do?
4.20	Moganedi	Ooh, We were, we were helping each other with quadratic and linear

When I inquired from the class on what they had done in my absence, there were two messages across the class (4.2). Some students said that they had done quadratic patterns, while on the other hand some said no in the background. However, those who said the former were louder than those who said the latter. As a result of the contradictory responses, Mahlako made an assertion (4.5), which for me was an indication that she was certain that quadratic patterns had been done. Correspondingly, I later learned that she was the one who initiated the whole class discussion on quadratic patterns. As a matter of fact, this was her way of fighting for her initiatives not to just pass unnoticed. Subsequently, I decided to ask individual students on what they had done. However, I targeted those who were opposing that quadratic patterns had been completed.

Amongst the individual students was Thato, who acknowledged that the discussion had taken place, however he made it clear that he did nothing (4.8 and 4.10). Having said that, I was surprised he did nothing, meanwhile he was aware of the discussion. In contrast, I didn't question him further; instead, I carried on inquiring from other students. Unlike Thato, Thwala mentioned nothing about quadratics, but he instead said they had done linear only (4.16). However, the class still insisted on quadratics having being treated as well. As such, it can be concluded that these students were not engaged in the discussion with the whole class. Therefore, their reasons for the disengagement they portrayed here will be tackled in the discussion of excerpt 5, as I interrogate them further.

Contrary to Thato and Thwala, Moganedi and other students confirmed they had experienced collaborative learning on both linear and quadratics (4.12 and 4.20). Their use of "we" is an indication that they saw themselves as part of the discussions which took place. Furthermore, Moganedi went on to highlight what he had learned about quadratic patterns. Correspondingly, other students were often adding and rectifying what he said.

At this stage I was convinced that the discussion had absolutely taken place. As a result, I went back to ask Thato, Thwala, and Billy about what exactly they were doing when others were engaged in the discussion. Excerpt 5 below captures my interaction with these particular students, based on their disengagement.

Excerpt 5

5.1	Teacher	And then when all this was happening where were you?
5.2	Thato	I was in the class I was listening to them.
5.3	Teacher	So, why are you saying you did nothing?
5.4	Thato	Eh, ah, Sir ok. I did not come forth and explain…
5.5	Teacher	So you did not learn anything?
5.6	Thato	Ah at least I did catch up something.
5.7	Teacher	Thwala, when all this was happening where were you?
5.8	Thwala	I was in the class?
5.9	Teacher	You didn't see these things happening?
5.10	Thwala	I saw them.
5.11	Teacher	So why are you saying you never spoke about quadratic patterns?

Pause

5.12	Teacher	Talk! (*loud*)
5.13	Thwala	Uhm meneer actually I did not understand them.
5.14	Teacher	Did you tell them? The people who were leading the discussion, did you tell them?
5.15	Thwala	Yah
5.16	Teacher	What did you say?
5.17	Thwala	I told them where I did not understand and then Chris explained to me.
5.18	Teacher	Still you did not understand?
5.19	Thwala	I did understand.
5.20	Teacher	Eh!

Laughter

5.21	Teacher	Billy, what transpired yesterday?
5.22	Billy	They were doing quadratic patterns
5.23	Teacher	And what were you doing?
5.24	Billy	I was doing linear.
5.25	Teacher	So when they were discussing quadratic patterns you did not become part of the discussion?
5.26	Billy	Yes
5.27	Teacher	Why?
5.28	Billy	No, uh, actually I was not yet clear with linear patterns.
5.29	Teacher	Then you decided that even when they were discussing something new you did not want to be part of the discussion?
5.30	Billy	No, Sir, I cannot just carry on learning while leaving other things behind.

Surprisingly, from this excerpt Thato was not disengaged as it appeared in excerpt 4. However, for him doing something meant that he actively took part by contributing and not only by listening; in other words, acting out. Even though he agreed that he had learned something (5.6), there was a sense of dissatisfaction, since he used the word “at least”. As noted in excerpt 3 before, Thato’s responsibility of learning lay with him. Thus, through how he engaged in social interactions, he positioned himself as having done or not done something. As a result, it appeared that collaborative interactions in which he had minimal agency, for him these were not situations he could say he did something. However, engaging in co-construction collaborations was what he regarded as having done something.

Thwala’s (dis)engagement on the other hand seemed to be dissatisfaction about the teacher’s absence in the discussions. Even though he finally agreed that quadratics had been discussed (5.10), yet he claimed that he did not understand (5.13). Furthermore, he mentioned that it had been made clear to him (5.17) and he agreed that he had only understood after the clarification (5.19),

notwithstanding, that he denied having done quadratics before. For this reason, it would appear that he wanted an expert assurance, and in this case that of the teacher to confirm what was discussed the previous day. While on the other hand it could still be argued that it was his way of saying that he was not ready to move to the next section. Or he just did not want to be called up and requested to share what he had learned. These were some of the questions that his contradiction raised and unfortunately remained unanswered at that stage.

Just like Thato and Thwala, Billy divorced himself from the discussion which had taken place (5.22), however his reasons were different. He claimed that he had not been part of the class discussion because he was still unclear about the linear patterns (5.28). Thus, his insistence (5.30) could have meant that he did not see how quadratic patterns build on linear, something which could have disabled him from making connections and hence, he concluded that he did not understand. It is however interesting to note that students mentioned linear and quadratics, so it would be expected that he said “We did linear”, before he mentioned not being part of quadratic. Thus, it can also be concluded that his revision of linear patterns was an individual task (5.24) even though the class mentioned that they had done linear patterns.

At that stage I was overwhelmed by the initiative that the students had made, which was engaging in discussion neither guided nor supervised. On the other hand, however, I was a bit disappointed that the lesson I had prepared for that period was not going to materialise anymore. As an appreciation of the students’ efforts it was only right to build on what they had started already, this practice is highly recommended in constructivists’ classrooms (Kilpatrick). As a result I was faced with two tasks which I had not prepared for. Firstly, I had to carry out the questioning in order to understand how the students had done quadratic patterns. Secondly, I had to generate and give a short activity to assess what they had learned and to broaden, and build on what they had started already. That was

yet again another stage of letting go, since the students gave the content already discussed direction, I had to let go of the direction I wished for.

Indeed, I carried out the questioning and then generated an exercise which fitted well and built on the students' initiatives. That exercise was directly related with how they had worked on quadratic patterns the previous day. As students worked through the exercise in their groups, I was observing some of the groups. However, in one group I placed an audio recorder, which captured their discussion as it unfolded. Due to the length and repetitive nature of the interactions which took place, I did not capture any excerpt from that group. However, a full transcription of the conversation is enclosed in appendix B. As a result, I only analysed the interaction and cited direct quotes in order to support some of the claims I made where necessary.

Listening to the discussion, it was clear that all the students, but Potego, engaged in co-construction collaboration. As such, their negotiations were in a back and forth manner. Furthermore, the mathematical arguments were not sustained by an individual student. That is, no student took a lead and roles such as writing were interchanged time and again. In the same way there were instances of engagement in modification collaboration evident when the students attempted to clarify each other. The following utterances lead to this view.

> Thoma: "As per Chris yesterday…"
> Koketso: "You know Mahlako yesterday said…"
> Hope: "I am using Patrick's method"

Here the students were attempting to make sense of an argument and clarify each other. Though no one mentioned being confused, but throughout their conversation they strove for clear and justified explanations. The people they all referred to were not members of their group; however, from the previous day's social interaction I observed that the students continued to make sense through

those interactions. That again, resonated with the sociocultural practice adopted by this class.

Conversely, Potego's was not heard at all from the beginning of the discussions. Even though she was part of the group, she remained silent such that one could conclude that the group consisted of only three members. As mentioned previously in excerpt 2, even here she was not agentic. However, that point as the discussion proceeded she started responding to questions which the other members asked aloud as they tried to respond to the exercise. Her first utterance in the conversation was "mmm", an indication that she was in agreement with what had been said. After that, she was heard responding continuously for some time. Until she then made an error of addition and after that she was corrected. She remained quiet immediately as she had previously done.

For Potego, it seemed that she had only wanted to talk if and when she was entirely sure of what she was saying. The question that remained was, was that of an enactment of mathematical agency? For me it was, since she listened to other students as they interpreted mathematical arguments and then she became informative. The latter is an example of integration collaboration where both the interpreter and informer were agentic. Furthermore, she reappeared in the conversation saying "I think we made a mistake it must be 4a + 2b + c". Such a contribution informs that the student was trying to interpret; hence I say it was integration. Additionally, her use of "we" showed that she saw herself as part of everything going on in the group even in her silence. The period also came to an end, and later I called up the students who had initiated the class discussion.

From the group conversation it can be concluded that students do not enact mathematical agency by taking the role of the (traditional) teacher. Students continue working as equals. Furthermore, it enables them to manage each other in the process. Hence, I heard Hope and Koketso repeatedly reprimanding Thomas

and telling him to remain focused on the exercise. Students strove to make sense of what they were doing and ensured that they could account for the procedure they had used.

4.2.3 Whole – Class Interactions

. The data analysed here were generated through unstructured interviews with the students who initiated the whole class discussion and made presentations. The idea was to find out how they had made the decision. In this section of the analysis I show how agency enabled through whole class discussion to initiate and facilitate a whole class discussion. Since a full transcription of the interview is enclosed is available, throughout the analysis I only included direct quotes of what had been said, which I found substantial to the narrative. For this interview, the participants were, Mahlako, Chris and Fridah.

Mahlako, the student who had initiated the whole class discussion, explained that she was doing quadratic patterns with Chris. Then, she realised they had understood them. As a result, during the mathematics period she suggested to Chris that they should share what they had done with the rest of class. Again, this initiation also shows elements of human agency as discussed in excerpt 1 earlier in this chapter. Enactment of mathematical agency here was defined by a positive self-concept of her ability to make sense of quadratic patterns (Meyer, 2012). Through this positive self-concept, her sense of agency then afforded other students the opportunity to learn.

As part of her explanation, she mentioned that she had written a sequence of numbers which was worked out earlier (with Chris), on the board. Then, she took the students through the extension of the sequence which formed a quadratic pattern. Additionally, Chris took the class through the process of determining the general formula (nth term) of the quadratic number pattern. That was co-

construction collaboration at a different level, because Mahlako could have done the whole presentation alone. Then again, the collaboration in presentation could be an indication of the discussion they held before they got in to the class discussion. Here Chris maintained his sense of agency using Mahlako's initiative. Therefore, one can conclude at this point that the rest of the class engaged in integration collaboration as they attempted to interpret the presentation.

Furthermore, Mahlako explained that the class was confused by how Chris had carried out the generalisation. Their confusion was not on the algorithm rather on the approach used. For example, the class wanted to know where the '2a' ; '3a+b' which he was using for generalisation came from. Even though Chris could not explain where they came from, he did not stop the presentation. "Chris then said to them you (the teacher) will come and teach them these things" and carried on with the calculation, Mahlako reported. This was evidence that the students in the class were agentic since there were interactions during the presentation. This could have stopped the presentation altogether. However, since he acknowledged that he was not more knowledgeable than the rest of the class, which assisted him, he mentioned the teacher would come and clarify. Instead Fridah clarified the approach.

Fridah who was not part of the initiation engaged in modification collaboration with the whole class. She mentioned "I said since you are already using them; let me show them where they come from". This collaboration came in handy to clarify Chris's mathematical argument. Here Fridah once again shows achievement of mathematical agency. With further interrogation Fridah mentions that she learned where they came from as she was preparing for that day's lesson. No one knew she had prepared them, as a result she could have just kept quiet. However, her sense of agency allowed her to use her colleagues' confusion as an invitation for her to make a presentation.

From this analysis it can be concluded that mathematics learning in classrooms where agency was engendered do not depend more on the teacher. Students are able to carry on with work and strive for understanding. In case where consensus is not reached the issue is noted down for consultation with the teacher.

As I concluded the interviews, I then asked these students their preferred way of doing maths. That is, the teacher is explaining first then the students working in groups or working in their groups first, then whole class discussion. They said they preferred to first work on their own before engaging in a whole class discussion. Some of the reasons included the following:

> Chris: When you explain first, when you give us questions, we would like to do them exactly as you explained.

> Fridah: We are no longer doing mathematics, we are just doing your way. That is why we want to do it on our own so that we can apply our minds.

I reinterpret these quotes above to mean that if the teacher explains first then students are not agentic. As a result they do not feel they owned the learning. For Fridah following predetermined steps is not doing mathematics, which implies doing mathematics is being agentic. She sees mathematics learning as working in groups and applying one's mind.

4.3 SUMMARY

In this chapter I presented, analysed and interpreted data which was collected during students' small groups discussions without the teacher's intrusion and later with the teacher. I also analysed data which was collected during a whole class discussion and lastly data which was collected through a semi-structured interview. Throughout the analysis I discussed ways in which students were (or not) agentic as they engaged in various classroom interactions. In the next chapter

draw conclusions based on the discussion, answer research questions and give recommendations.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The previous chapter gave an analysis of the data generated through the various methods outlined in chapter 3. This chapter is a concluding section of this study; therefore the research question which the study sought to address is answered here. Then, the limitations of the study are outlined and finally recommended area(s) for further research in the field of agency in mathematics classrooms are given as well.

The research question which the study sought to address was: how is agency enacted through students interactions in a mathematics classroom? Since model 1 of chapter 2 outlined the different interactions in which students engage during their learning experience, this research question is therefore addressed under the different interactions as headings. However, focus will be on (a) Student-Material Interaction, (b) Student-Student Interaction and (c) Student-Teacher Interaction only. This is the case as whole class discussion is taken to occur as pioneered by students only or with the teacher. Furthermore, it should be noted that these interactions are not linear in nature and as such do not occur in the order in which they are discussed here.

5.2 CONCLUSION

5.2.1 Student-Material Interactions

When the students engaged with the learning material, agency was enacted in three ways. One, when the students needed to decide on whether or not to learn the new approach or continue with their old approach. Two, when the

students had to really make a choice without the teacher giving them an instruction on what to do with regard to the task they were faced with. For these first two, the students were engaged in the “dance of agency”. Three, when the students initiated group discussions based on the concepts in the material, this led to others going into the material as well, in order for them to engage meaningfully in the group discussions. Therefore, for student material engagement to foster enactment of mathematical agency, the material should incorporate different views of the same mathematical idea. The material should present open questions which will allow students to come up with their own schemas, as this will lead to initiation of student-student interactions.

5.2.2 Student – Student Interactions

Agency was enacted through student-student interactions when the students initiated group or whole class discussions, even though the initiatives differed in that some were for seeking clarity, while others were for offering explanations. Subsequently, the students engaged in modification and integration collaborations and others in co-construction collaborations. Thus, agency was enacted with the students having minimal agency and being principal agents interchangeably, and all of these initiations enhanced mathematics learning.

The interactions discussed here were possible because the teacher allowed the students to engage without his intrusion. This is not a call for the removal of the teacher from the classroom for the students to enact agency; however, it is a call for the teacher to play his or her roles differently. In this case the teacher’s ideas and voice were presented through the learning material which the students engaged in. This in turn allowed the students to think back and forth, and eventually took stance without checking with the teacher or being questioned by the teacher.

One other striking feature of student-student interaction is that, the students take accountability of their learning as they work through mathematical tasks. That is, they justify their choice of approach and their answers as well. Similarly, in whole class discussion students still argue on the basis of understanding concepts. It is during this stage of interaction that students become autonomous. Autonomy here refers to the opportunity and ability to make own choices without being controlled by anyone else. The students enacted agency when they showed autonomy in their learning. Some students moved to other groups to seek for help, to offer assistance or to work together. Agency is therefore enacted as students make and implement decisions during a learning experience.

5.2.3 Student-Teacher Interactions

From the excerpts captured in this study, student -teacher interactions were based on students' actions. During these interactions the teacher asked the students about their decisions and learning experiences. The students' enactment of agency during these interactions was that they were able to explain their actions and share their experiences. Furthermore, they were able to explain their decision making process, in such a way that one could see the decision was good for their learning. There were no student-teacher interactions captured purely based on a mathematical concept; this was the case as the classroom adopted a student-centred type of practice.

5.3 LIMITATIONS

This study was conducted in a classroom where student participation was a norm, though it was never viewed as enactment of agency. Similar studies can be conducted in classes where student participation is not a norm prior to entry into the study.

5.4 RECOMMENDATIONS

This study was conducted in a classroom where student participation was a norm, though it was never viewed as enactment of agency. Similar studies can be conducted in classes where student participation was not a norm prior to entry into the study. The study was conducted in one mathematics classroom, even though qualitative research does not seek generalisability, it will be interesting to see what results can be obtained with more classes and taught by different teachers. Through literature I learned that agency and identities are always related, a similar study which incorporates students' (re) formulation of mathematical identities can be conducted.

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ANNEXURES

ANNEXURE A: LETTER FOR REQUESTING PERMISSION TO CONDUCT RESEARCH

Enquiries
Mr. Lekwa L. Mokwana

P O Box 709
Glen Cowie
1061
16 May 2016

Limpopo Department of Education
Private Bag X9489
Polokwane
0700
Attention: Head of Department

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The matter above bears reference.
2. I am a Mathematics Education master's student at the University of Limpopo – Turfloop Campus, as part of the requirements for the fulfilment of the degree I need to conduct a research and produce a dissertation.
3. This letter serves to request for permission to conduct research at **Secondary School** in the **Circuit** of **Sekhukhune District**. The school has been purposively selected
4. The topic of the proposed research is **"Enactment of Mathematical Agency: A narrative analysis of classroom interactions."** Even though the research will be classroom based, my research agenda will not affect the day to day academic activities of the classroom and the school. Relevant research ethics will be adhered to.
5. Before I can commerce with the research the University's ethical committee will issue a certificate for ethical clearance of the proposed research. I have already applied for ethical clearance and received a conditional approval. Amongst others I need to submit a permission letter from the Department (**see attached**) in order to receive a full approval.
6. I will be glad if the department can grant me the permission to research.

Kind regards

Mokwana LL (Mr)

ANNEXURE B: PERMISSION LETTER FROM LIMPOPO DEPARTMENT OF EDUCATION

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

Ref: 2/5/6/1 Enq: MC Makola PhD Tel No: 015 290 9448 E-mail: MakolaMC@edu.limpopo.gov.za

Mokwana LL
P O Box 709
Glen Cowie
1061

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal; **"ENTACTMENT OF MATHEMATICAL AGENCY: A NARRATIVE ANALYSIS OF CLASSROOM INTERACTIONS."**
3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the schools concerned.

3.3 The conduct of research should not anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

Request for permission to Conduct Research: Mokwana LL

CONFIDENTIAL
Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa - development is about people!

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

23/05/2016

MUTHEIWANA NB
HEAD OF DEPARTMENT (ACTING)

DATE

ANNEXURE C: PERMISSION LETTER FROM THE SCHOOL

24 May 2016

Lekwa Mokwana
PO Box 709
Glen Cowie
1061

RE: Request for permission to conduct research

1. The matter above bears refers.
2. It is with pleasure to inform you that permission to conduct research has been granted as requested for the research titled: **"Enactment of Mathematics Agency: A narrative analysis of classroom interactions"**
3. Throughout your research period it is recommended that you ensure that your research does not interfere with the academic activities of the school.
4. You are once again reminded to adhere to all relevant ethical considerations issues as recommended by the Department of Education and your institution.

Kind regards.

DEPARTMENT OF EDUCATION
24 -05- 2016
LIMPOPO PROVINCE

24/05/2016
DATE

(Principal)

ANNEXURE D: RESEARCH ETHICAL CLEARANCE CERTIFICATE

University of Limpopo
Department of Research Administration and Development
Private Bag X1106, Sovenga, 0727, South Africa
Tel: (015) 268 2212, Fax: (015) 268 2306, Email:noko.monene@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE CLEARANCE CERTIFICATE

MEETING:	**05 July 2016**
PROJECT NUMBER:	**TREC/58/2016: PG**
PROJECT:	
Title:	Enactment of Mathematical Agency: A narrative analysis of Classroom interactions
Researcher:	Mr LL Mokwana
Supervisor:	Dr KM Chuene
Co-Supervisor:	N/A
Department:	Mathematics, Science and Technology Education
School:	Education
Degree:	Masters in Education

PROF TAB MASHEGO
CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE

The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: **REC-0310111-031**

Note:
i) **Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee.**
ii) **The budget for the research will be considered separately from the protocol.**
PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.

Finding solutions for Africa

ANNEXURE E: CONFIRMATION LETTER FROM LANGUAGE EDITOR

To	:	**Dr K. Chuene** Senior Lecturer Department of Maths, Science & Technology Education
From	:	**Dr L.D. Mkuti** Department of Languages
Date	:	
Subject	:	**EDITORIAL WORK FOR MR L.Z. MOKWANA RESEARCH DISSERTATION**

This letter serves as proof that I edited the research dissertation of **Mr L.Z. Mokwana** entitled "***ENACTMENTOF MATHEMATICAL AGENCY: A NARRATIVE ANALYSIS OF CLASSROOM INTERACTIONS***".

Kind regards

DR. L.D. MKUTI
DEPARTMENT OF LANGUAGES

Finding solutions for Africa

ANNEXURE F: LEARNING MATERIAL

mathcentre

Factorising quadratics

An essential skill in many applications is the ability to factorise quadratic expressions. In this unit you will see that this can be thought of as reversing the process used to 'remove' or 'multiply-out' brackets from an expression.

You will see a number of worked examples followed by a discussion of special cases which occur frequently with which you must become familiar.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that all this becomes second nature, and you eventually carry out the process of factorising simply by inspection. To help you to achieve this, the unit includes a substantial number of such exercises.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- factorise quadratic expressions
- understand what is meant by, and factorise, a 'difference of two squares'
- understand what is meant by, and factorise, a 'complete square'

Contents

1. Introduction

In this unit you will learn how many quadratic expressions can be factorised. Essentially, this is the reverse process of removing brackets from expressions such as $(x+2)(x+3)$.

2. Multiplying-out brackets, and quadratic expressions

You will be familiar already with the well-known process of multiplying-out brackets. For example, you will have seen expressions like $(x+2)(x+3)$ and then expanded these terms to arrive at a quadratic expression. Let us see how this works. The arrows in the figure below show how each term in the first pair of brackets multiplies each term in the second pair.

$$(x+2)(x+3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Then the like-terms, $3x$ and $2x$, are collected together to give $5x$. The result of multiplying-out the brackets is the **quadratic expression** $x^2 + 5x + 6$.

Key Point

A **quadratic expression** has the general form $ax^2 + bx + c$ where a, b and c are numbers. Note that in a quadratic expression the highest power of x is 2. The number a is called the **coefficient** of x^2, b is called the **coefficient** of x, and c is called the **constant term**. These numbers can be positive or negative. The numbers b and c can also be zero.

Some examples of quadratic expressions:

$$x^2 - 7x + 11, \qquad 4x^2 + 3x - 1, \qquad x^2 + 8x, \qquad 3x^2 + 2$$

Like many processes in mathematics, it is useful to be able to go the other way. That is, starting with the quadratic expression x^2+5x+6, can we carry out a process which will result in the form $(x+2)(x+3)$? The answer is: yes we can! This process is called **factorising the quadratic expression**. This would help, for example, if we wanted to solve a quadratic equation. Such an equation is formed when we set a quadratic expression equal to zero, as in

$$x^2 + 5x + 6 = 0$$

This is because the equation can then be written

$$(x+2)(x+3) = 0$$

and if we have two expressions multiplied together resulting in zero, then one or both of these must be zero. So, either $x+2=0$, or $x+3=0$, from which we can conclude that $x=-2$, or $x=-3$. We have found the solutions of the quadratic equation $x^2+5x+6=0$.

So, the ability to factorise a quadratic is a useful basic skill, which you will learn about in this unit. However, be warned, not all quadratics will factorise, but a lot do and so this is a process you have got to get to know!

Exercises 1

Prepare yourself for factorising quadratic expressions by multiplying out the brackets in each of the following cases:

a)	$(x+1)(x+2)$	b)	$(x+2)(x+3)$	c)	$(x+1)(x-3)$
d)	$(x+2)(x-4)$	e)	$(x-3)(x+7)$	f)	$x(x-7)$
g)	$(x-3)(x-2)$	h)	$(x-7)(x-7)$	i)	$(x-2)(x-2)$
j)	$(x+2)(x-2)$	k)	$(x-3)(x+3)$	l)	$(2x+1)(x+1)$
m)	$(x-5)(2x-1)$	n)	$(3x-1)(3x+1)$	o)	$4x(x-2)$

3. Factorising quadratics

To learn how to factorise let us study again the removal of brackets from $(x+3)(x+2)$.

$$(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

Clearly the number 6 in the final answer comes from *multiplying* the numbers 3 and 2 in the brackets. This is an important observation. The term $5x$ comes from *adding* the terms $2x$ and $3x$.

So, if we were to begin with $x^2 + 5x + 6$ and we were going to reverse the process we need to look for two numbers which multiply to give 6 and add to give 5.

$$\begin{aligned} ? \times ? &= 6 \\ ? + ? &= 5 \end{aligned}$$

What are these numbers ? Well, we know that they are 3 and 2, and you will learn with practise to find these simply by inspection.

Using the two numbers which add to give 5 we split the $5x$ term into $3x$ and $2x$. We can set the calculation out as follows.

$$\begin{aligned} x^2 + 5x + 6 &= x^2 + 3x + 2x + 6 \\ &= x(x+3) + 2x + 6 && \text{by factorising the first two terms} \\ &= x(x+3) + 2(x+3) && \text{by factorising the last two terms} \\ &= (x+3)(x+2) && \text{by noting the common factor of } x+3 \end{aligned}$$

The quadratic has been factorised. Note that you should never get these wrong, because the answer can always be checked by multiplying-out the brackets again!

Example

Suppose we want to factorise the quadratic expression $x^2 - 7x + 12$.

Starting as before we look for two numbers which multiply together to give 12 and add together to give -7. Think about this for a minute and you will realise that the two numbers we seek are -3 and -4 because

$$-4 \times -3 = 12, \qquad \text{and} \qquad -4 + -3 = -7$$

So, using the two numbers which add to give -7 we split the $-7x$ term into $-4x$ and $-3x$. We set the calculation out like this:

$$\begin{aligned} x^2 - 7x + 12 &= x^2 - 4x - 3x + 12 \\ &= x(x-4) - 3x + 12 && \text{by factorising the first two terms} \\ &= x(x-4) - 3(x-4) && \text{by factorising the last two terms extracting} \\ & && \quad \text{a factor of } -3 \text{ in order to leave } x-4 \\ &= (x-4)(x-3) && \text{by noting the common factor of } x-4 \end{aligned}$$

Once again, note that the answer can be checked by multiplying out the brackets again.

Example

Suppose we wish to factorise the quadratic expression $x^2 - 5x - 14$.

Starting as before we look for two numbers which multiply together to give -14 and add together to give -5. Think about this for a minute and you will realise that the two numbers we seek are -7 and 2 because

$$-7 \times 2 = -14, \qquad \text{and} \qquad -7 + 2 = -5$$

So, using the two numbers which add to give -5 we split the $-5x$ term into $-7x$ and $+2x$. We set the calculation out like this:

$$\begin{aligned} x^2 - 5x - 14 &= x^2 - 7x + 2x - 14 \\ &= x(x-7) + 2x - 14 && \text{by factorising the first two terms} \\ &= x(x-7) + 2(x-7) && \text{by factorising the last two terms} \\ &= (x-7)(x+2) && \text{by noting the common factor of } x-7 \end{aligned}$$

So the factorisation of $x^2 - 5x - 14$ is $(x-7)(x+2)$.

4. Factorising by inspection

Clearly, when you have some experience of this method, it is possible to avoid writing out all these stages. This is rather long-winded. What you need to do now is carry out the process **by inspection**. That is, simply by looking at the given expression, decide in your head which numbers need to be placed in the brackets to do the job. Look at the following two examples and see if you can do this.

Example

Suppose we want to factorise the quadratic expression $x^2 - 9x + 20$ by inspection.

We write

$$x^2 - 9x + 20 = (\quad)(\quad)$$

and try to place the correct quantities in brackets. Clearly we will need an x in both terms:

$$x^2 - 9x + 20 = (x \quad)(x \quad)$$

We want two numbers which multiply to give 20 and add to give -9. With practice you will be able to do this in your head. The two numbers are -4 and -5.

$$x^2 - 9x + 20 = (x-4)(x-5)$$

The answer should always be checked by multiplying-out the brackets again.

Example

Suppose we want to factorise the quadratic expression $x^2 - 9x - 22$ by inspection.

We write

$$x^2 - 9x - 22 = (\quad)(\quad)$$

and try to place the correct quantities in brackets. Clearly we will need an x in both terms:

$$x^2 - 9x - 22 = (x \quad)(x \quad)$$

We want two numbers which multiply to give -22 and add to give -9. The two numbers are -11 and $+2$.

$$x^2 - 9x - 22 = (x - 11)(x + 2)$$

The answer should always be checked by multiplying-out the brackets again.

If you can't manage to do these by inspection yet do not worry. Do it the way we did it before. Your ability to do this will improve with practice and experience.

All of these examples have involved quadratic expressions where the coefficient of x^2 was 1. When the coefficient is a number other than 1 the problem is more difficult. We will look at examples like this in the next section.

Exercises 2.

Factorise the following.

a)	$x^2 + 8x + 15$	b)	$x^2 + 10x + 24$	c)	$x^2 + 9x + 8$	d)	$x^2 + 15x + 36$
e)	$x^2 + 2x - 3$	f)	$x^2 + 2x - 8$	g)	$x^2 - 2x - 3$	h)	$x^2 + 7x - 18$
i)	$x^2 - 12x + 35$	j)	$x^2 - 11x + 10$	k)	$x^2 - 13x + 22$	l)	$x^2 + 12x + 27$

5. Expressions where the coefficient of x^2 is not 1.

Suppose we wish to factorise the expression $3x^2 + 5x - 2$.

As we did before we look for two numbers which add to give the coefficient of x; so we seek two numbers which add to give 5.

However, instead of looking for two numbers which multiply to give -2 we must look for two numbers which multiply to give -6, (that is, the coefficient of x^2 multiplied by the constant term, 3×-2). This is entirely consistent with the method we applied before because in those examples the coefficient of x^2 was always 1.

So

$$? \times ? = -6 \qquad ? + ? = 5$$

By inspection, or trial and error, two such numbers are 6 and -1.

$$6 \times -1 = -6 \qquad 6 + -1 = 5$$

We use these two numbers to split the $5x$ term into $6x$ and $-1x$.

$$\begin{aligned} 3x^2 + 5x - 2 &= 3x^2 + 6x - x - 2 \\ &= 3x(x + 2) - x - 2 && \text{by factorising the first two terms} \\ &= 3x(x + 2) - (x + 2) && \text{factorising the last two terms by extracting } -1 \\ &= (x + 2)(3x - 1) && \text{by noting the common factor of } x + 2 \end{aligned}$$

Example

Suppose we wish to factorise $2x^2 + 5x - 7$.

As we did before we look for two numbers which add to give the coefficient of x; so we seek two numbers which add to give 5.

We look for two numbers which multiply to give -14, (that is, the coefficient of x^2 multiplied by the constant term, 2×-7).

So

$$? \quad \times \quad ? = -14 \qquad ? \quad + \quad ? = 5$$

By inspection, or trial and error, two such numbers are 7 and -2.

$$7 \times -2 = -14 \qquad 7 + -2 = 5$$

We use these two numbers to split the $5x$ term into $7x$ and $-2x$.

$$\begin{array}{rcll} 2x^2 + 5x - 7 & = & 2x^2 + 7x - 2x - 7 & \\ & = & x(2x + 7) - 2x - 7 & \text{by factorising the first two terms} \\ & = & x(2x + 7) - (2x + 7) & \text{factorising the last two terms by extracting } -1 \\ & = & (2x + 7)(x - 1) & \text{by noting the common factor of } 2x + 7 \end{array}$$

Example

Suppose we wish to factorise $6x^2 - 5x - 4$.
We look for two numbers which multiply to give -24, (that is, the coefficient of x^2 multiplied by the constant term, 6×-4).

We look for two numbers which add to give the coefficient of x; so we seek two numbers which add to give -5.

So

$$? \quad \times \quad ? = -24 \qquad ? \quad + \quad ? = -5$$

By inspection, or trial and error, two such numbers are -8 and 3.

$$3 \times -8 = -24 \qquad 3 + -8 = -5$$

We use these two numbers to split the $-5x$ term into $3x$ and $-8x$.

$$\begin{array}{rcll} 6x^2 - 5x - 4 & = & 6x^2 + 3x - 8x - 4 & \\ & = & 3x(2x + 1) - 8x - 4 & \text{by factorising the first two terms} \\ & = & 3x(2x + 1) - 4(2x + 1) & \text{factorising the last two terms by extracting } -4 \\ & = & (2x + 1)(3x - 4) & \text{by noting the common factor of } 2x + 1 \end{array}$$

Example

Suppose we wish to factorise $15x^2 - 3x - 18$.

In this example note that the coefficients share a common factor of 3. This can be extracted:

$$3(5x^2 - x - 6)$$

Now we are looking at factorising $5x^2 - x - 6$.

We look for two numbers which multiply to give -30, (that is, the coefficient of x^2 multiplied by the constant term, 5×-6).

We look for two numbers which add to give the coefficient of x; so we seek two numbers which add to give -1.

So

$$? \times ? = -30 \qquad ? + ? = -1$$

By inspection, or trial and error, two such numbers are 5 and -6.

$$5 \times -6 = -30 \qquad 5 + -6 = -1$$

We use these two numbers to split the $-x$ term into $5x$ and $-6x$.

$$\begin{aligned} 3(5x^2 - x - 6) &= 3(5x^2 + 5x - 6x - 6) \\ &= 3\left(5x(x+1) - 6x - 6\right) && \text{by factorising the first two terms} \\ &= 3\left(5x(x+1) - 6(x+1)\right) && \text{factorising the last two terms by extracting } -6 \\ &= 3\left((x+1)(5x-6)\right) && \text{by noting the common factor of } x+1 \end{aligned}$$

Exercises 3

Factorise the following.

a) $2x^2 + 11x + 5$ b) $3x^2 + 19x + 6$ c) $3x^2 + 17x - 6$ d) $6x^2 + 7x + 2$
e) $7x^2 - 6x - 1$ f) $12x^2 + 7x + 1$ g) $8x^2 + 6x + 1$ h) $8x^2 - 6x + 1$

6. Special case 1 - the difference of two squares

A special case concerns what is known as **the difference of two squares**. What does this look like ? Typically, $x^2 - 9$. Note that there is no x term and that the number 9 is itself a square number. A square number is one which has resulted from squaring another number. In this case 9 is the result of squaring 3, $(3^2 = 9)$, and so 9 is a square number.

Hence $x^2 - 9$ is the difference of two squares, $x^2 - 3^2$.

When we try to factorise $x^2 - 9$ we are looking for two numbers which add to zero (because there is no term in x), and which multiply to give -9. Two such numbers are -3 and 3 because

$$-3 \times 3 = -9, \qquad \text{and} \qquad -3 + 3 = 0$$

We use these two numbers to split the $0x$ term into $-3x$ and $3x$.

$$\begin{aligned} x^2 - 9 &= x^2 - 3x + 3x - 9 \\ &= x(x-3) + 3x - 9 && \text{by factorising the first two terms} \\ &= x(x-3) + 3(x-3) && \text{factorising the last two terms by extracting } +3 \\ &= (x-3)(x+3) && \text{by noting the common factor of } x-3 \end{aligned}$$

So

$$x^2 - 9 = (x-3)(x+3)$$

Recall that 9 is a square number, $(9 = 3^2)$. So this example is a difference of two square. The result of this example is true in more general cases of the difference of two squares.

It is always the case that $x^2 - a^2$ factorises to $(x-a)(x+a)$.

Key Point

The difference of two squares, $x^2 - a^2$, always factorises to

$$x^2 - a^2 = (x - a)(x + a)$$

Example

Suppose we want to factorise $x^2 - 25$.

Note that $x^2 - 25$ is the difference of two squares because 25 is a square number ($25 = 5^2$). So we need to factorise $x^2 - 5^2$. We can do this by inspection using the formula above.

$$x^2 - 5^2 = (x - 5)(x + 5)$$

Example

Suppose we want to factorise $2x^2 - 32$.

This time there is a common factor of 2 which can be extracted first.

$$2x^2 - 32 = 2(x^2 - 16) = 2(x - 4)(x + 4)$$

Example

A slightly different form occurs if we now include a square number in front of the x^2 term. For example, suppose we wish to factorise $9x^2 - 16$. Note that 9 is a square number, and so the term $9x^2$ can be written $(3x)^2$. So we still have a difference of two squares

$$(3x)^2 - 4^2$$

To factorise this we write

$$9x^2 - 16 = (3x - 4)(3x + 4)$$

Exercises 4

Factorise the following.

a) $x^2 - 144$ b) $x^2 - 36$ c) $x^2 - 1$ d) $x^2 - 121$
e) $9x^2 - 1$ f) $16x^2 - 9$ g) $49x^2 - 1$ h) $25x^2 - 16$

7. Special case 2 - complete squares

A second important special case is when we need to deal with complete squares. This happens when the answer can be written in the form $(\quad)^2$, that is as a single term, squared.

Consider the following example.

Example

Suppose we wish to factorise $x^2 + 10x + 25$.

As before we want two numbers which multiply to give 25 and add to give 10. Two such numbers are 5 and 5 because

$$5 \times 5 = 25, \qquad \text{and} \qquad 5 + 5 = 10$$

We use these two numbers to split the $10x$ term into $5x$ and $5x$.

$$\begin{array}{rcll} x^2 + 10x + 25 & = & x^2 + 5x + 5x + 25 & \\ & = & x(x+5) + 5x + 25 & \text{by factorising the first two terms} \\ & = & x(x+5) + 5(x+5) & \text{factorising the last two terms by extracting } +5 \\ & = & (x+5)(x+5) & \text{by noting the common factor of } x+5 \end{array}$$

The result can be written as $(x+5)^2$, a **complete square**.

Example

Suppose we want to factorise $x^2 - 8x + 16$.

We seek two numbers which add to give -8 and multiply to give 16. The two required numbers are -4 and -4 and so, by inspection,

$$x^2 - 8x + 16 = (x-4)(x-4)$$

The result can be written as $(x-4)^2$, a **complete square**.

More complicated examples can occur, for example when there is a number in front of the x^2. Work through the following example.

Example

Suppose we want to factorise $25x^2 - 20x + 4$.
Note that $25x^2$ can be written as $(5x)^2$, a squared term. Note also that $4 = 2^2$. In this case, by inspection,

$$25x^2 - 20x + 4 = (5x-2)(5x-2)$$

The result can be written as $(5x-2)^2$, a **complete square**.

Do not worry if you have difficulty with this last example. The skill will come with practice. Even if you did not recognise the complete square, you could still use the previous method:

To factorise $25x^2 - 20x + 4$

As before we want two numbers which multiply to give 100 (the coefficient of x^2 multiplied by the constant term) and add to give -20. Two such numbers are -10 and -10 because

$$-10 \times -10 = 100, \qquad \text{and} \qquad -10 + -10 = -20$$

We use these two numbers to split the $-20x$ term into $-10x$ and $-10x$.

$$\begin{array}{rcll} 25x^2 - 20x + 4 & = & 25x^2 - 10x - 10x + 4 & \\ & = & 5x(5x-2) - 10x + 4 & \text{by factorising the first two terms} \\ & = & 5x(5x-2) - 2(5x-2) & \text{factorising the last two terms by extracting } -2 \\ & = & (5x-2)(5x-2) & \text{by noting the common factor of } 5x-2 \end{array}$$

So $25x^2 - 20x + 4 = (5x-2)^2$ as before.

8. Special case 3 - the constant term is missing

There is one more special case that you ought to be familiar with. This arises in examples where the constant term is absent, as in $3x^2 - 8x$. How do we proceed ?

The way forward is to look for common factors. In the case of $3x^2 - 8x$ note that each of the two terms has a common factor of x. (Note that this is not the case when dealing with a quadratic expression containing a constant term).

The common factor, x, is written outside a bracket, and the contents of the bracket are chosen by inspection so that they multiply-out to give the correct terms.

$$3x^2 - 8x = x(3x - 8)$$

Example

Suppose we wish to factorise $10x^2 + 5x$.

This time we note the common factor of $5x$. This is extracted and we adjust the contents of the brackets accordingly:

$$10x^2 + 5x = 5x(2x + 1)$$

Exercises 5.
Factorise the following.

a) $x^2 + 18x + 81$ b) $x^2 - 4x + 4$ c) $x^2 - 22x + 121$ d) $25x^2 + 40x + 16$
e) $64x^2 + 16x + 1$ f) $x^2 + 6x$ g) $3x^2 - x$ h) $5x^2 - 20x$

Answers

Exercises 1

a) $x^2 + 3x + 2$ b) $x^2 + 5x + 6$ c) $x^2 - 2x - 3$ d) $x^2 - 2x - 8$
e) $x^2 + 4x - 21$ f) $x^2 - 7x$ g) $x^2 - 5x + 6$ h) $x^2 - 14x + 49$
i) $x^2 - 4x + 4$ j) $x^2 - 4$ k) $x^2 - 9$ l) $2x^2 + 3x + 1$
m) $2x^2 - 11x + 5$ n) $9x^2 - 1$ o) $4x^2 - 8x$.

Exercises 2

a) $(x + 3)(x + 5)$ b) $(x + 4)(x + 6)$ c) $(x + 1)(x + 8)$ d) $(x + 3)(x + 12)$
e) $(x + 3)(x - 1)$ f) $(x + 4)(x - 2)$ g) $(x - 3)(x + 1)$ h) $(x + 9)(x - 2)$
i) $(x - 7)(x - 5)$ j) $(x - 1)(x - 10)$ k) $(x - 11)(x - 2)$ l) $(x + 9)(x + 3)$

Exercises 3

a) $(2x + 1)(x + 5)$ b) $(3x + 1)(x + 6)$ c) $(3x - 1)(x + 6)$ d) $(2x + 1)(3x + 2)$
e) $(7x + 1)(x - 1)$ f) $(3x + 1)(4x + 1)$ g) $(2x + 1)(4x + 1)$ h) $(2x - 1)(4x - 1)$

Exercises 4

a) $(x - 12)(x + 12)$ b) $(x - 6)(x + 6)$ c) $(x - 1)(x + 1)$ d) $(x - 11)(x + 11)$
e) $(3x + 1)(3x - 1)$ f) $(4x + 3)(4x - 3)$ g) $(7x + 1)(7x - 1)$ h) $(5x + 4)(5x - 4)$

Exercises 5

a) $(x + 9)^2$ b) $(x - 2)^2$ c) $(x - 11)^2$ d) $(5x + 4)^2$
e) $(8x + 1)^2$ f) $x(x + 6)$ g) $x(3x - 1)$ h) $5x(x - 4)$