THE ENACTMENT OF ASSESSMENT FOR LEARNING TO ACCOUNT FOR LEARNERS' MATHEMATICAL UNDERSTANDING

By

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DISSERTATION

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DEDICATION

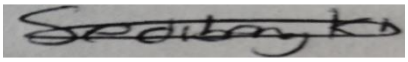
I dedicate this dissertation to the following people:

My daughter, Ofentse Sedibeng, your presence in my life is motivation in and of itself.

My family - Mom, dad and siblings may this be a certitude that we serve a living God.

DECLARATION

I, Sedibeng Khutso Makhalangaka, declare **THE ENACTMENT OF THE FIVE KEY STRATEGIES OF ASSESSMENT FOR LEARNING TO ACCOUNT FOR LEARNERS' MATHEMATICAL UNDERSTANDING** dissertation hereby submitted to the University of Limpopo, for the degree of Master of Education in Mathematics Education has not previously been submitted by me for a degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.



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ABSTRACT

The purpose of this study was to document my enactment of the five key strategies of assessment for learning in my mathematics classroom to account for learners' mathematical understanding. I used a constructivism teaching experiment methodology to explore learners' mathematical activities as they interacted in the classroom. Twenty-five learners from my Grade 10 mathematics class took part in the study. Data were gathered through classroom observations, written work samples from learners, and the teacher's reflective journal. My enactment of the five key strategies enabled learners to participate in classroom discussions, collaborate with their peers, and use self-assessment tools while engaging in classroom interactions. The major findings revealed that, through my enactment of the five key strategies, learners developed conceptual understanding, procedural fluency and strategic competence of the concepts taught. In addition, practices such as the development of lesson plans detailing how the five key strategies will be enacted in the classroom, use of comment – only feedback for grading learners' work, creating a conducive learning environment to allow the use of peer and self-assessment allowed for a meaningful enactment of assessment for learning in my classroom. Strategies four and five, whose primary goal is to encourage learners' participation in the lesson, were critical in promoting learners' mathematical understanding.

KEY CONCEPTS

Enactment, Assessment for learning, Account, Learners' mathematical understanding

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CHAPTER 1: INTRODUCTION AND BACKGROUND TO THE STUDY

1.1 INTRODUCTION

This dissertation presents the findings of a qualitative research study conducted based on the enactment of assessment for learning in a mathematics classroom to account for learners' mathematical understanding. It focuses on how the five key strategies of assessment for learning were enacted during the teaching and learning of mathematics, as well as how the teacher (the researcher in this study) accounted for the learners' mathematical understanding throughout the process. In this introductory chapter, this section discusses the motivation of the study as well as an overview of the dissertation. It begins by presenting the background and motivation of study, key concepts, statement of the research problem, purpose and research questions. The section also provides a brief overview of the research methodology, ethical considerations, the research setting, and the significance of the study.

1.2 KEY CONCEPTS

Enactment: An instance of acting out or putting something into action or enactment.

Assessment for learning: A planned process that includes teaching strategies to promote learning by inviting learners to participate and take ownership of the process.

Account: Report or description of an experience.

Mathematical understanding: The mathematical understanding that was accounted for in this study was conceptual understanding of concepts in mathematics, along with strategic competence and procedural fluency (Kilpatrick, Swafford & Findell, 2001).

CAPS: Curriculum and Assessment Policy Statement. Further education and training refers to the education training that is provided from Grade 10 to Grade 12. According to Education's (2011) CAPS for FET mathematics, CAPS is a single, comprehensive and concise policy document introduced by the Department of Basic Education and was developed for the subjects listed in the National Curriculum Statement.

FET: Further Education Training (Grade 10-12). Further education and training refers to the education training that is provided from Grade 10 to Grade 12.

Functions: Mathematics topic from FET mathematics CAPS document.

1.3 BACKGROUND AND MOTIVATION

Learning is a process in which learners construct knowledge by scrutinising the world around them and by forming associations between new concepts and prior understandings (Vygotsky, 1986). According to Minarni, Napitupulu and Husein (2016), the main goal of teaching and learning mathematics in high school is for learners to understand mathematical concepts, describe the relationships between concepts, and know how to apply the concepts efficiently in solving mathematical activities. One of the major challenges I have encountered as a mathematics teacher in Limpopo Province for over seven years is that learners fail to understand and make connections between mathematical concepts.

Korn (2014) mentioned that conceptual understanding of mathematics provides learners with a more holistic education in the sense that learners do not only know the procedures involved in solving mathematics activities, but they are also able to make connections between the concepts. Developing mathematical understanding is critical because "a student who possesses only procedural knowledge of mathematics is not as valuable as a student who possesses a conceptual understanding of mathematics" (Korn, 2014, p.5). Mwakapenda (2004) concurs with the author above by mentioning that a learner who understands mathematics conceptually knows more than isolated facts and methods. As a result, the conceptual understanding of mathematics will be the mathematical understanding accounted for in this study along with strategic competence and procedural fluency (Kilpatrick, Swafford & Findell, 2001).

Assessment is one of the tools that can help learners advance their mathematical understanding by identifying their difficulties in the subject. According to Moyosore

(2015), assessment is the use of various strategies to gather information about teaching and learning. They go on to say that assessment can be formative or summative, with the former being more commonly known as assessment for learning. Black and William (1998) observed that if teachers incorporate assessment for learning into their daily lessons as part of their teaching, learners can learn at roughly twice the rate. Assessment for learning, according to Leary (2013), is an assessment that is specifically designed to facilitate learning. Black and William (2009) contend that assessment for learning is the process that entails seeking and interpreting evidence to be used by teachers and learners to ascertain how far the former are in their learning, where they need to go and how best to get there. Furthermore, "assessment for learning is the process of seeking and interpreting evidence for use by learners and teachers in determining where learners are in their learning, where they need to go, and how best to get there" (Black & William, 2009, p.4). According to Naihsina and Jessica (2018), assessment for learning is noticeable in classroom interactions between the teacher and learners as well as among learners themselves.

The following are five key strategies in assessment for learning (Black & William, 2018, p.10):

1. Clarifying, understanding and sharing learning intentions.
2. Engineering effective classroom discussion, tasks and activities that elicit evidence of learning.
3. Providing feedback that moves learners forward.
4. Activating learners as resources for one another.
5. Activating learners as owners of their own learning.

In this study, I defined assessment for learning as a planned process that included teaching strategies to promote learning by inviting learners' participation and empowering them to take ownership of their own learning. Researchers such as Carless (2011) and Pham (2011) recognised that assessment could help promote learners' learning; but they also noted that there were numerous questions about how to effectively apply assessment for learning. One of the questions concerns the credibility of the questions and methods employed by teachers in the enactment of assessment for learning in their classrooms. According to Black and William (1998), teachers rarely consult their colleagues or seniors to see if their methods and questions are assessing what they were intended for. They also assert that assessment tasks administered by teachers with the goal of enacting assessment for learning result in rote and superficial learning. As a result, the majority of teachers are unaware that they are encouraging rote and superficial learning rather than developing learners' understanding.

The quality of teachers' enactment of assessment for learning in their classrooms influences learners' learning (Johnson, Sondergeld & Walto, 2019). They go on to say that the research base is limited in terms of how teachers should enact the five key strategies of assessment for learning in their classrooms. As a result, the purpose of this research was to use the teaching experiment methodology to document my enactment of the five key strategies of assessment for learning in my classroom to account for learners' mathematical understanding. With greater emphasis on strategies four and five, activating learners as resources for one another and activating learners as owners of their own learning, whose main goal was to invite learners' participation in their learning.

1.4 STATEMENT OF THE RESEARCH PROBLEM

The importance of assessment for learning in gathering detailed information for improving instruction and learning as it occurs cannot be overstated (Phelan, Choi, Vendlinski, Baker & Herman 2011; Ho, 2015). According to Moyosore (2015), assessment for learning is critical for both learners and teachers. It aids in diagnosing learners' learning challenges and employing alternative remedial methods to improve learning the subject. He adds that assessment for learning helps teachers to identify the specific challenges that learners face in specific concepts of the subject and to predict the appropriate strategies to help learners learn with understanding.

According to Black and William (1998), who observed that sound recommendations from research findings are often not considered by teachers if they are conveyed as general concepts, enacting assessment for learning may be difficult for teachers. Saddler (1998) emphasises this point by observing that the role of assessment for learning is often inadequately conceptualised by teachers in research studies. Black and William (2018) concur that intentions, actions, plans, classroom dialogue and methods for enacting the key strategies of assessment for learning by teachers in classrooms are some of the recurring factors that are missing or poorly explained in the literature.

Unfortunately, without the enactment of assessment for learning in mathematics classes, teachers are deprived of the opportunity to provide feedback. Saddler (1998) suggests that the provision of feedback may aid teachers to remediate, reinforce, motivate and hopefully obtain high achievement. Complicating matters is the fact that most teachers find it difficult to incorporate assessment for learning into their daily lessons because they fail to thoroughly conceptualise assessment for learning before beginning to enact it in the classroom (Harrison, 2013). It is then necessary for them to conduct a classroom study in order to improve their practice as a result of their findings. As a result, in the reported study, I attempted to document my enactment of the five key strategies of assessment for learning in my classroom to account for learners' mathematical understanding.

1.5 PURPOSE OF THE STUDY AND RESEARCH QUESTIONS

The purpose of this study was to document my enactment of the five key strategies of assessment for learning in my mathematics classroom in order to account for learners' mathematical understanding.

1.5.1. Research Questions

The main question followed by the sub-questions are stated below.

- How can I enact the five key strategies of assessment for learning to account for learners’ mathematical understanding?
- What are teaching strategies that allow for meaningful enactment of assessment for learning?
- What mathematical understanding is accounted for during the enactment of assessment for learning?

1.6 RESEARCH METHODOLOGY

In this section, I present the research methodology that I used in this study, which is discussed in detail in Chapter 3. Since the purpose was to gain a better understanding of the phenomenon under investigation, I used the qualitative research approach (Creswell, 2009) because it would allow me to investigate and comprehend the impact of some educational interventions (Mills, 2003). The approach was appropriate for my research because I used the five key strategies of assessment for learning in my mathematics classroom to account for learners' mathematical understanding, allowing me to investigate what learners were saying or doing while engaging in mathematical activities (Steffe & Thompson, 2000).

1.6.1. Research Design

As an appropriate research design, I used the constructivist teaching experiment methodology. According to Steffe and Ulrich (2014), the constructivist teaching experiment allows the researcher to participate in the teaching process as a teacher/researcher while carrying out the planned teaching episodes. On the same issue, Özdemir (2017) asserts that the constructivist teaching experiment methodology allows teacher-researchers to assess learners' progress through mathematical communication. They further indicate that this type of research design allows teacher-researchers to integrate teaching with research, theory, and practice. I used classroom interactions to organise and guide participants in experiencing the impact of enacting the five key strategies of assessment for learning on learners' mathematical understanding (Steffe & Thompson, 2000). In order to account for learners' mathematical understanding by exploring and explaining their mathematical activities as they engaged in classroom interactions (Muthelo, 2010; Mabotja, 2017).The teaching experiment methodology provided me with the opportunity to enact the five key strategies of assessment for learning in my mathematics classroom.

1.6.2. Sampling

Participants for this study came from my Grade 10 mathematics class, which had 25 learners. All 25 learners were exposed to the exploratory teaching methodology used in the teaching experiment methodology. According to Mabotja (2017), exposing all of the learners in the class to exploratory teaching helps to ensure that they are not disadvantaged by selection bias. I reported data based on daily classroom interactions and learners' activities samples. The reported classroom interactions and learners' samples supported the purpose of documenting the enactment of assessment for learning to account for learners' mathematical understanding. Furthermore, my Grade 10 mathematics classroom was convenient for me because it was situated in the school that I am currently attached to in Limpopo Province. I used convenient sampling to sample learners from my Grade 10 mathematics classroom because of their easy accessibility and willingness to participate in the study and their availability at any given time. Their geographical proximity simplified my work (Etikan, Musa & Alkassim, 2016).

1.6.3. Data Collection

I used classroom observations captured on video, learners' written work samples and the teacher's reflective journal (teacher's note-taking) as data collection methods. According to Steffe and Thompson (2000), one of the primary goals of the teaching experiment methodology is for researchers to focus on what learners are doing or saying in order to better understand their mathematical learning as they engage in mathematical activities. The data collection methods I used were consistent with the research design, and allowed me to investigate learners' mathematical learning and to analyse their understanding of mathematical concepts throughout the teaching process.

1.6.4. Data Analysis

The first stage of data analysis entailed selecting information-rich interactions from the entire classroom interactions where enactment of assessment for learning was observed. The selection was shaped by William and Thompson's (2007) framework of the five key strategies of assessment for learning, which serves as the theoretical framework of this study.

In the second stage, I translated and interpreted the information-rich interactions from classroom discussions that had been transcribed. Kilpatrick et al. (2001) guided the interpretations with strands of mathematical proficiency. As I enacted the five key strategies of assessment for learning, my interpretations of the transcribed information-rich interactions focused on the three strands of mathematical understanding, which are conceptual understanding, procedural fluency and learners' strategic competence in mathematics.

Learners' written activities work samples were analysed in the third stage according to the strands of mathematical proficiencies identified by Kilpatrick et al. (2001): conceptual understanding, procedural fluency and strategic competence.

1.6.5. Quality Criteria

a. Credibility

According to Anney (2014), credibility is when the researcher can confidently declare that the findings of study were true and represented information gathered from participants' original data. According to Davies (2011), triangulation occurs when a researcher employs more than one method of conducting research in order to obtain richer data. To ensure credibility, I used data triangulation in this study.

b. Triangulation

According to Noble and Heale (2019), triangulation is the use of multiple approaches to answer a research question in order to increase the credibility of the research findings. Triangulation plays an important role in the teaching experiment methodology (Castro, Castro & Molina, 2007). Hence, I gathered data from various sources to answer the research question. I collected data through a video recorder to record classroom interactions. I also used learners' written activities from their classwork books, and a teacher's reflective journal (teacher's note-taking) (Steffe & Thompson, 2000).

c. Transferability

When findings can be applied in contexts other than the study situation, such as different contexts or other groups, they are said to be transferable (Polit & Beck, 2012; White, 2005). The transferability criterion, on the other hand, is dependent on the intention of the qualitative study and may be appropriate only if the researcher wants to generalise the findings. As previously stated, the intention of this study was not to generalise the findings, so this criterion did not apply (Cope, 2014).

d. Confirmability

Confirmability ensured that the responses reflected participants' experiences rather than the researcher's preferences, which were influenced by the researcher's biases. Conformability was achieved in this study by ensuring that findings were derived directly from the video data by using participants' direct quotes and raw data from learners' written work (Cope, 2014).

1.7 ETHICAL CONSIDERATIONS

According to Nkosi (2014), ethical considerations are critical components of all research studies, and researchers must ensure that they are followed. First and foremost, I sought permission from the University of Limpopo's Turfloop Research Ethics Committee (TREC) to conduct my research. In addition, I requested approval from the Limpopo Department of Education by sending request letters (attached in Appendix C) to the Limpopo head office and the school principal. To avoid being dishonest about the purpose of my study, I also submitted the outline of my study to the aforementioned authorities. I also requested permission from parents of participants since they were minors. To ensure confidentiality and anonymity, I used pseudonyms to protect the identities of participants and the research site.

Furthermore, I explained the purpose of the study to participants. Machaba (2013) states that it is the researcher's responsibility to explain the study to participants and what is required in terms of participation. I also made certain that I distributed written consent forms to each participant, assuring them that their participation was voluntary and that they could opt out at any time. Participants' parents were given informed consent forms (attached in Appendix E), which guaranteed participants' rights and permission to take part in the study. By signing the informed consent forms, participants indicated that they understood what was explained to them, that they were willingly participating in the study, and that their parents had allowed them (the minors) to be part of the study. Furthermore, I guaranteed complete confidentiality and anonymity.

1.8 SIGNIFICANCE OF PROPOSED RESEARCH

The majority of assessment for learning research has been conducted to advocate the importance and benefits of using assessment for learning in the classroom. This study contributed to the scarcity of information about the enactment of the five key strategies of assessment for learning in mathematics classrooms. The findings of the study could motivate teachers to learn how to enact the five key strategies of assessment for learning in their mathematics classrooms in order to improve learners' mathematical understanding. Furthermore, when assessment for learning is used effectively in teachers' classroom practices, learners learn optimally and their learning outcomes are significantly impacted (Johnson, Sondergeld & Walto, 2019). The findings of the study could significantly contribute to the body of knowledge in the enactment of assessment for learning, and the findings could aid the Department of Basic Education in their quest to improve mathematics performance nationally.

1.9 RESEARCH SETTING

To adhere to the ethics of this study, I used pseudonyms rather than learners' real names. According to Mabotja (2017), the purpose of using pseudonyms is for anonymity as well as school confidentiality. After being granted permission to conduct the study by the Limpopo Department of Education, I submitted a letter of permission to the principal of the school to which I am attached. The principal granted me permission to conduct the study at the school.

The school is located in one of Ga-Shongoane villages about 89 kilometers from Lephalale town in Limpopo Province. The school has 300 learners who speak Sepedi as their first language. The learners at the school come from the village where the school is located as well as neighbouring villages. The school specialises in physical science and agriculture and has two Grade 10 classes - categorised as A and B, the first of which has 25 learners studying mathematics and physical science, and the second has 25 learners studying mathematical literacy and agricultural science. The teaching experiments were carried out in Grade 10A because it was the only class that did mathematics, and therefore, relevant to the study.

1.10 OUTLINE OF THE STUDY

This dissertation is divided into five chapters, which are outlined below. The first chapter provides an overview of the context of the study. It includes the purpose of the study and the research questions, as well as a brief introduction to methodology and data analysis. The second chapter contains a review of the literature as well as a theoretical framework pertinent to the study. The methodology of the study is outlined in chapter three. The chapter discusses the rationale for selecting the qualitative research paradigm, the research design of the teaching experiment methodology, data collection and analysis, and rigour issues in teaching experiments. Ethical consideration issues were also discussed and documented. The fourth chapter includes data-driven analyses of classroom interactions between learners and the teacher-researcher. The final chapter, Chapter 5, discusses the findings in relation to the research questions that guided the study. The final chapter also includes recommendations and limitations of the study.

1.11 CHAPTER SUMMARY

This chapter served as a synopsis of the dissertation. I began by providing the background of the study based on my experience as a mathematics teacher, highlighting some of the previous researches on assessment in mathematics, mathematical understanding, and how these prompted me to conduct this study. Furthermore, the research questions, purpose of the study, overview of the research methodology, significance of the study, ethical considerations, introduction of key concepts, and research setting were all documented. Finally, the chapter provided the structure of the dissertation.

CHAPTER 2: LITERATURE REVIEW

2.1. INTRODUCTION

The previous chapter focused on the introduction of the study, including the outline of the dissertation. This chapter reviews literature around the enactment of assessment for learning. The theoretical framework adopted in this study as proposed by William and Thompson (2007) of the five key strategies of assessment for learning was described. The literature review was organised around the following themes: assessment in education, the difference between assessment of learning and assessment for learning, the benefits of assessment for learning in mathematical understanding, studies on enactment of assessment for learning, and challenges in the enactment of assessment for learning in mathematics. The section concludes with a presentation of mathematical understanding by various authors.

2.2. ASSESSMENT IN EDUCATION

According to Shepard (2000), assessment is one of the most important educational tools that is used for a variety of purposes. Assessment, among other things, is used to motivate learners to learn more effectively and to improve their performance. Over time, assessment has aided teachers in evaluating the performance of their learners through informal and formal assessment tasks such as assignments, quizzes, tests, and examinations. Yambi (2018) further states that assessment is used to recognise learners' strengths and weaknesses for educators to administer specific educational or social support. According to Onyiloye and Imenda (2019), assessment forms a pivotal part of teaching and learning. It is also one of the most important aspects of schooling. They further state that assessment consists of all the activities that can be used by both teachers and learners to gather information that can adjust teaching and learning. According to Ramsden (1992), "Assessment is a way of teaching more effectively by understanding exactly what learners know and do not know" (p. 182). As a result, assessment should not only serve to evaluate learners' content knowledge when they are required to apply or reconstruct, but also to assist lecturers in modifying their teaching in order to target specific challenges that impede learners' learning

(Nkealah, 2019). As a result, assessment is divided into two types: assessment for learning and assessment of learning (Black & William, 2009; Bennett, 2011).

According to Hargreaves (2001), assessment of learning is the evaluation of what or how much knowledge the learner has acquired for summative purposes. They go on to say that this type of assessment is frequently used to keep or promote a learner to the next grade for the benefit of stakeholders outside the learner's classroom, such as the school, parents or school authorities. On the same issue, Gouws (2013) agrees that assessment of learning includes various methods of determining whether learners have achieved the intended learning outcomes of instruction. It is also concerned with the number of assessment tasks specified in the curriculum and assessment documents. According to Gouws (2013), assessment of learning provides a summary judgement of learning and understanding but does not allow for corrective actions to eliminate potential learning barriers. Feedback on learning assessment is delayed in the sense that it arrives at the end of the year or term. Feedback does not assist learners in correcting their mistakes; instead, it takes the form of symbols or marks that provide learners with only a bare minimum of direction in terms of strengths and weaknesses (Chappuis & Stiggins, 2002; Bennett, 2011; Black & William, 1998).

In assessment for learning, on the other hand, the teacher gathers, analyses and uses data, including state and district assessment data, to measure learner progress, guide instruction, and provide timely feedback in assessment for learning. The primary goal of assessment for learning is to improve learning by changing how learners learn and teachers teach (Gouws, 2013). I believe that while assessment of learning is important, it does not provide adequate opportunities for both teachers and learners to improve their teaching and learning, respectively. Although we are required by curriculum policy to use both assessment for learning and assessment of learning in the teaching and learning of mathematics in South Africa, I have observed that assessment of learning causes teachers to rush through the syllabus for a particular term or year, rather than teach for understanding. This, in my opinion, has a detrimental impact on the learning of mathematics because teachers are denied ample opportunities to modify their teaching strategies to meet the needs of the learners. As a result, the emphasis of this study was on assessment for learning, which I discussed extensively in the following paragraphs.

2.3. ASSESSMENT FOR LEARNING

Assessment for learning is a planned process that includes teaching strategies to elicit an indication of learners' understanding that is used by both learners and teachers to improve instruction and learning (Furtak, Kiemer, Circi, Swanson, de León, Morrison & Heredia, 2016; Kingston & Nash, 2011). According to Ramsey and Duffy (2016), assessment for learning motivates teachers to identify gaps in knowledge and how to make adjustments in teaching to improve learners' learning of specific concepts or skills. According to Black and William (2009), “any evidence of formative interaction must be analysed as reflecting a teacher's chosen plan to develop learning, the formative interactions that the teacher carries out contingently within the framework of that plan” (p.30). According to Black and William (2018), the term "assessment for learning" refers to the process by which teachers use assessment evidence to inform their teaching. According Ramsey and Duffy (2016), assessment for learning is an essential component of daily effective instruction and should not be done on a sporadic basis. However, if teachers do not have a plan in place to enact key strategies of assessment for learning in their classrooms, they will be unable to develop conceptual understanding in their learners.

Assessment for learning encourages teachers to have a conceptual understanding of the main content they are teaching. When teachers use assessment for learning effectively, they develop and magnify their understanding of common misconceptions (Ramsey & Duffy, 2016). I agree with the preceding statement because when teachers understand their own misconceptions, it becomes easier for them to prevent their learners from making the same mistakes. According to Ramsey and Duffy (2016), the goal of assessment for learning is to discover learners' thinking and to give teachers the opportunity to develop existing learners' conceptualisations, and to incorporate these ideas into their instruction, allowing them to gain in-depth understandings of concepts. According to Christopher (2012), assessment for learning has been lauded as one of the most effective pedagogical approaches for improving learners' learning.

It is also stated that assessment for learning improves learners' achievement, stimulates metacognition, and encourages motivated learning and self-efficacy. According to Chappuis (2017), not everything considered to be assessment for learning as practised by teachers in the classrooms is equally effective. According to Andika, Sari, Ningsih, Masniladevi and Helsa (2019), the enactment of assessment for learning can propel learners to achieve all of the lesson objectives because the nature of its design allows learners to be actively involved in all learning activities, including interacting with their peers and teachers, resulting in a positive learning environment. When learners actively participate in the lesson, their chances of understanding the topic or concept increase because their focus shifts from memorising to actually understanding the work. This is supported by Ernst (2014), who asserts that when strategies of assessment for learning are enacted in the classroom, learners' focus shifts from completing work to understanding mathematical concepts. I fully share the same sentiments with the authors above. The learning environment that promotes the learning of mathematical concepts over learning to be graded is more encouraging to both learners and teachers, in the sense that when learners are preoccupied with learning to finish the work before evaluation, they indeed shift their focus from learning with understanding to completing the work. In my over 7 years of teaching mathematics in secondary school, I have noticed that the pressure to finish the work during examination is detrimental to leaners' mathematical understanding. The essence of learning mathematics concepts turns into memorisation and ultimately poor performance.

2.4. BENEFITS OF ASSESSMENT FOR LEARNING IN MATHEMATICAL UNDERSTANDING

Assessment for learning is beneficial to both learners and teachers in the learning and teaching of mathematics subject because it allows them to assess their achievement and understanding using learners' responses (Herbert, Demskoi & Cullis, 2019). Chapman (2017) concurs that assessment for learning has numerous benefits for learners' mathematical understanding. Through assessment for learning, teachers are able to involve learners in teaching and learning activities to elicit information, analyse the information, and determine learners' levels of understanding.

A strategy such as providing feedback that moves learners forward is critical in improving learners' skills and understanding. According to Chapman (2017), one of the primary purposes of feedback is to correct a misconception, miscalculation, or lack of understanding. However, descriptive feedback is more useful in mathematical understanding because it identifies discrepancies between a learner's understanding and the learning objective, as well as the appropriate modification required to meet the learning objective. According to Wylie and Lyon (2015), in order to play the intended role, the teacher's descriptive feedback must be of high quality. Similarly, if learners use the information provided by the teacher to improve their skills and understanding, they will benefit greatly (Ernst, 2014), and their mathematical understanding will improve as a result.

Furthermore, in mathematics, strategies such as activating learners as resources for one another and taking responsibility for one's own are critical. According to Chapman (2017), peer and self-assessment enables learners to determine their level of understanding and, as a result, they are able to modify their strategies and knowledge to aid their process towards achieving their learning goals. Peer feedback allows learners to communicate their ideas to one another, giving the teacher an ample opportunity to find out what learners do not understand and to provide them with feedback that will improve their mathematical understanding.

Peer-assessment is one of the strategies of assessment for learning that improves learners' mathematical understanding by eliciting and providing feedback during the learning process (Almujtahid, Hasih & Mardiyana, 2018). In other words, rather than grading their work, peer assessment encourages the provision of feedback information in order to improve learning in mathematics. Furthermore, this assessment strategy encourages learners to use the results of peer feedback to help them learn mathematics. The results also help learners to be corrected and reassured about their mathematical understanding.

Activating learners as owners of their own learning has a significant impact on mathematical understanding. According to Wylie and Lyon (2015), “strategies such as self-assessment allow learners to identify which concepts they understood and which they still need more time with, writing their current understanding of a main topic or idea and indicating at the end of a lesson what they still have questions about." In other words, learners who have a thorough understanding of the success criteria can easily self-assess their level of comprehension, whereas learners who are unaware of the success criteria cannot (Chapman, 2017).

2.5. THEORETICAL FRAMEWORK

The study employed William and Thompson's (2007) framework, which defines the five key strategies of assessment for learning as a theoretical framework. The five key strategies defined below were used to adapt my instructional practices in a way that promoted the enactment of assessment for learning in my classroom. According to Ramsey and Duffy (2016), the framework integrates the role of learners and their peers with teachers to create a learning environment that assumes collaborative responsibility for learners' learning. Balan (2012) is in agreement with the latter authors by stating that the key strategies of assessment for learning go further than just changing the assessment practices to include changing the entire teaching and learning environment in the classroom. The five key strategies of assessment for learning have the potential to transform the traditional learning environment of a teacher-centred to a learner–centred one. According to ample research on assessment for learning, when learners and their peers are involved in the lesson, learning becomes productive. Each of the five key strategies of assessment for learning is mentioned and thoroughly explained below.

The following are the five key strategies of assessment for learning

1. Clarifying and sharing learning intentions and criteria for success
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding
3. Providing feedback that moves learners forward
4. Activating learners as instructional resources for one another
5. Activating learners as owners of their own learning

1. Clarifying and sharing learning intentions

Various studies such as those by Good and Brophy (2003) and Natriello (1987) have indicated that learners fail to understand the reasons behind activities they are carrying out in the classroom, or what is being evaluated by the assessment because teachers do not share the goals and criteria for the activities in question. Balan (2012) further asserts that learners' understanding of assessment criteria plays a paramount role in teaching and learning. According to William and Thompson (2007), this strategy means sharing the learning objectives and criteria for success by demonstrating examples of good and poor work to learners. The teacher should explain the objectives of each lesson to be attained in a learner-friendly language, and learners should internalise success criteria and discuss them among themselves before the beginning of the assignment/project (Chapman, 2017; Ramsey & Duffy, 2016). Involving learners in clarifying and sharing learning intentions with them enables them to become self-regulated learners and to take ownership of their learning (Peters & Kitsantas, 2009). This strategy of sharing learning intentions and success criteria does not benefit learners only, but also builds a consensus in terms of communication between both teachers and learners (Sadler, 1989).

2. Engineering effective classroom discussion, tasks and activities that elicit evidence of learning

According to William and Thompson (2007), the purpose of eliciting evidence of learning is to reveal various aspects of learners' thinking and understanding. The information revealed can assist educators in modifying their instruction or providing more support to what they are already doing. Balan (2012) further states that engaging learners in classroom discussions, tasks and activities can attain this. The purpose is not the answers learners give, but the thinking behind their answers is of paramount importance. Through this strategy, various aspects of learners' thinking and various views are revealed, not only for the educator but also for the learners. William and Thompson (2007) further contend that effective use of questioning techniques influence learners to participate willingly in classroom discussions and cooperate with their peers. It includes using questioning strategies to assist in uncovering prior knowledge, misconceptions and stimulating new understandings in learners. Black and William (1988) further indicate that the questions posed in classroom discussions should intrigue learners' interest in taking part and that proper wait time should be observed to afford enough time for learners to think about the solutions to questions and respond. Teachers should avoid providing learners with the correct answers before they attempt to answer the questions posed. Johnson et al. (2019) assert that the questions should elicit evidence of learning as well as promoting critical thinking in learners.

3. Providing feedback that moves learners forward

The third strategy involves providing feedback that moves learners forward in the sense that it should be aligned to learning goals. According to Chapman (2017), feedback can be provided to an individual learner, a small group, or an entire class. Hattie and Timperley (2007) recommend that feedback should be tied to one of the instructional processes (i.e., where the learner is going, where the learner is right now, and how to get there) and be at the task, process, or self-regulation level depending on its purpose. Feedback should promote self-reflection which propels learners 'self-efficacy and provide information to the teacher that can, in reciprocity, be utilised to inform instruction. Wylie and Lyon (2015) assert that feedback supplies information about gaps in learners' knowledge about the content and information on how to close the identified gaps to improve learning and enhance understanding. One of the effective ways to use feedback that moves learners is the use of feedback that is descriptive.

Chapman (2017) states that descriptive feedback that is aligned with learning goals and success criteria is more effective and contributes to increased learner achievement rather than evaluative–only feedback. Furthermore, comment-only feedback should be used more frequently than grades because it boasts learners' self–efficacy and ignites intrinsic motivation for lower achievers. Ernst (2014) contends that even though comment-only feedback is time-consuming encourage learners to analyse the comments about their work rather than focusing on scores which might demoralise them.

4. Activating learners as resources for one another

In this strategy, learners should be encouraged to work with each other and make use of their peers as learning resources via peer feedback or peer assessment. Johnson et al. (2019) state that there is ample evidence that when learners engage with each other and reflect on one another's work through peer assessment or peer feedback, they become increasingly reflective about their own work, too. They further indicate that learners are more responsive to their peers 'feedback rather than their teachers. Learners must be guided throughout for positive gains. Chapman (2017) states that teachers should support and scaffold learners by incorporating routines of collaborative teaching into their instruction to enforce this strategy of learners using each other as learning resources for one another. Peer feedback ignites learners 'confidence concerning their achievement and arouses their interest in the topic (Rakoczy, Pinger, Hochweber, Klieme, Schütze & Besser, 2019).

Kagan (1992), Slavin (1995) and Webb (2007) are among the number of studies that illustrate that when learners work together, it has a positive impact on their learning. Goos, Galbraith and Renshaw (2002) further ascertained that learners learn collaboratively by sharing activities, same objectives, ongoing communication, constructing their understanding together by exploring each other' s views. The strategy of activating learners as resources for one another plays a vital role in mathematics education. Since, in this field collaborative learning propels learners to come up with "acceptable methods, solutions and interpretation that require learners to present and defend their ideas and to request their peers to clarify and justify their ideas" (Balan, 2012, p.38).

5. Activating learners as owners of their own learning

The strategy encourages learners to become owners of their learning; learners ought to be self-regulated and possess metacognitive strategies. According to Zimmerman (2000), learners who are self-regulated can manage their learning via different strategies such as time management on their studies, setting goals, and self-evaluation. Johnson et al. (2019) further indicate that teachers can stimulate learners' self-awareness of their learning by allowing learners to evaluate their work by the use of self-assessment. Furthermore, the teacher should act as a facilitator or co-learner, refrain from being the sole provider of knowledge, and create a learner-centred environment. The use of learners 'self-assessments will propel learners into taking ownership of their learning.

Chapman (2017) concurs that self-assessment practices such as journals; reflections, traffic light cards, and self-evaluation are of paramount importance in activating learners to be owners of their learning. Ernst (2014) asserts that the use of self–assessment is the most effective in improving learning because learners receive feedback instantly and teachers gain insights into learners 'knowledge. Furthermore, teachers will act on the information gathered from learners' self-assessments to remedy their challenges before formal tasks.

2.6. STUDIES ON ENACTMENT OF ASSESSMENT FOR LEARNING

Studies such as Enrst (2015), Chapman (2017), Andika, Sari, Ningsih, Masniladevi and Helsa (2019), Almujtahid, Hasih and Mardiyana (2018), Wylie and Lyon (2015), Johnson et al. (2019), Chapman (2017), and Rakoczy, Pinger, Hochweber, Klieme, Schütze and Besser (2019) researched the enactment of the key strategies of assessment for learning . Most of the studies used surveys, action research, experimental research design, quasi-experimental design and case studies as methods of research, and William and Thompson's (2007) framework was widely used as a theoretical framework. The studies emphasised the importance of using strategies of assessment for learning in mathematics while also highlighting some challenges, prompting me to conduct a study on my enactment of strategies of assessment for learning in mathematics to account for learners' mathematical understanding. Assessment for Learning is used by teachers who want to know their learners' mathematical strengths and weaknesses before or during math instruction (Ernst, 2014).

Learners will benefit greatly as well if they apply the information to improve their skills and understanding. Thus, Ernst (2014) conducted action research titled "Assessment for learning in secondary mathematics" with the goal of enacting strategies of assessment for learning in mathematics to determine learners' strengths and weaknesses in the subject prior to sitting for formal tasks. Surveys and interviews were used to collect information from teachers and learners. The findings of the study lauded the use of strategies of assessment for learning in mathematics classrooms because it was determined that feedback from assessment for learning was found to be interesting and challenging to mathematics classrooms. Teachers enjoyed collaborating on lesson planning, which supported learners' learning. What I have gathered and learned from this study is that the enactment of the key strategies of assessment for learning in mathematics play an important role in arousing learners' interest in mathematics. Similarly, teachers developed an interest in lesson planning, which is normally a daunting task to do for most of them.

Helsa et al. (2019) conducted a qualitative study on assessment for learning in mathematics. The intention was to investigate the impact of assessment for learning on the process of learning mathematics in order to detail the attainment of learning objectives such as conceptual understanding, problem-solving, reasoning and communicating ideas. According to their findings, the enactment of assessment for learning in mathematics provided information to teachers about their learners who are still struggling and in need of remediation, as well as learners who have mastered the learning objectives in mathematics. In cases where learners have achieved the learning objectives optimally, the teacher assigned additional work for enrichment. The effective enactment of the key strategies of assessment for learning in mathematics classroom allows the teacher to cater for both struggling and gifted learners equally. It is critical that teachers become familiar with the assessment for learning repertoire. Strategies of assessment for learning such as peer assessment is one of the most important strategies for improving understanding during the learning process. Mardiyana et al. (2018) determined this after conducting their qualitative research using questionnaires, observations, and interview methods as data collection instruments. They went on to say that when learners use peer–assessment to evaluate their peers, they also reflect on their own work, which is an important tool of improving their mathematical understanding.

Balan (2012) used a quasi-experimental research design to introduce formative assessment, also known as assessment for learning in mathematics classrooms, with focus on both teachers and learners. The study was founded on William and Thompson's (2007) framework of the five key strategies of assessment for learning. The quasi-experiment design included an intervention and control group, as well as data collection methods such as the pre and post-test. According to the findings of the study, learners that were in the intervention group where the five key strategies of assessment for learning were enacted performed well. They were able to use relevant mathematical skills to solve problems, their problem-solving performance improved. Furthermore, their (learners) reasoning improved as well, as they were able to present their solutions concisely. The findings also revealed that learners in the intervention group shifted their non-productive mathematical-related beliefs to beliefs that were productive and conducive to learning mathematics. What I found more intriguing was that the learners in the intervention group who were subjected to the enactment of the key strategies of assessment for learning also changed their beliefs about their mathematical understanding. In summary, Balan (2012) discovered that the enactment of key strategies of assessment for learning stimulates learners' mathematical understanding. What I learnt from the study that was aimed at both teachers and learners is that the latter were receptive and appreciated of the strategies of assessment for learning as resources for their learning.

The primary focus of this study was on learners' problem–solving abilities rather than other mathematical skills. The pre- and post-tests given to learners as part of data collection focused solely on problem-solving. There is no evidence that what learners learnt will transfer to other mathematical skills. The study was conducted by a researcher on other mathematics educators. However, the teachers did not have complete control over how the research was carried out. According to Liu (2013), a study conducted in China on enacting assessment for learning in English writing has aided in improving learners' writing when creating thesis statements. According to the study, the use of assessment for learning pushed learners to develop other important writing skills such as using relevant vocabulary, writing paragraphs in an orderly manner, and constructing complex sentences. On the same topic, Wilson (2014) stated that enacting assessment for learning improved the teaching of English as a foreign language in a conversation class in Japan.

The aforementioned studies were conducted on a global scale. In Africa, the literature reviewed included fewer studies on the enactment of the key strategies of assessment for learning. Among the studies reviewed in this study are those by Moyosore (2015) and Otieno (2020). Moyosore (2015) conducted a study on assessment for learning with the goal of "investigating the effect of formative assessment on learners' achievement in secondary school Mathematics." The experimental research design was used by the researcher to carry out the study. Purposive random sampling was used to select two public secondary schools with a total of 120 learners in Iseyin Local Government, Oyo State, Nigeria. The data was collected and statistically analysed. The results revealed that formative assessment has a significant impact on mathematics achievement. Learners who were exposed to formative assessment before taking their summative tests performed significantly better than those who were not. Furthermore, the findings revealed that when teachers incorporate formative assessment into their teaching, learners gain a better understanding of subject content. Formative assessment assists teachers to identify challenges faced by learners on subject content, and provides them with an ample opportunity to provide remediation and modification measures to enhance learners' understanding of the subject content in order to improve subject achievement. According to the study, school officials should emphasise the use of formative assessment by all educators, and allow teachers to attend workshops and seminars on how to enact formative assessment in their classrooms.

In their research on assessment for learning and mathematics achievement in public secondary schools in Nairobi, Kenya, Otineo (2020) used a quasi-experimental research design. The goal of the study was to see if "assessment for learning has a positive impact on learners' mathematics performance." One of their major findings is that teachers do enact assessment for learning in their classrooms, but some have indicated that there are a number of challenges that impede their effective enactment of assessment for learning in their classrooms, which in turn affects learners' performance in mathematics. Teachers attributed their inability to use assessment for learning in their classrooms to a lack of resources, insufficient time, larger classes, learners' unpleasant attitudes toward assessment for learning and a lack of adequate support from school authorities, as well as a lack of independence to be creative in their classrooms. However, the findings of the study revealed that the reasons mentioned by teachers are not the only ones preventing them from effectively enacting strategies of assessment for learning. The findings also revealed that learners' negative attitudes toward assessment for learning, not coming to school on a daily basis, truancy, and their copying of each other's solutions during the assessment are some of the reasons teachers fail to effectively enact assessment for learning in their classrooms. This finding differs from findings by Balan (2012), who found that learners had a positive attitude toward the enactment of assessment for learning.

In reviewing the literature, I discovered that there are fewer studies on the enactment of key strategies of assessment for learning in South Africa. Nkealah's (2019) study is one of the studies that enacted assessment for learning in their studies. The study was conducted with learners in their third year of an English poetry class at the University of Limpopo. An experimental design was used to investigate the practical enactment of strategies of assessment for learning in the teaching of poetry. Information was gathered using both qualitative and quantitative methods. One of the primary goals of the study was to improve learners' understanding of English content knowledge and the pass rate. Learners received detailed feedback on how to improve their essays, in–text feedback on their essays, as well as verbal feedback in class, as part of their application of the key strategies of assessment for learning. The findings indicated that the practical enactment of assessment for learning propelled learners to act on the feedback that was provided to them, which led to their high academic success. For this reason, the researcher asserted that incorporating assessment for learning into teaching could boost transformation in universities. However, Nkealah (2019) stated that while enacting assessment for learning during teaching and learning in the classroom is beneficial, it takes time. Furthermore, they stated that universities do not have time slots set aside for feedback sessions to be integrated into lectures. The time allotted to lectures is limited in order to complete the curriculum, making it difficult to enact assessment for learning to its full potential.

Oyinloye and Imenda (2019) conducted a study with the purpose of "investigating the impact of enacting instruction that applies assessment for learning principles on learner performance in life science" (p.3). The study was conducted in South Africa's KwaZulu-Natal Province, with four schools chosen at random from the King

Cetshwayo District. The authors used a quasi-experimental, pre-test-post-test comparison group design that included four randomly selected schools. The first two schools were the 'treatment condition,' while the other two were the 'comparison group.' To participate in the study, 160 learners in Grade 11 were chosen from the schools. Two educators went through training to learn how to use assessment for learning as a teaching and learning approach, while the other two educators in the comparison group continued to use their traditional teaching and learning approaches. This section, in which teachers were required to attend training on how to enact assessment for learning in their classrooms before the researchers could begin their study, indicates that there is a lack of practical steps for teachers to follow in order to enact assessment for learning in their classrooms. Oyinloye and Imenda (2019) discovered that learners who were taught using an assessment for learning teaching and learning approach outperformed those who were taught using a traditional classroom teaching and learning approach. Furthermore, their findings revealed that incorporating assessment for learning into the teaching and learning process not only inspired learners to look forward to the next lesson, but also improved the relationship between educators and learners.

2.7. DRAWBACKS ON THE ENACTMENT OF ASSESSMENT FOR LEARNING AND RESEARCH GAP

According to Johnson et al. (2019), many teachers do not perceived assessment for learning as an integral part of their mathematical instruction, but rather as a separate task. They go on to say that one of the challenges is that teachers do not have a diverse repertoire for enacting key strategies of assessment for learning, and that training provided by the district is frequently insufficient. According to Wylie and Lyon (2015), some of the significant barriers preventing teachers from enacting assessment for learning in their classrooms are time and class sizes; conceptual confusions related to assessment of learning; systematic imbalances between system priorities and classroom assessment practices; and a lack of effective models for teachers to employ assessment. They conducted a qualitative study in which they examined the breadth and quality of formative assessment enactment among 202 mathematics and science teachers who took part in a two-year, school-based professional development programme focused on formative assessment. Using surveys, logs, and baseline data as data collection methods, their findings indicated that while teachers made significant improvements in some areas, certain aspects of formative assessment are underemphasised, and there are some patterns around enactment quality that suggest that more targeted professional development is warranted.

According to Ramsey and Duffy (2016), even though there is sufficient evidence relating to the effectiveness of assessment for learning in supporting learners' learning, the majority of teachers do not enact the key strategies of assessment for learning in their classrooms. The authors used a variety of qualitative and quantitative methods, including interviews with district and school administrators, surveys on teachers on their formative practices, observations of teachers and material collection. The study was carried out to investigate common patterns, strengths and challenges in the enactment of assessment for learning in classrooms across districts. The findings revealed that teachers frequently used assessment for learning practices that are specifically teacher-centred, such as clarification of assessment tasks and the provision of feedback, but less frequently used learning goals, success criteria, peer and self-assessment.

Johnson et al. (2019) concur that while teachers use some strategies of assessment for learning, they are still lacking in areas where they must invite learners' participation to take ownership of their own learning. Johnson et al. (2019) and Ramsey and Duffy (2016) made significant contributions to the field of assessment for learning by outlining critical practices to follow when enacting strategies of assessment for learning in their questionnaires and observation tools. Even though the researchers sampled master teachers as their participants in the study, the teachers still did not enact all the five key strategies of assessment for learning meticulously. There was still a need for research to be done on the enactment of the five key strategies of assessment for learning, with more emphasis on the last two strategies.

Chapman (2017) concedes that based on their dissertation findings on assessment for learning, it was determined that engineering effective classroom discussions and tasks that elicit evidence of learning was the most frequently enacted assessment for learning strategy as reported on the questionnaire and observed in the classrooms, followed by providing feedback that moves learners forward. However, teachers did not routinely employ the last two strategies to encourage learners to take ownership of their own learning. They also attributed the failure to enact all five key strategies of assessment for learning to teachers' refusal to give up their perceptions about the purpose of assessment and the nature of mathematics teaching and learning. They go on to say that it was discovered that teachers' formative feedback in mathematics is also a challenge in secondary schools because it is evaluative in nature. Another factor is that the majority of teachers continued to use the traditional teaching approach when teaching and assessing mathematics. According to findings by Rakoczy, Pinger, Hochweber, Klieme, Schütze and Besser (2019), the enactment of assessment for learning has the potential to influence how learners view or perceive feedback. However, teachers must advocate for the importance of feedback to learners because research has shown that they (learners) valued the feedback strategies provided but did not use them.

The study also confirmed the importance of effective feedback, and demonstrated that formative assessment is more effective when teachers enact it effectively. Prior to the start of the study, teachers who participated were trained on how to enact formative assessment, which was one of the discoveries that revealed a gap on how to enact the five key strategies of assessment for learning. According to Oyinloye and Imenda (2019), teachers were required to attend training on how to enact assessment for learning prior to the start of the research. One of the gaps I discovered was that the majority of studies on assessment for learning were conducted by external researchers on mathematics educators and their learners. I believe that mathematics teachers should conduct similar studies in their own classrooms to gain insight into the enactment of the five key strategies of assessment for learning. Reflecting on my work and learners' work made sense to me as a constructivist teacher to conduct a study in my own classroom because I created knowledge by interacting with my learners. According to Cobb (2000), teachers should move from traditional approaches where theory is viewed to stand apart from the practice of learning and teaching of mathematics whereby teachers are regarded as users of research findings that are created outside the context of the classroom. This is supported by Muthelo (2010), who further states that “teachers should be in the fore front in terms of doing research that addresses problems and issues of mathematics education more especially those that are very close to what happens in the classroom”(p.3). Hence, in this study I found being in mathematics classroom as a teacher-researcher would enable me to try and close the gap of most studies that were conducted by other researchers who were not mathematics teachers.

2.8. MATHEMATICAL UNDERSTANDING

Mathematical understanding occurs when a learner can justify why a particular mathematical rule makes sense, or why a given mathematical claim is correct (Council of Chief State School Officers, 2010). According to NCTM (2000), the purpose for learning mathematics is to equip and strengthen learners' understanding of mathematical concepts and their relationships. According to Nakamura and Koyama (2018), it is especially important for mathematics educators to strive to comprehend learners' mathematical understanding because it makes no sense for mathematics teaching and learning to take place without learner's mathematical understanding.
The concept of understanding in mathematics has been extensively researched (Pirie & Kieren, 1994; Hiebert & Carpenter, 1992; Sierpinska, 1994; Skemp, 1976). According to Pirie and Kieren (1989, p. 2), "Mathematical understanding can be described as levelled but non-linear. It is a recursive phenomenon that occurs when thinking moves between levels of sophistication". They also proposed a recursive model composed of eight levels in the mathematical understanding process. Pirie and Kieren (1994) define mathematical comprehension as "a whole, dynamic, levelled but non-linear, transcendently recursive process" (p. 166). The eight levels in the Pirie-Kieren model of the growth of mathematical understanding are primitive knowing, image making, image having, property noticing, formalising, observing, structuring and inventing.

According to Carpenter and Hiebert (1992), learning mathematics with understanding is one of the most widely held beliefs in the mathematics education community around the world. According to Hiebert and Carpenter (1992), the number of connections and the strength of those connections inform learners' level of mathematical understanding. They also stated that learners understand mathematical concepts when they are broadly connected among a variety of connections. Furthermore, they claim that mathematical understanding improves as the connections become stronger and more organised. However, if the connections are weak, mathematical understanding is limited. Sierpinska (1994) in Mabotja (2017), on the other hand, “discusses three different ways of looking at understanding: act of understanding, understanding, and processes of understanding” (p.31). To begin with, an act of comprehension is defined as the mental experiences in relation to what is to be comprehended within the framework of comprehension. Second, understanding is attained as a result of understanding acts. Finally, a series of comprehension as represented by connections form between mental concepts.

Skemp (1976) classified mathematical understanding into two categories: relational understanding and instrumental understanding. He defined relational understanding as “knowing both what to do and why to do it” (p. 2). Instrumental comprehension, on the other hand, was simply defined as “rules without reasons” (p. 2). This is in contrast to relational understanding, which provides pathways for retrieving information from learners' memories, and fosters the development of mathematical understanding. On the other hand, instrumental understanding entails rule memorisation. Rensaa (2018) went on to say that relational learning is intended for learners who are intrinsically motivated, which Skemp attributes to knowing the structure of a concept, whereas instrumental learning is superficial, rules-based, and tied to a specific set of tasks to be completed. Again, the learning strategy known as instrumentalism propels instrumental understanding.

On the same issue, Utomo (2019) discovered that learners with solid knowledge of their mathematical abilities had ample relational understanding because they met all of the indicators of relational understanding after conducting her study on analysing learners’ relational and instrumental understanding on addition of integers. Low achievers, on the other hand, only met half of the indicators of relational understanding. Finally, learners with average understanding were thought to understand the concepts, whereas low achievers have procedural mistake misconceptions. A primary goal of teaching for understanding, according to Walle, Karp and Bay-Williams (2016), is to assist children in developing a relational understanding of mathematical ideas. Learners with an instrumental understanding of mathematics are prone to forgetting what they have learned because they are taught concepts and procedures in isolation. Furthermore, relational understanding develops over time and becomes more complicated as a student makes numerous connections between ideas. Teaching for relational understanding should be a goal for daily teaching and learning, even if it takes time.

According to Rensaa (2018), the connection between concepts is critical in mathematical understanding. Furthermore, Hiebert and Lefevre (1986) described a similar distinction between conceptual knowledge and procedural knowledge of mathematics in Rensaa (2018). They defined conceptual knowledge as knowledge that is densely connected, implying that it cannot exist in isolation. “Conceptual knowledge requires the learner to be active in thinking about relationships and making connections, as well as making adjustments to accommodate the new learning with previous mental structures,” (Korn, 2014, p.5).

Procedural knowledge, on the other hand, includes the fundamental linear connections or step-by-step instructions that govern how to carry out tasks. The type of information embraced by procedural knowledge is symbolism and knowledge of procedures or rules for solving classes of tasks. In contrast, “meaningful knowledge of procedures and concepts is obtained when interrelating the categories and realising how one may lead to the other” (Rensaa, 2018, p.9). However, Hiebert and Lefevre (1986) concluded that the relationship between conceptual knowledge and procedural knowledge is frequently bidirectional because when one improves, the other often improves.

Korn (2014) further contends that mathematics education should place a strong emphasis on teaching the conceptual understanding of mathematics because a student has conceptual knowledge in mathematics when he or she “understands the meaning and underlying principles of mathematical concepts” (Frederick & Kirsch, 2011, p. 94). According to Malatjie and Machaba (2019), conceptual understanding is the connections between mathematical concepts that are connected to each other. They add that when a learner has conceptual understanding, he or she will be able to describe, explain and apply the same concept in different situations. Furthermore, conceptual understanding is one of the strands of mathematical proficiency that enables learners to gain a thorough understanding of mathematical ideas, concepts, and operations, as well as the relationships between them (Department of Education, 2018). According to Barmy (2007), the strength and number of mental representations influence the level of understanding in mathematics. They also stated that a mathematical procedure is better understood if it is linked to an existing network with a large number of connections.

According to Korn (2014), knowing mathematics conceptually is linked to knowing mathematics procedurally; so it is equally important to teach mathematics with meaning in order to stimulate learners' conceptual understanding. Procedural proficiency was once the primary focus of mathematics teaching and learning, and remains so today. But conceptual understanding is an equally important goal (National Council of Teachers of Mathematics, 2000; National Research Council, 2001; CCSSO, 2010). This is supported by findings of Rahmawati (2016), who argue that the procedural level of learners in the study that they have conducted was higher than the level of conceptual understanding. However, there is a positive relationship between conceptual and procedural understanding. Gordon and Gordon (2006) are of the view that stressing conceptual understanding and the meaning of mathematics should be accomplished using authentic examples and problems that force learners to think rather than manipulate symbols. They also claim that if this is not done, learners will be unprepared to continue with mathematics as a career. As a result, they will be unprepared to use mathematics in their jobs and throughout their lives.

According to the preceding arguments, it is critical that learners develop a conceptual understanding of mathematics, which I found relevant to my study. Kilpatrick et al.'s (2001) theory of mathematics proficiency was used in this study to assess learners' mathematical understanding. The theory of mathematics proficiency is divided into five parts: "conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition" (Department of Education, 2018, p.8). However, in this study, I analysed learners' mathematical understanding by focusing on conceptual understanding, procedural fluency and strategic competence. Because of the topic that I taught learners during the data collection process, the three strands were relevant to my study.

I taught Grade 10 learners about "functions," but the topic was still in its early stages because Grade 10 is an entry grade into the FET phase. "The concept of a function, in which one quantity (output value) uniquely depends on another quantity (input value), work with relationships between variables using tables, graphs, words, and formulae, and convert flexibly between these representations" (Department of Education, 2011, p.24). As a result, the first three strands of mathematical proficiency were appropriate for their (Grade 10 learners) level of understanding. I analysed learners' mathematical understanding using the three strands of mathematical proficiency mentioned above because they are the foundation of mathematical understanding (Department of Education, 2018). On the same issue, the Department of Education's (2011) CAPS for FET mathematics indicate that the specification of content progression from Grade 10 to 12 in all topics, that is, topics that progress in complexity, is hierarchical.

Conceptual understanding: A thorough understanding of mathematical concepts, operations and ideas, as well as the relationships between them. Learners who have conceptual understanding understand why a particular idea in mathematics is important and the different contexts in which it is used.

Procedural fluency: Learners must be able to carry out mathematical procedures correctly, effectively, appropriately and relevantly.

Strategic competence: Learners should be able to make sound decisions about which strategies to use, or to devise their own strategies for working out mathematical activities and problems.

2.9. CHAPTER SUMMARY

Topics covered in the literature review included assessment in education, assessment for learning, benefits of assessment for learning in mathematics, studies on enactment of assessment for learning, challenges to enactment of assessment for learning and mathematical understanding. The theoretical framework that served as the foundation for this study was also described. According to the literature that I reviewed, the use of key strategies of assessment for learning in classrooms is beneficial to both learners and teachers, but I discovered that there is a gap in the enactment of key strategies of assessment for learning in mathematics classrooms. As a result, my proposed study will fill the gap by documenting my enactment of assessment for learning to account for learners' mathematical understanding. Assessment for learning was used in this study as a planned process that entails teaching strategies to promote learning by inviting learners to participate and to take ownership of their own learning.

CHAPTER 3: RESEARCH METHODOLOGY

3.1. INTRODUCTION

The previous chapter focused on the literature and theoretical framework about the reported issues of the study. This section describes the research process, which includes the qualitative research paradigm, and details on other aspects of the research process such as participant selection, data collection, and analysis processes. The chapter further explains how the teaching experiment methodology was used to enact the five key strategies of assessment for learning during mathematics classroom interactions. The chapter concludes with a discussion of rigour in the teaching experiment methodology as well as ethical considerations.

3.2. RESEARCH PARADIGM

A research paradigm, according to Guba and Lincoln (1994), is a fundamental belief or worldview that guides research. According to Johnson and Christensen (2005), a research paradigm is a point of view that is based on a set of shared assumptions, beliefs, concepts and practices. Such beliefs become the guiding principles for the activities of community members (Engler, 2009). I had to comprehend the worldview through which knowledge is developed in the process of learning for this reported study by documenting the enactment of the five key strategies of assessment for learning in my mathematics classroom to account for learners' mathematical understanding. It was because I needed a set of beliefs to guide my instruction.

There are various paradigms, according to Guba and Lincoln (1994), such as positivism, constructivism, and others. As a worldview, I used the constructivism paradigm in this study. In order for me to enact the five key strategies of assessment for learning in my classroom, I needed to interact with the learners during the lessons in order to assess their mathematical understanding and to identify their difficulties. According to Mabotja (2017), knowledge in the constructivist paradigm is created through interaction between the researcher and participants.

Despite the fact that constructivism emphasises knowledge development as a result of interactions, Mabotja (2017) suggested that “one should not assume to include situations where information is transferred directly from teachers to learners. Instead, teachers' roles are to assist learners in modifying their mathematical activities” (p.50). Steffe and Thompson (2000) add that in this scenario, teachers who are also researchers should learn how to use mathematical knowledge to teach and interact with learners in the classroom. According to Muthelo (2010), “Constructivists reject the notion of reality as accessible and interpretable outside our own experiences, and instead attribute our knowing to experiential reality” (p.8). As a result, the constructivism paradigm was appropriate for my study in the sense that I had to be present in my mathematics classroom to conduct this study and to construct knowledge with the learners through our interactions. The constructivism paradigm enabled me to describe the implementation of the five key strategies of assessment for learning in order to account for learners' mathematical understanding during classroom interactions. Knowledge was not directly transferred to learners, but was also created through classroom interactions between myself and the learners. According to Mabotja (2017), multiple realities in the constructivism paradigm are discovered through interactions between the researcher and participants. For this reason, the qualitative research approach which was chosen for this study will be discussed further in the following section.

3.3. QUALITATIVE RESEARCH APPROACH

According to Denzin and Lincoln (2005), qualitative research has multiple realities that arise from an interpretative, naturalistic approach to the subject under investigation. According to Creswell (2009), qualitative research is a research method used to investigate and understand the meaning that individuals attach to a social or human problem. The qualitative research approach was appropriate for this study because it strengthened an understanding and interpretation of meaning as well as the intentions beneath the human interactions (Mukwevho, 2018). In this study, I wanted to investigate the enactment of assessment of learning in my mathematics classroom so that I could account for learners' mathematical understanding. As a result, using a qualitative approach in this study enabled me to describe what occurred, and to comprehend the impact of some educational interventions (Mills, 2003). Furthermore, Creswell (2003) asserts that qualitative researchers gather enough information about the individual or location in order to be deeply involved in the actual experiences of participants. According to Sharon (2009), qualitative researchers are interested in how people interpret their experiences, how they construct their world, and meanings that they attribute to their experiences. As a result, in this study, I used the five key strategies of assessment for learning in my mathematics classroom to account for learners' mathematical understanding, allowing me to investigate what learners were saying or doing while engaging in mathematical activities (Steffe & Thompson, 2000). Finally, the qualitative research approach is exploratory and descriptive. The descriptive nature of this approach allowed me to outline the description of participants' experiences, allowing me to either sustain or challenge the theoretical framework on which this study was based (Meyer, 2001). For this reason, I selected the constructivist teaching experiment methodology as a research design, which will be discussed further in the following section.

3.4. RESEARCH DESIGN

According to GAMA (2015), a research design is a plan for a study which provides a framework for attaining research goals. The design is the main guide that provides the framework for collecting and exploring data. Since qualitative research is exploratory and descriptive, the constructivist teaching experiment methodology was chosen as the best research design of this study. I used the exploratory teaching experiment methodology, which allowed me to investigate what learners were doing or saying as I enacted the five key strategies of assessment for learning during classroom interactions. According to Hajra (2013), "a teaching experiment is a living methodology designed to explore and explain learners' mathematical activities" (p.9). According to Muthelo (2010), this methodology is contextual rather than prescriptive. As a result, the teaching experiment methodology allowed me to assess learners' progress through mathematical communications while also combining teaching with research, theory and practice (Özdemir, 2017). The constructivist teaching experiment methodology was appropriate for my study because, as I enacted the five key strategies of assessment for learning in the mathematics classroom, I was able to account for learners' mathematical understanding by exploring and explaining their mathematical activities as they engaged in classroom interactions.

According to Steffe and Thompson (2000), “the primary purpose of using teaching experiment methodology is for researchers to experience, first-hand, learners' mathematical learning and reasoning” (p. 267). As a result of the enactment of the five key strategies of assessment for learning, learners were given an opportunity to participate in classroom discussions and to collaborate with their peers using peer and self-assessment as they engaged in mathematical activities, giving me an opportunity to experience their way of learning and reasoning in mathematics. The constructivist teaching experiment methodology includes planning a teaching experiment, continuous experimentation in the classroom and retrospective analysis.

3.4.1. Rationale for constructivist teaching experiment methodology

The constructivist teaching experiment methodology was appropriate for this study because my intention was to enact the five key strategies of assessment for learning in my mathematics classroom to account for learners' mathematical understanding. Furthermore, I wanted to determine how the enactment of the five key strategies of assessment for learning in mathematics classroom can contribute towards learners' mathematical understanding during their participation in mathematical activities. The constructivist teaching experiment methodology afforded me an opportunity to document what transpired during classroom interactions. I was able to "experience, first-hand, learners' mathematical learning and reasoning" as I enacted the five key strategies of assessment for learning (Steffe & Thompson, 2000, p.268). This enabled me to participate in my learners' learning processes and, ultimately, their mathematical knowledge construction.

Muthelo (2010) adds that determining what learners say and do in an attempt to discover their mathematical realities is an important part of the constructivist teaching experiment methodology. Learners participated in classroom discussions, tasks and activities that elicited evidence of learning during the enactment of assessment for learning. Learners collaborated with one another and used their peers as learning resources through peer feedback. They took ownership of their own learning by conducting self-assessments, which provided information to the teacher, which was used to inform instruction. Therefore, the constructivist teaching experiment methodology gave me plenty of opportunities to do so. During classroom discussions, peer discussions and questioning, I was able to determine what learners were saying when responding to the questions posed, as well as their engagement with one another during peer feedback.

3.4.2. Elements of constructivist teaching experiment methodology

A constructivist teaching experiment consists of a series of teaching episodes (Muthelo, 2010; Mabotja, 2017), each of which includes a teaching agent, one or more learners, a witness to the teaching episodes, and a method of recording what happens during the episode (Hajra, 2013). There were five teaching episodes in total, all of which took place in a grade 10 mathematics classroom and were recorded with a video recorder. Steffe and Thompson (2000) state that records are used to prepare subsequent episodes as well as to conduct a retrospective analysis of the teaching experiment. I collaborated with my colleague, who was a witness throughout the teaching experiment, to plan lessons before enacting each teaching episode. In each teaching episode, we based our planning and learning materials design on the five key strategies of assessment for learning in the sense that the lesson plans for all teaching episodes included information on how I planned to incorporate the five key strategies of assessment for learning practices throughout my mathematics instruction in each teaching episode. My entire teaching episodes were planned around the five key strategies of assessment for learning.

However, to plan the teaching episodes, we also used the Curriculum Assessment Policy Statement (CAPS) document for mathematics 10-12 and the Siyavula textbook for Grade 10 mathematics. In each teaching episode, I, as the teaching agent enacted the five key strategies of assessment for learning developed by William and Thompson (2007), beginning with strategy number one and progressing to strategy number five. I was able to “diagnose learners' progress, identify gaps in knowledge and understanding, and determine how to make immediate adjustments in instruction to improve learners' learning of specific concepts, skills, or standards” by enacting these key strategies of assessment for learning in my classroom (Ramsey & Duffy, 2016, p.6). According to Steffe and Thompson (2000), “the most important aspect of the teaching experiment is the modelling of the learners' responses into a coherent picture of the learners' progress over an extended period” (p. 54).

Conducting a constructivist teaching experiment necessitates the presence of a witness who can observe all of the events of each teaching episode (Steffe & Thompson, 2000). This is done so that the observer can provide an unbiased perspective of the interactions that occurred during the teaching episodes, which assisted me, the teacher-researcher, in planning for the next teaching episode (Steffe & Thompson, 2000). My observer and I discussed what happened during each teaching episode at the end of the episode, which informed our planning for future teaching episodes.

3.4.3. Participants/sampling

The participants in this study came from my Grade 10 mathematics classroom which had 25 learners. All 25 learners were exposed to the exploratory teaching used in the teaching experiment methodology. According to Mabotja (2017), exposing all of the learners in the class to exploratory teaching helps to ensure that they are not disadvantaged by selection bias. From the 25 learners, I reported data based on daily classroom interactions and learners’ activities samples that supported the purpose of documenting the enactment of assessment for learning to account for learners’ mathematical understanding. According to Steffe and Thompson (2000), this act of selecting participants from a group is acceptable in the teaching experiment methodology because researchers can choose one or a few participants to participate in a study.Steffe and Thompson (2000) add that this aspect of the teaching experiment methodology guides and organises the teacher-researchers' experience of learners doing mathematics.

Furthermore, Etikan, Musa and Alkassim (2016) concede that this method of selecting participants goes hand in hand with convenient sampling because researchers are cautioned that “convenience sampling should not be taken to be representative of the population” (p.2). Furthermore, my grade 10 mathematics classroom was convenient for me because it was located in the school to which I am currently assigned, and the geographical proximity made my work easier (Alkassim et al., 2016). As a result, as a teacher-researcher, I was able to witness first-hand learners' mathematical learning and reasoning. This aided me in gathering the necessary data to document my enactment of the five key strategies of assessment for learning to account for learners' mathematical understanding (Muthelo, 2010).

3.4.4. Data Collection

In this study, I used a video recorder to record classroom observations, learners' written work samples, and the teacher's reflective journal (teacher's note-taking) as data collection tools. These methods aided me in documenting my use of the five key strategies of assessment for learning to account for learners' mathematical understanding. One of the main goals of the teaching experiment methodology, according to Steffe and Thompson (2000), is for researchers to focus on what learners are doing or saying in order to better understand their learning of mathematics as they engage in mathematical activities. In order to achieve the aforementioned purpose of the teaching experiment methodology, studies (Mabotja, 2017; Muthelo, 2010; Özdemir, 2017) advocated for the use of multiple data collection methods mentioned above. Cobb (2000) concurs that it is critical to investigate learners' mathematics in depth within appropriate learning environments during the process of the teaching experiment methodology.

My data collection methods were consistent with William and Thompson's (2007) theoretical framework of the five key strategies of assessment for learning, as they encouraged teacher and learner participation in the teaching and learning of mathematics. The framework of the five key strategies of assessment for learning developed by William and Thompson (2007) includes the learner, their peers, and the teacher as participants in teaching and learning. The five key strategies motivated the aforementioned participants to actively participate in the lesson.

3.4.5. Classroom Observations

I used a video recorder to record classroom interactions between myself and the learners, as well as between the learners themselves. As I enacted the five key strategies of assessment for learning in my classroom, the video recorder assisted me in capturing all the classroom interactions, even when learners were engaged in mathematical activities. Since the study was about enacting the key strategies of assessment for learning to account for learners' mathematical understanding, the video recorder was appropriate for data collection because I needed to record what learners said or did as they engaged in classroom interactions and mathematical activities (Özdemir, 2017; Mabotja, 2017; Uygan, 2019).

In Mabotja (2017), Steffe and Thompson (2000) state, "In order to learn learners' mathematics, the researcher could create situations and ways of interacting with learners that encourage the learners to modify their current thinking" (p.56). In this study, learners were encouraged to share ideas with one another as they engaged in mathematical activities (Muthelo, 2010). Furthermore, by enacting the key strategies of assessment for learning in my scenarios, learners were compelled to share their ideas. As a result, Steffe and Thompson (2000) argue that recording all classroom interactions with a video recorder provides researchers with insight into learners' actions and interactions that researchers could not notice during the class. According to Mabotja (2017), the video recorder generates a large amount of data in a short period, and researchers will continue to revisit the videos even after the teaching episode has ended. The video recorder data provided me with an opportunity to repeatedly observe all of the interactions in the classroom. I ensured that learners' artificial behaviour in front of the camera was eliminated by recording my lessons two months prior to the start of my data collection to ensure that they became familiar with the presence of the camera in their classroom (Castro, Castro & Molina, 2007).

3.4.6. Learners' Written Activities

Activities written in learners' class-work books aided me in assessing their mathematical understanding in terms of Kilpatrick et al.'s (2001) mathematical proficiencies, conceptual understanding, procedural fluency and strategic competence of the concepts taught. The teaching experiment methodology allowed me to observe what learners were doing through their written mathematical activities. As part of the five key strategies of assessment for learning, I was able to discover what they were doing, assess their mathematical understanding, and provide written feedback that moved learners forward.

Learners' reflective journals greatly helped me in monitoring their understanding of mathematics concepts at the end of each teaching episode. Furthermore, reflective journals were used to assess learners' ability to take responsibility for their own learning. Moreover, data collected through learners' reflective journals in conjunction with strategy number five of "activating learners as owners of their own learning" demonstrated that learners took ownership of their understanding of mathematics concepts. Studies by Özdemir (2017), Mabotja (2017), Uygan (2019), Muthelo (2010), Masha (2004), and Molina, Castro and Castro (2007) also collected data using learners' samples in order to see what learners were doing or writing in order to gain insight into their thinking and learning of mathematics, which in turn aided in class-interventions.

3.4.7. Note-taking through teacher's reflective journal

Data was also collected by taking notes in a reflective journal. I was able to jot down key points that came up during each teaching episode. These notes were used during the reflections that took place in-between teaching episodes. As I enacted the five key strategies of assessment for learning during the course of the teaching experiment to account for learners' mathematical understanding, researchers' reflections in-between teaching episodes played an important role in decision-making throughout the process of the study (Castro et al., 2007).

3.5. DATA ANALYSIS

Data from teaching experiments were analysed as soon as data collection began in order to inform decisions or to generate new learning trajectories, which aided in planning for the next teaching episode through retrospective data analysis (Steffe and Thompson, 2000). Steffe and Thompson (2000) continue on the same issue, stating that "through retrospective analysis, we attempt to bring to the fore the activity of model building that was present throughout the teaching episodes" (p. 297). Transcriptions from video data and samples of learners' written activities were used in the data analysis process.

The video data was analysed in a number of stages. In the first stage, I began by watching video recordings of the entire classroom discussions of each teaching episode, which included classroom interactions as well as learner-to-learner interactions as they engaged in mathematical learning activities. I listened to the entire classroom discussions several times, which allowed me to make sense of the video data. Second, I transcribed the entire classroom discussion, which allowed me to produce a written text of the data on the video recordings. Furthermore, I read the transcriptions several times to make sense of the text data. Finally, information-rich interactions from whole-class discussions in which assessment for learning was observed were chosen. William and Thompson’s (2007) framework of the five key strategies of assessment for learning influenced the selection. In the following chapter, the selected information-rich interactions were presented in the form of teaching episodes that included classroom interactions and mathematical learning activities.

In the second stage, I translated and interpreted the information-rich interactions from classroom discussions that had been transcribed. The interpretations were guided by learners' mathematical understanding as informed by Kilpatrick et al. (2001) strands of mathematical proficiencies. Thus, the interpretations of transcribed information-rich interactions focused on the three strands of mathematical proficiencies, which are conceptual understanding, procedural fluency and learners' strategic competence towards mathematics during classroom interactions as I enacted the five key strategies of assessment for learning. Furthermore, I wanted to know which strategies of assessment for learning elicited the three strands of mathematics proficiencies of the concepts taught to learners from the transcribed information-rich interactions video data and written text.

Second, three strands of mathematical proficiencies were identified from the transcribed information-rich interactions. I then used William and Thompson's (2007) framework of the five key strategies of assessment for learning, which I discussed in the previous chapter, to categorise the transcribed information-rich interactions based on their shared characteristics. Themes were created based on the five key strategies of assessment for learning (Pope et al., 2000). Because the themes in this study were generated based on the theoretical framework of five key strategies of assessment for learning, deductive thematic analysis was used in the process to analyse the video data and transcribed data (Fereday & Muir-Cochrane, 2006). Learners' written work samples which comprised classwork books with written activities, and their reflective journals were analysed in the third stage (see Appendix A).The samples were analysed in the third stage based on the strands of mathematical proficiencies identified by Kilpatrick et al. (2001); that is, conceptual understanding, procedural fluency and strategic competence of the concepts taught to learners. I examined learners' mathematical understanding using the three strands of mathematical proficiency mentioned above because they are the foundation of mathematical understanding (Department of Education, 2018). On the same subject, the Department of Education's (2011) CAPS for FET mathematics indicate that the specification of content progression from Grade 10 to 12 in all topics, that is, topics that progress in complexity, is hierarchical.

During the data collection period for this study, I taught Grade 10 learners about functions, but the topic was still in an introductory phase because Grade 10 is an entry grade into the FET phase. "The concept of a function, in which one quantity (output value) uniquely depends on another quantity (input value), work with relationships between variables using tables, graphs, words, and formulae, and convert flexibly between these representations" (Department of Education, 2011, p. 24). As a result, the first three strands of mathematical proficiencies in this study were appropriate for their (Grade 10 learners) level. Learners' responses to activities aided in the analysis of their mathematical understanding. Finally, all of the data from each teaching episode were analysed to fulfil the purpose of the study of documenting the enactment of the five key strategies of assessment for learning to account for learners' mathematical understanding.

3.6. QUALITY CRITERIA

3.6.1. Rigour in constructivist Teaching Experiment Methodology

It is just as important to ensure rigour in quantitative research as it is in qualitative research. The teaching experiment methodology cannot be separated from issues of research rigour because it (research rigour) serves the purpose of establishing trust and confidence in the findings of the study (Castro et al., 2007). Generalisability and reliability are used to demonstrate rigour in the teaching experiment methodology. Despite the fact that Steffe and Thompson (2000) argue that teaching experiments should not be replicated. In this study, I demonstrated how I enacted the five key strategies of assessment for learning in such a way that it contributed to learners' mathematical understanding in the following chapter. According to Mabotja (2017), “Researchers who make a claim about what learners know are required to make records of the living models of learners' mathematics that demonstrate features of the claim available to the interested public” (p.60).

As I previously stated, the participants (eight learners) from whom I sampled their work samples and conversations for reporting do not represent the entire Grade 10 class of 25 learners in terms of the purpose of the study. They did, however, provide me with the opportunity to confirm that the enactment of assessment for learning can elicit learners' mathematical understanding. Steffe and Thompson (2000) reckon that expecting teaching experiments to "generalise" in such a way that claims deemed true about random samples represent the entire population from which the sample was drawn is pointless. As a result, my goal was not to generalise to the population, but to enact the five key strategies of assessment for learning to account for learners' mathematical understanding. As a result, the purpose of teaching experiments is not to generalise findings hypothetically, but rather to use the findings to guide teachers' experiences with learners learning mathematics (Steffe & Thomson, 2000).

3.6.2. Trustworthiness in Qualitative Research

3.6.2.1. Credibility

According to Anney (2014), credibility is when the researcher can confidently declare that the findings of the study were true and represented information gathered from the participants' original data. Treharne and Riggs (2014) indicate that activities such as spending a prolonged time engaging with participants and triangulation are more likely to ensure that the research will produce credible results. In this study, I spent the prolonged time engaging with participants because they were my learners from Grade 10 mathematics classroom that I was assigned to. Therefore, I had sufficient time to become acquainted with the context. According to Davies (2011), triangulation occurs when a researcher employs more than one method of conducting research in order to obtain richer data. To ensure credibility, I used data triangulation.

3.6.2.2. Triangulation

According to Noble and Heale (2019), triangulation is the use of multiple approaches to answer a research question in order to increase the credibility of the research findings. Triangulation plays an important role in the teaching experiment methodology (Castro et al, 2007). Hence, I gathered data from various sources to answer the research question. I collected data by video recording classroom interactions. In addition, I used learner written activities from their classwork books, and a teacher's reflective journal (teacher's note-taking) (Steffe & Thompson, 2000). The following chapter describes how Cope's (2014) suggestion was enacted in qualitative research.

3.6.2.3. Transferability

When findings can be applied in contexts other than the study situation, such as different contexts or other groups, they are said to be transferable (Polit & Beck, 2012; White, 2005). The transferability criterion, on the other hand, is dependent on the intention of the qualitative study and may be appropriate only if the researcher wanted to make generalisations about the findings. As previously stated, the intention of this study was not to generalise findings, so the criterion did not apply (Cope, 2014). However, I provided thick descriptions of my experiences, behaviours and contexts in order for the findings to make sense to an outsider in case they want to use the findings in their classrooms (Korstjens & Moser, 2018).

3.6.2.4. Confirmability

Confirmability ensured that the responses reflected participants' experiences rather than the researcher's preferences, which were influenced by the researcher's biases. Conformability was achieved in this study by ensuring that findings were derived directly from the video data by using participants' direct quotes and raw data from learners' written work, which then prompted me to describe the subsequent chapter on how conclusions and interpretations were established as suggested by Cope (2014).

3.7. ETHICAL CONSIDERATIONS

According to Nkosi (2014), ethical considerations are critical components of all research studies, and researchers must ensure that they are followed. First and foremost, I sought permission from the University of Limpopo's Turfloop Research Ethics Committee (TREC) to conduct my research. In addition, I requested approval from Limpopo Department of Education by sending request letters (attached in Appendix C) to the Waterberg district director and the school's principal. To avoid being dishonest about the purpose of my study, I also submitted the outline of my study to the aforementioned authorities. Because the participants were minors, I also requested permission from their parents. To ensure confidentiality and anonymity, I used pseudonyms to protect the identities of participants and the research site.

Furthermore, I explained the purpose of the study to participants. Machaba (2013) states that it is the researcher's responsibility to explain the study to participants and what is required in terms of participation. I also made certain that I distributed written consent forms to each participant, assuring them that their participation was entirely voluntary and that they could opt out at any time. Participants' parents were given informed consent forms (attached in Appendix E), which guarantee participants' rights and permission to participate in the study. Signing the informed consent forms indicated that participants understood what was explained to them, that they were willing to participate in the study, and that they had parental permission to do so. Furthermore, I guaranteed complete confidentiality and anonymity.

3.8. CHAPTER SUMMARY

The chapter provided justification for using the teaching experiment methodology as a research design of this study, and outlined the rationale for the qualitative research paradigm. The chapter also described how participants were chosen, as well as how the data was collected and analysed. The section also addressed issues such as ethical considerations and quality criteria of the study. The analysis of the data collected is presented in the following chapter.

CHAPTER 4: THE DATA PRESENTATION AND ANALYSIS

4.1. INTRODUCTION

The chapter reflects on three teaching experiments in a Grade 10 mathematics classroom that I conducted over a period of two to three months. The first two teaching experiments were conducted during the first two months, and the final one during the third month. The three teaching experiments provided me with the opportunity to account for learners' mathematical understanding after enacting the five key strategies of assessment for learning in my mathematics classroom according to William and Thompson's (2007) framework. The framework comprises five key strategies: (i) clarifying, understanding and sharing learning intentions; (ii) engineering effective classroom discussions, tasks and activities that elicit evidence of learning; (iii) providing feedback that moves learners forward; (iv) activating learners as resources for one another; and (v) activating learners as owners of their own learning. My observer and I created an environment conducive to enacting the five key strategies of assessment for learning. The table below defines what it means to enact each key strategy of assessment for learning.

Table 1: Defining 5 key strategies of assessment for learning

Strategy	Description
Clarifying and sharing learning intentions.	Disclose the learning objectives and criteria for success by demonstrating examples of good and poor work to learners.
Engineering effective classroom discussion, tasks and activities that elicit evidence of learning.	Effective use of questioning techniques in order to influence learners to participate willingly in classroom discussions and cooperate with their peers. To assist in uncovering prior knowledge, misconceptions and stimulating new understandings in learners.

Providing feedback that moves learners forward.	Using feedback that is tied to learning goals that promote self-reflection, which propels learners' self-efficacy and gives information to the teacher that can, in reciprocate, be utilised to inform instruction. Use of comment-only feedback more often. It is more effective than giving feedback using scores.
Activating learners as resources for one another.	Learners should be encouraged to work with each other and make use of their peers as learning resources via peer feedback.
Activating learners as owners of their own learning.	The teacher should act as a facilitator and refrain from creating a teacher-centred learning environment where s/he is the only provider of information. The use of learners 'self-assessments will propel learners into taking ownership of their learning.

Note: Adapted from "Integrating assessment with instruction: What will it take to make it work?" by William, D., and Thompson, M. (2007).p.15

Furthermore, throughout the teaching experiments, I used the five key strategies of assessment for learning in order to account for learners' mathematical understanding. Kilpatrick's (2001) first three strands of mathematical proficiencies, conceptual understanding, procedural fluency and strategic competence of the concepts taught in the teaching experiments informed the mathematical understanding I accounted for. Learners with conceptual understanding should understand mathematical concepts, operations and ideas, as well as the relationships between them. Furthermore, learners with conceptual understanding comprehend why a particular idea in mathematics is important and the types of contexts in which it is used. Learners who have procedural fluency should be able to carry out mathematical procedures correctly, effectively, appropriately and relevantly. Learners with strategic competence should be able to make sound decisions about which strategies to use or to devise their own strategies for working out mathematical activities and problems.

4.2. TEACHING EXPERIMENTS

Three teaching experiments were carried out in this study. This was done to document my enactment of assessment for learning in order to account for learners' mathematical understanding. Teaching experiment one focused on the basics of parabolic functions. The teaching experiment consisted of two episodes, the first of which focused on the basics of parabolic functions, such as the shape of the parabola and the effect of “a” in a parabolic function $y = ax^2 + q$. The second teaching episode concentrated on sketching the parabolic function using intercepts and a turning point. The second teaching experiment with two teaching episodes focused on developing learners' conceptual understanding and procedural fluency in finding square roots, emphasis on parabolic functions and working with hyperbolic functions. The first teaching episode focused on the revision of finding square roots in order to build conceptual understanding in the sketching of parabolic functions. The second teaching episode focused on sketching hyperbolic functions and determining their intercepts. One teaching episode was included in Teaching Experiment Three. The emphasis in this episode was on developing strategic competence with parabolas and hyperbolas. I also included an exponential graph, which was unfamiliar to the learners.

I together with my observer created lesson plans for each teaching episode using the Grade 10 Siyavula textbook and the CAPS document for FET mathematics, which provided main details on how I will incorporate the five key strategies of assessment for learning during the teaching and learning of mathematics. All of the happenings in all of the teaching episodes were video recorded. These included the interactions between learners themselves and with me while participating in mathematical activities. The teaching episodes included were the interactions that occurred while I enacted the five key strategies of assessment for learning during the teaching and learning of mathematics.

4.3 TEACHING EXPERIMENT 1: INTRODUCING PARABOLIC FUNCTIONS

Background of Teaching Experiment 1

Mathematical activities on parabolic functions were assigned to learners in this teaching experiment. The primary objective was for them to learn about the shape of a parabola, how to sketch a parabolic function using the table method, as well as how to use intercepts and a turning point to sketch the parabola. My facilitation during the teaching experiment was guided by the five key strategies of assessment for learning as defined in William and Thompson's (2007) framework. My objective for this teaching experiment was also to account for learners' mathematical understanding of parabolic functions according to Kilpatrick, Swafford and Findell's (2001) theory of mathematical proficiency, which includes conceptual understanding, procedural fluency and strategic competence, by exploring and explaining what learners say or do while engaging in mathematical activities (Steffe and Thompson, 2000).

4.3.1 Teaching Episode 1: Parabolic Standard Form and Sketching Parabola Using a Table

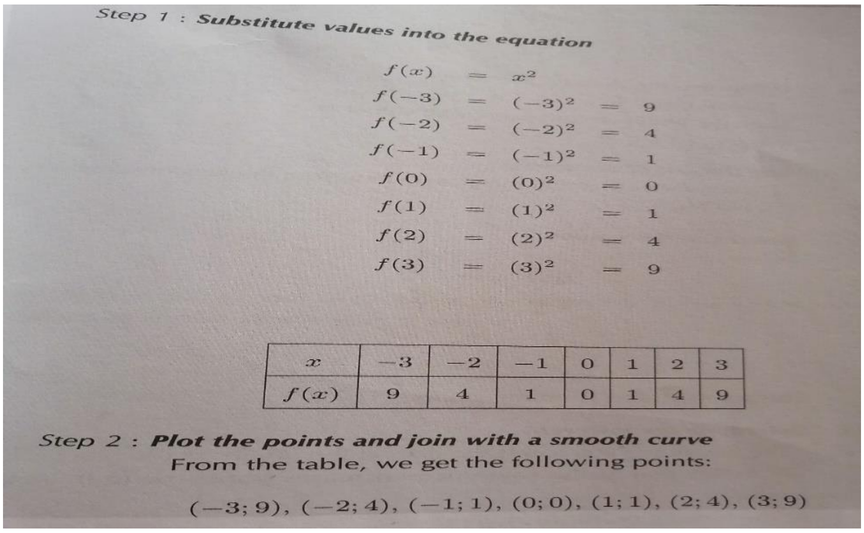
In this teaching episode, I showed learners examples of parabolic functions, their shapes, and the standard form of a parabolic function $y = ax^2 + q$ from the Siyavula mathematics Grade 10 textbook. In addition, in figure 4, I provided the learners learning activity 1 in which they were supposed to deduce the effect of "a" in a parabolic function, and sketch the parabolic functions using the table method first and then the Intercepts method. I facilitated the lesson using William and Thompson's (2007) five key strategies of assessment learning. The strategies were critical in inviting learners' participation throughout the teaching episode so that I could account for their mathematical understanding by discovering what learners were doing or saying as they engaged in mathematical activities (Steffe and Thompson, 2000).

This is what transpired in the classroom:

Teacher (myself):

The learning objectives for today's lesson are on parabolic functions, which include the shape of a parabola by deducing the effect of "a", sketching the parabola using the table method as well as using the intercepts and turning the point. Furthermore, you are going to write the two learning activities starting with learning activity 1 in figure 4. Right now, open your Siyavula textbooks to see an example of a parabolic function, which is in the form of $y = ax^2 + q$.

Learners followed the my lead and flipped through their textbooks looking for examples of parabolic functions. They saw the shapes of parabolic functions and what to expect when sketching a parabola. Figures 1 and 2 show the shape of the parabola that learners saw in their textbooks as an example of an acceptable shape for a parabolic function.



Step 1 : ***Substitute values into the equation***

$$
\begin{aligned}
f(x) &= x^2 \\
f(-3) &= (-3)^2 = 9 \\
f(-2) &= (-2)^2 = 4 \\
f(-1) &= (-1)^2 = 1 \\
f(0) &= (0)^2 = 0 \\
f(1) &= (1)^2 = 1 \\
f(2) &= (2)^2 = 4 \\
f(3) &= (3)^2 = 9
\end{aligned}
$$

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

Step 2 : ***Plot the points and join with a smooth curve***

From the table, we get the following points:

$(-3; 9)$, $(-2; 4)$, $(-1; 1)$, $(0; 0)$, $(1; 1)$, $(2; 4)$, $(3; 9)$

Figure 1: An example of using the table method to find the values of a parabolic function

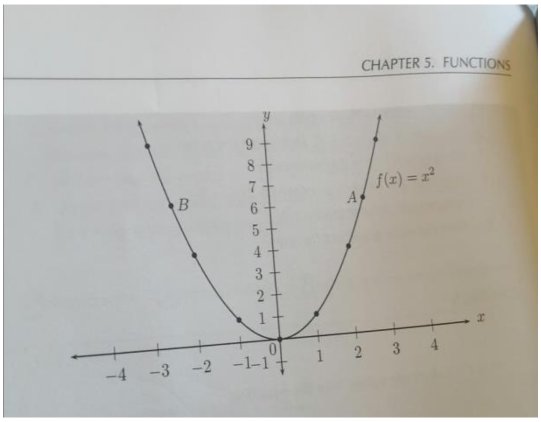


Figure 2: the shape of a parabolic function

However, before the learners write the learning activity in figure 4 below by deducing the effect of "a" in the parabolic function standard form $y = ax^2 + q$, I began by asking them straight-line function-based questions in order to uncover their prior knowledge of straight-line graphs leading to parabolic functions. I asked them questions in order to persuade them to willingly participate in classroom discussions, and to collaborate with their peers to assist in uncovering their prior knowledge in order to stimulate their new understandings before they could proceed with learning activity 1 in figure 4, a practice supported by Black and William (2018).

Classroom discussion between myself and the learners

Teacher*:* How is the shape of a straight-line graph?

Prince: It is just a straight-line plotted on the Cartesian plane

Teacher: Do you all agree with what he just said?

Class: Yes.

Teacher: Can someone come and make a rough sketch of straight-line graphs on the board?

Figure 3: A rough sketch of a straight-line graph

Teacher: Is the sketch correct?

Class: Yes.

Teacher: What is the standard formula of a straight-line graph?

Makgabo:$y = mx + c$

Teacher: What is "m" in the formula?

Class: Gradient.

Teacher: How do you sketch a straight-line graph?

Adelaide: In Grade 9 we used ordered pairs to sketch a straight-line graph.

Kholo: Yes, if you have (1; 2), (3; 4), (5; 6) you plot them on the Cartesian plane and we draw the graph.

Teacher: How did you find ordered pairs?

Pharcily: Sometimes the ordered pairs will be given to us and we will plot.

Monica: What if ordered pairs are not given?

Prince: We will complete the table as we did with the flow diagram and use input (x) values to find output values (y).

Learners appeared to have solid prior knowledge of straight-line graph sketching, so I told them to sit in groups of three and answer learning activity 1 in figure 4. They were expected to complete the table using the given parabolic functions, plot all the graphs on the same set of axes, and deduce the effect of "a " in a parabolic function $y = ax^2 + q$. I wanted to find out what they would say or write when deducing the effect of "a" on parabolic functions before intervening and guiding them. According to Steffe and Thompson (2000), during the responsive interaction with learners in the teaching experiment, the teacher-researcher should try to comprehend what they can do when engaging in mathematical activities rather than dismissing their knowledge as immature or incorrect.

Learning activity 1: Complete the table and plot the following graphs on the same system of axes and use your results to deduce the effect of "a" for the graphs of $y = ax^2 + q$

x	-2	-1	0	1	2
$y_1 = -x^2 - 2$					
$y_2 = x^2 - 1$					
$y_3 = x^2$					
$y_4 = -2x^2$					

Figure 4: Learning activity 1: deducing the effect of "a"

Benad's group discussion for learning activity 1 in figure 4 above.

Adelaide: Where do we start?

Benad: Since we must complete the table first, let's use the table method the same way as in the straight-line graph.

Monica: Okay, we use the equation of $y_1 = -x^2 - 2$ and where we see "x" we substitute with the values of x on the table to find y values.

Adelaide: Okay, let's do it, I will be the scriber,

Monica: When I substitute $x = -2$ into $y_1 = -x^2 - 2$, then $y_1 = - - 2^2 - 2 = -2$ and when I substitute $x = -1$ then $y_1 = - - 1^2 - 2 = -1$

Benad: No, Monica that is not correct.

Monica: Why? I used the calculator to get the answer.

Benad: Okay, but you did not include the -2 inside the bracket.

Adelaide: Oh yah, Benad is correct. The negative number should be inside the bracket like this $y = -(-2)^2 - 2 = -6$.

Monica: Okay, let me do it the right way, so when I substitute $x = -1$ then $y_1 = -(-1)^2 - 2 = -3$.

Benad: Yes that's correct, let's calculate like this for all the graphs.

The excerpt above demonstrated how learners could accept criticisms from their peers and to correct their mistakes after they have been corrected by their peers. Benad's group completed the table and sketched all of the graphs by working as peers. Their solution is shown in figure 5 below. Furthermore, the group correctly used their prior knowledge to assist each other in completing tables of straight-line graphs and sketched the correct parabolic functions in figure 5.

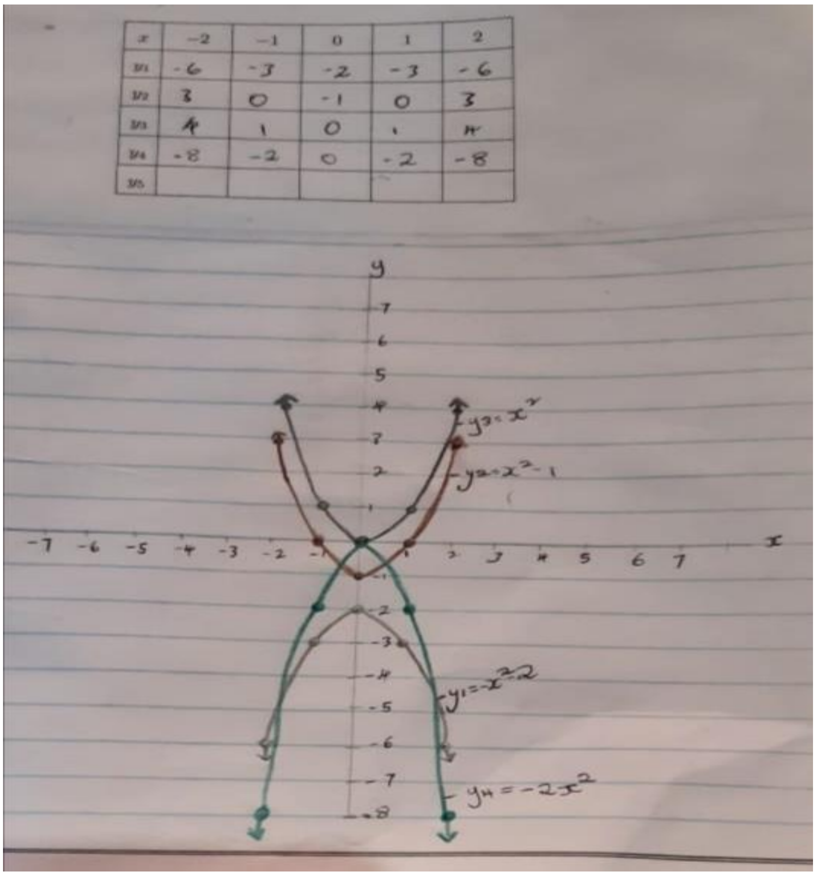


Figure 5: Benad's group plotted graphs for Learning Activity 1

Despite the fact that Benad's group had not yet written their conclusion about the effect of "a" in the parabolic function, I provided feedback on what they had already written.

Teacher: Your table is correctly completed and your graphs are correctly sketched, what is your conclusion about the effect of "a"? What have you noticed while sketching?

Benad: Eish ma'am, we are still trying to figure out the effect of "a".

Teacher: Okay, let me give you a chance to find out.

I was pleased with Benad's group's performance because they correctly sketched parabolic functions, and I gave them time to figure out their conclusion on the effect of "a" before moving on to other groups. As I moved around the classroom, I noticed that the majority of the groups, like Benad's, were correctly completing the tables and sketching the graphs with the assistance of their peers. However, Kholo's group did not complete all of the values in the table and their graphs in figure 6 below were not the same as the graphs of the other groups. I then initiated a conversation to assist them in identifying their mistakes prior to sketching all of the graphs.

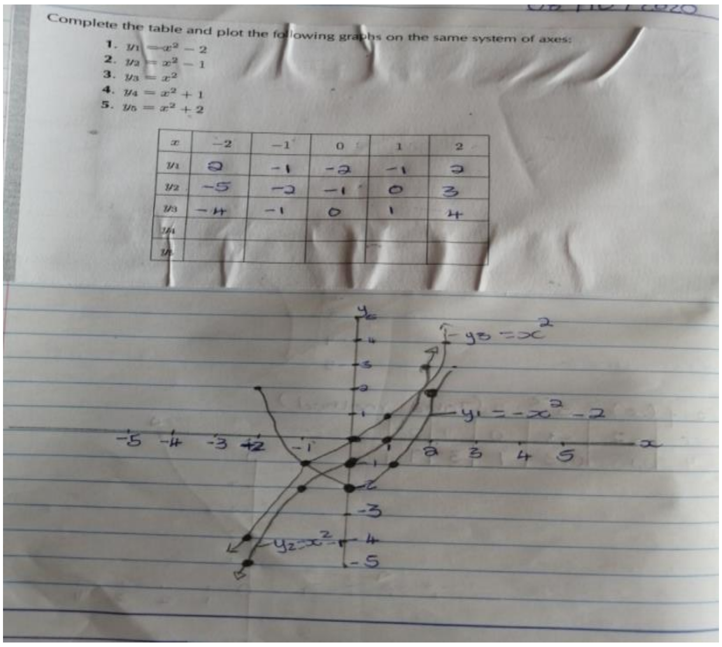


Figure 6: Kholo’s group plotted graphs

A discussion with Kholo’s group concerning solution in Figure 6

Teacher: Your graph of y_1 is correct. My concern is what happened to the graph y_2 and y_3, they do not look like parabolic functions.

Lesego: I am also surprised, ma’am.

Pharcily: But we found the values using a calculator to complete our tables and sketched the graphs.

Kholo: Yes but still the two graphs ma’am is talking about do not look like parabolic functions.

Lesego: I think we made the mistakes somewhere because the shape of parabolic functions do not look like the graphs we have drawn for the graphs of y_2 and y_3.

Teacher: Then find out what could be the mistake and fix it.

Pharcily: We used the calculator to find the values and used the values to plot the graphs. I do not know what went wrong.

Teacher: Recalculate your values and see if there is no mistake in your values.

.

According to the discussion above, Kholo's groups struggled to draw the correct graphs, and all learners in the group could not see what went wrong or how to fix their mistakes. I then asked Benad to assist Kholo's group in determining their mistakes and assisting them in sketching the correct graphs as part of peer assessment.

A discussion between Benad and Kholo's group concerning their solution in Figure 6

Benad: Okay guys let me see your parabolic functions.
Lesego: Here they are (Lesego pointed at the graphs that she and the group sketched in figure 6).
Pharcily: We are failing to see where we went wrong because we used the values we calculated and sketched the graphs.
Benad: How did you calculate the value -5 in your table for the graph of $y_2 = x^2 - 1$?
Kholo: We substituted -2 into $y_2 = x^2 - 1$ which became $y = -2^2 - 1$ and the answer is -5.
Lesego: And we calculated all the values like what Kholo did using our calculators.
Benad: Oh ya, that's where you went wrong, you put -2 inside the bracket like this $y = (-2)^2 - 1$ and the answer will be 3 and you continue like this for all the values.
Pharcily: I see, we should include the negative numbers in the brackets.
Lesego: When substituting by -1 the answer will be like this $y = (-1)^2 - 1 = 0$.
Benad: Yes that's correct, rework your calculations and you will be able to sketch the correct parabolic functions with correct shapes.

Benad was able to assist Kholo's group to identify the mistakes they made when calculating their values to sketch the parabolic functions using the table method based on the extract above. Finally, Kholo's group completed their table and correctly sketched all the graphs, which resembled Benad's group graphs in figure 5 above. My observer and I used the method of requesting learners from groups that completed their tables and correctly sketched all of the parabolic functions to assist other groups that had difficulty working with negative numbers.

I observed that although the majority of learners, with the assistance of their peers, were able to correctly sketch the graphs, they had difficulty deducing the effect of "a" in a parabolic function. I returned to Benad's group to see if they had reached a conclusion, but they were still unable to deduce the effect of "a." I then intervened in an attempt to motivate learners to deduce the effect of "a" without imposing any opinions on them. I intervened by directing learners to their Siyavula textbook to examine the image in figure 7 below, and then proceeded to engineer effective questioning to stimulate new understandings in learners so that they could deduce the effect of “a ” on their own while guiding them. Because learning in a constructivism class entails acquiring knowledge through interactions with the teacher, learners must be active participants (Guba & Lincoln, 1994; Mabotja, 2017).

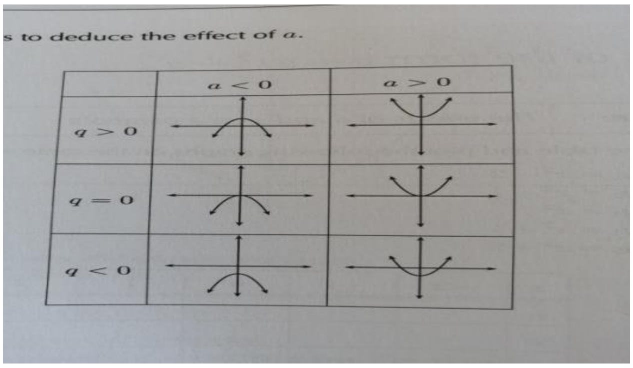


Figure 7: Parabolic functions showing the effect of “a”

Teacher: When you look at figure 7 with pictures of parabolic functions, what do you notice about the shapes of the graphs?

Teacher: What is the difference between the graphs on your left versus those shapes on your right?

Monica*:* The graphs on the left are facing downward and the ones on the right are facing upward.

Teacher: Why do you think this is the reason?

Monica: I do not know.

Teacher: Anyone who wants to try?

Teacher: The graphs on the left are facing down because of the effect of “a” in the parabolic function standard form $y = ax^2 + q$. When “a” is positive, the graph faces upwards and when “a” is negative, the graph faces downward.

Prince: Oh that makes sense, even on our solutions in the graphs where the values of "a" are negative, the graphs are facing down, and where the values of "a" are positive, the graphs are facing upwards.

Teacher: On the board, we have the following two functions, determine the signs of "a" and tell me what will happen to the shape of the graph?

Functions on board

1. $Y = x^2 + 1$
2. $Y = -2x^2 + 1$

Figure 8: Parabolic functions on board

Lesego: The value of "a" in graph number one is positive 1 and the graph will face upward.

Benad: The value of "a" in graph number two is negative 2 and the graph will face downward.

The learners were given reflective journals (Appendix A) to complete just 5 minutes before the period ended.

Analysis of episode 1

In this episode, I shared and clarified the lesson's learning objectives, and instructed the learners to consult their textbooks to see examples of parabolic functions in figures 1 and 2 above and how their shapes should look like. Learners were shown the success criteria for a parabolic function as well as how good the parabola should look like. The strategy of clarifying and sharing learning intentions with learners appeared to work in this episode because they seemed to pay attention to what I said and what they saw in their textbooks. I also asked effective probing questions to learners in order to uncover their prior knowledge of functions. During questioning, learners were able to recall straight-line graphs from previous lessons and grades. In addition, I assigned mathematics learning activity 1 in figure 4 to learners to complete in groups in order to elicit evidence of learning. Learners were expected to complete the table with parabolic functions, plot all graphs on the same set of axes, and deduce the effect of "a" during the learning activity in a parabolic function of $y = ax^2 + q$ thereafter. The majority of the learners knew what to do when it came to filling out the tables because they had previously learned how to draw straight-line graphs using the table method. Nevertheless, most of them were struggling to work with negative numbers, specifically when substituting -2 in the equation $y = -x^2 - 2$, some of the learners wrote that the answer is -2 instead of -6. This was a clear indication that they lacked basic algebra knowledge and a conceptual understanding of working with negative numbers in equations. As a result, they were unable to create the conceptual relationships required to sketch the parabolic function. They were unable to make connections between negative number substitutions into the quadratic equation in this scenario. However, to address the difficulties that learners encountered when working with negative numbers, I probed them to identify their mistakes, and requested that those who were able to work with negative numbers, such as Benad, assist those who were unable to obtain correct answers when working with negative numbers. Collaborative learning with peers produced positive results because the majority of learners did learn how to substitute negative numbers into their parabolic functions and correctly sketched the parabolas.

During the classroom discussions, I provided learners with feedback that helped them progress. I further acknowledged the correct answers that they provided. In addition, as I passed by the groups while they were engaged in mathematical learning activity 1 in figure 4, I used comment-only feedback with no scores. My comments were task-focused because they prompted learners to think about and strive for the correct solution. For example, when I realised that Benad's group had completed the table and sketched all of the parabolic functions correctly, I gave them task-focused feedback in which they were told that they had sketched all of the functions correctly and that they should continue deducing the effect of "a" in the parabolic functions that they had drawn. Some groups, such as Kholo's, had more problems because they could not correctly sketch all of the parabolic functions. My attempt to provide them with feedback that moved them forward through comments and probing aided the group in determining that there was something wrong with their sketches because they did not resemble parabolic functions. This outcome is supported by Lee (2006), who indicated that giving learners useful comments when working on tasks that they deem difficult paves a way for them to move forward in their learning. In terms of my comments, learners in Kholo’s group were able to detect their mistakes in answering the learning activity 1 in Figure 4. At the same time, I realised that the group lacked conceptual understanding of certain concepts, which made it difficult for the learners to find ways to correct their mistakes. Learners lacked conceptual understanding because they did not have a solid understanding of the mathematical concepts that led to the sketching of parabolic functions, such as substituting negative numbers in quadratic equations and understanding the relationships between different concepts (Department of Education, 2018).

Otherwise, I incorporated the use of peer feedback, which proved to be more effective. I asked Benad's group to peer-assess Kholo's group in order to have learners work as resources for one another, and Benad did an excellent job of assisting Kholo's group in seeing their mistakes and fixing their work. Similarly, for other groups that had difficulty sketching graphs due to inability to work with negative numbers correctly, I picked learners who were able to sketch the correct parabolas in order to assist them, which yielded positive results. As a result, struggling learners were more open to criticisms from their peers and corrected their mistakes more easily because they were more comfortable taking criticisms from their peers than from me as their teacher.

As I read learners’ reflective journals, most of them indicated that they were struggling with sketching graphs y_1 and y_2 using the table method. Again, they struggled with finding a conclusion on the effect of “a” in parabolic functions of $y = ax^2 + q$. However, the majority of learners indicated that they learnt better through their peers as they were able to correct their mistakes in graphs y_1 and y_2. What learners wrote in their reflective journals resonated with what I observed during the classroom interactions and through their written work in learning activity 1. The use of reflective journals encouraged them to take ownership of their learning because, when asked, "what is next," they realised they were struggling with substituting negative numbers. Most of them wrote that they needed to learn more about how to complete tables given negative numbers, while others mentioned that they needed to learn more about working with graphs in which the value of "a" is a negative number.

Note taking –teacher’s reflective journal (see Appendix B)

During the teaching episode, I used all five key strategies of assessment for learning. The strategies appeared to make a difference in inviting learners to participate and engage in the lesson. The observer, on the other hand, indicated that I did not provide learners with adequate wait time during questioning, as required by William and Thompson's (2007) theory of the five key strategies of assessment for learning. The suggestion was that in the next teaching episode and going forward, I should be more careful during questioning and give learners enough time to answer the questions before jumping in to answer. For example, when they were struggling with the deduction of "a" in quadratic functions $y = ax^2 + q$, in my quest to make them understand, I asked them questions related to the effect of "a" in $y = ax^2 + q$. However, in questions where they took time to respond, I quickly answered the questions myself. For instance, when I asked them why they thought some parabolas were facing downwards and others were facing upwards, I quickly told them that when the value of "a" is positive, the graph will face upward, and when the value of "a" is negative, the graph will face downward without allowing adequate time for them to respond.

What I expected did not occur; as the plan was such that learners would complete all of the lesson objectives which were to sketch the parabolic functions using both the table and the intercepts methods. Since they had seen the table method in previous grades or lessons when sketching straight-line graphs, I thought they would be able to draw parabolic functions using the method with ease, and quickly move on to sketching parabolic functions using the intercepts method. However, I realised that their inability to work with tables fluently was due to a lack of conceptual understanding of working with equations. One of the strands of mathematical proficiency that allows learners to acquire a proper understanding of mathematical ideas, concepts and operations, as well as the relationships between different concepts, is conceptual understanding (Department of Education, 2018). When working with negative numbers, learners lacked conceptual understanding of how to complete the tables. This slowed their progress in the lesson, but what I found interesting was that they were able to grasp the areas where they were lacking through peer assessment.

The observer and I planned to continue with the learning objectives from teaching episode one in the next teaching episode, teaching learners how to sketch parabolic functions using intercepts and turning points. Moreover, to address the shortcomings that I observed in my facilitation for teaching episode one, such as not giving learners enough time to answer questions, in the next teaching episode and going forward, I am going to be more careful with questioning. I will also give learners enough time to answer questions before jumping to answer them myself. According to Black and William (1998), teachers should allow enough time for learners to attempt answering the questions. Lee (2006) further states that learners require three to five seconds wait time in whole-class questioning, or 30 seconds to one minute if they are to talk to their peers.

4.3.2 Teaching Episode 2: Sketching Parabolic Functions Using Intercepts and Turning Point

Background of teaching episode 2

In this teaching episode, I worked with learners to recap finding the x and y intercepts in a straight-line graph before moving on to finding the x and y intercepts in a parabolic function. I further allowed learners to work in pairs to learn how to sketch the parabolic function using the intercepts method. This was accomplished by guiding the pairs to use their Grade 10 Siyavula mathematics textbooks to go through the example of sketching the parabolic function using the intercepts and turning point in figures 10.1 and 10.2 below together. In addition, I assigned learners a mathematics learning activity 2 in figure 11 consisting of two parabolic function questions to elicit evidence of learning and to assess their mathematical understanding at the end of the lesson. My planning ensured that learners could correctly sketch the two parabolic functions in the learning activity.

This is what transpired in the classroom:

Teacher: Today we are going to learn how to sketch a parabolic function using intercepts and a turning point.
Teacher: How did we draw a parabolic function in the previous lesson?
Adelaide: Using the table method.
Matle: And plot the points using the table.
Teacher: Good, as we are going to find intercepts in a parabolic function, let's recap on how to find x and $y - intercepts$ in a straight-line graph.

Monica: Ma'am, I know, we let $x = o$ to find $y - intercept$.

Kholo: And we let $y = 0$ to find $x - \ intercept$.

Teacher: Now, we are going to learn how to draw the graph using x and $y - intercept$ and the turning point.

Teacher: Who is willing to come and draw the Cartesian plane on the board?

Prince: I will draw it ma'am (he drew the Cartesian plane with y and x-axis).

Figure 9 depicts the Cartesian plane that Prince drew on the board.

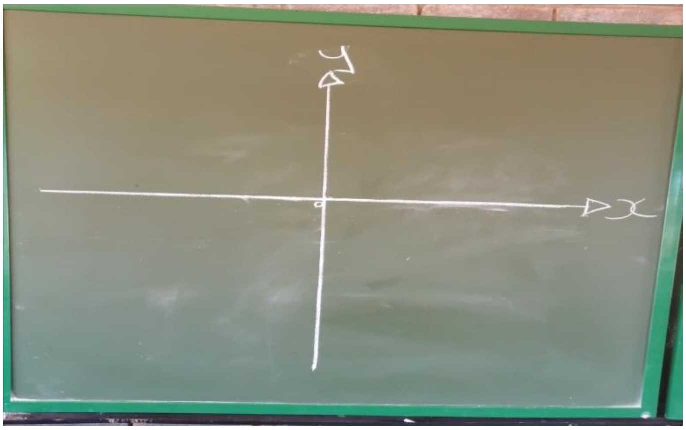


Figure 9: Cartesian plane on the board by Prince

Teacher: Look on the vertical axis of the Cartesian plane, what do you see?

Adelaide: It is just a straight line with y.

Teacher: On the horizontal axis, what do you see?

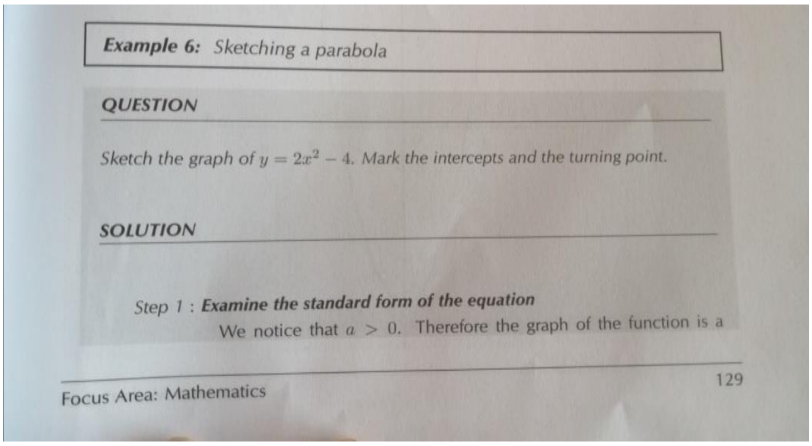
Benad: A line with arrow and x.

Teacher: What are these observations telling you?

Monica: I think it means you can never find x on a vertical axis and y on a horizontal axis.

Prince: Yes, that is why when we find $y - intercept$ we let $x\ = o$ and $x - intercept$ we let $y\ = 0$.

Teacher: That is correct, go to page 129 of your textbooks (with figures 10.1 and 10.2 below), and go through example 6 showing how to sketch a parabolic function and discuss it in pairs before we proceed with the learning activity for today.


Example 6: *Sketching a parabola*

QUESTION

Sketch the graph of $y = 2x^2 - 4$. Mark the intercepts and the turning point.

SOLUTION

Step 1 : ***Examine the standard form of the equation***
We notice that $a > 0$. Therefore the graph of the function is a

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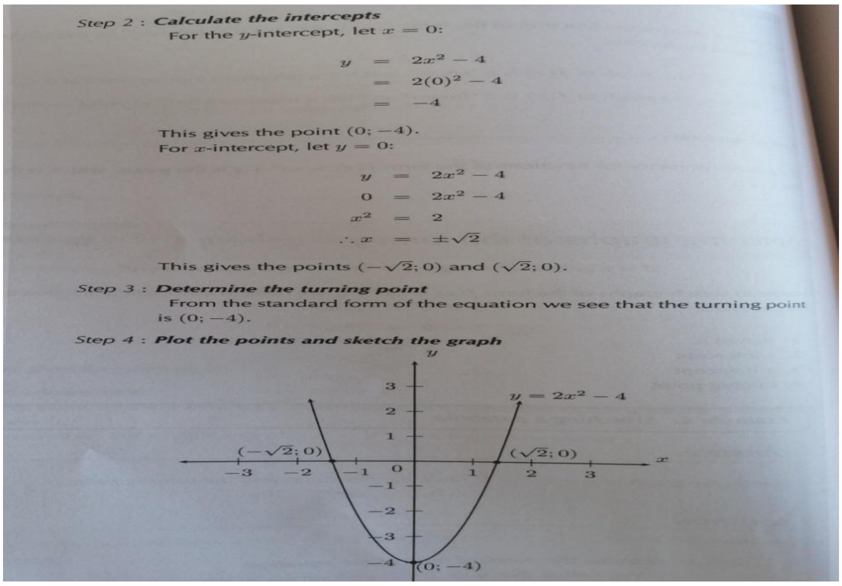
Figure 10.1: Example of sketching a parabola picture 1


Step 2 : ***Calculate the intercepts***
For the y-intercept, let $x = 0$:

$$y = 2x^2 - 4$$
$$= 2(0)^2 - 4$$
$$= -4$$

This gives the point $(0; -4)$.
For x-intercept, let $y = 0$:

$$y = 2x^2 - 4$$
$$0 = 2x^2 - 4$$
$$x^2 = 2$$
$$\therefore x = \pm\sqrt{2}$$

This gives the points $(-\sqrt{2}; 0)$ and $(\sqrt{2}; 0)$.

Step 3 : ***Determine the turning point***
From the standard form of the equation we see that the turning point is $(0; -4)$.

Step 4 : ***Plot the points and sketch the graph***

Figure 10.2: Example of sketching a parabola picture 2

Learners discussed example 6 of sketching a parabolic function in Figures 10.1 and 10.2 above. Furthermore, they used prior knowledge about finding intercepts as well as knowledge gained through classroom discussions to make sense of the scenario. When the groups had finished with the example, I offered them the learning task in figure 11 to complete. The activity consisted of two questions, with learners tasked with answering the first in pairs and the second individually.

Learning activity 2 -Class-work: Find the intercepts and turning point to sketch the following graphs.

1. $y = -x^2 + 4$

2. $y = \frac{1}{2}x^2 + 4$ **Criteria for success**: the correct values of x and y

Using correct algebraic methods, Sketching correct shapes of parabolas according their effects of "a".

Figure 11: learning activity 2 –class work

Learners worked in pairs, explaining to each other how to sketch the parabolic function in question one $y = -x^2 + 4$, and as my observer and I passed by the groups, we noticed that the majority of learners were doing well. They managed to find the correct intercepts, turning points and sketched the graph correctly.

A discussion between Adelaide and Monica for the graph of $y = -x^2 + 4$

Monica: We are given $y = -x^2 + 4$, so we are going to look for x-intercept by substituting y by 0, then $0 = -x^2 + 4$.

Adelaide: Okay.

Monica: Let us transpose 4 to the left and it will become negative, $-4 = -x^2$.

Monica: And then we divide by -1, $\frac{-4}{-1} = \frac{-x^2}{-1}$ and then we will have $4 = x^2$.

Adelaide: Why are you dividing by -1?

Monica: We divide by negative -1 because we are looking for x not – x.

Adelaide: Oh, okay I see and to find x we are going to look for the square roots on both sides of the equation.

Monica: Yes and the values of x will be ± 2.

Adelaide: Yes, you are right from example 6 it means $x = 2 \; or \; x = -2$.

Monica: The x-intercepts will be (2; 0) and (-2; 0).

Adelaide: That is correct, now to find y-intercept we let x to be zero.

Adelaide: $y = -(0)^2 + 4$ and then $y = 4$ then y intercept will be (0; 4).

Monica: Let us sketch the graph.

Figure 12 depicts Monica and Adelaide's ultimate solution to the graph of $y = -x^2 + 4$ with the teacher’s comments.

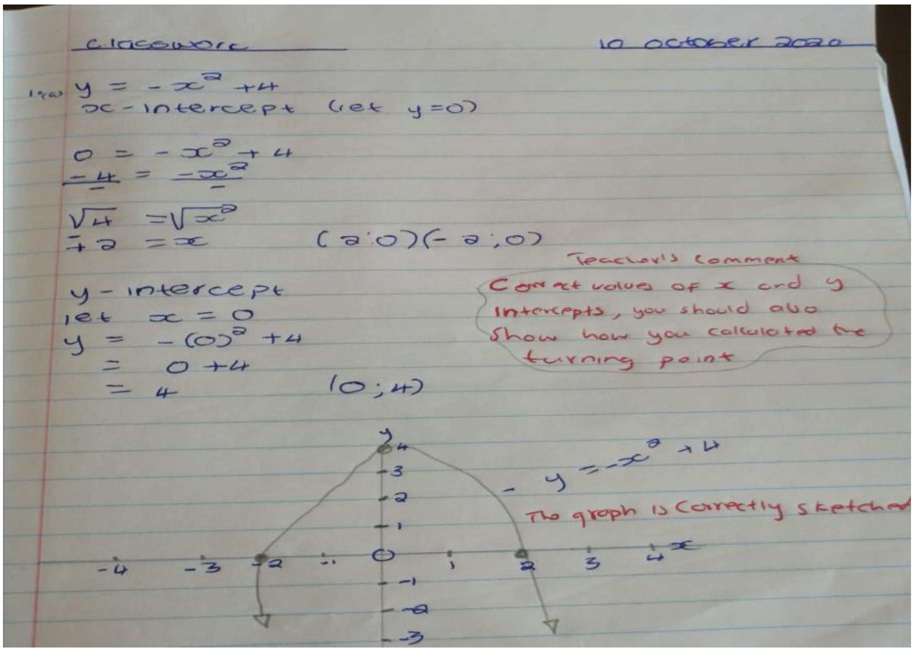


Figure 12: Adelaide and Monica's final solution to the graph of $y = -x^2 + 4$ with the teacher's comments.

The extract above in Figure 12 demonstrates how Adelaide and Monica worked as a team to accurately sketch the parabolic function $y = -x^2 + 4$. Because the value of "a" was negative, the shape of their graph phased down; nonetheless, the intercepts and turning point were appropriately depicted on their sketch. I then provided them with comment-only feedback that helped them progress. I then went to see what the other groups were up to. Prince and Makgabo were also on the right track. See Figure 13 for a part of their conversion and their sketch for the graph of $y = -x^2 + 4$ with the teacher's comments.

<u>A discussion between Prince and Makgabo for the graph of $y = -x^2 + 4$</u>

Prince: To find the y-intercept we substitute x by 0.
Makgabo: Yes, the y-intercept becomes 4 which is (0; 4).
Prince: And the turning point will be (0; 4).

Makgabo: Oh yes, so now let us check, will our parabolic function smile or frown?

Prince: The graph will face down because the value of "a" is negative.

Makgabo: You are correct, we have everything now, let us plot sketch the graph.

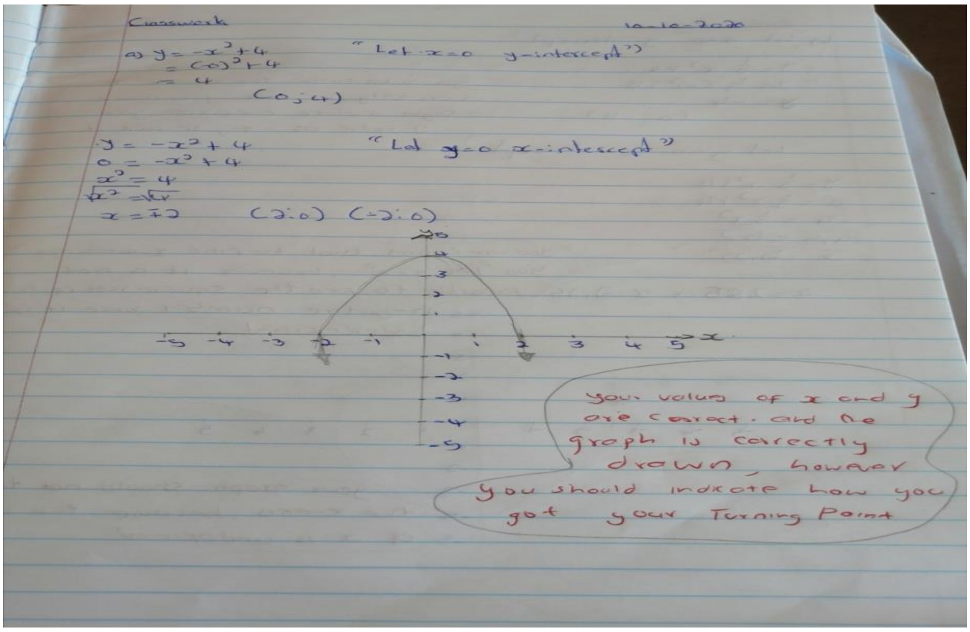


Figure 13: Prince and Makgabo's final solution to the graph of $y = -x^2 + 4$ with teacher's comments

From the comments I made on Prince and Makgabo's sketch in figure 13 above they proceeded with their discussions to address the comments.

Makgabo: Mem commented by saying that we should indicate how we got the turning point

Prince: Yes the turning point is (0; 4)

Makgabo: Why?

Prince: Look at example 6 that they used (o, q) to find the turning point.

Makgabo: Okay, but why (0; q).

Prince: I think it is because our graph turns at the y-intercept.

As I moved around the classroom, I noticed that all learners were able to correctly sketch the first question. Activating learners as resources for one another was critical in motivating them to sketch the graph of $y = -x^2 + 4$ without the help of the teacher. They were able to easily correct the minor mistakes they made while working in pairs, demonstrating that they were taking control of their learning. Figure 14 shows one of the graphs of $y = -x^2 + 4$ from one of the pairs, with the teacher's comments. They managed to sketch their graph correctly.

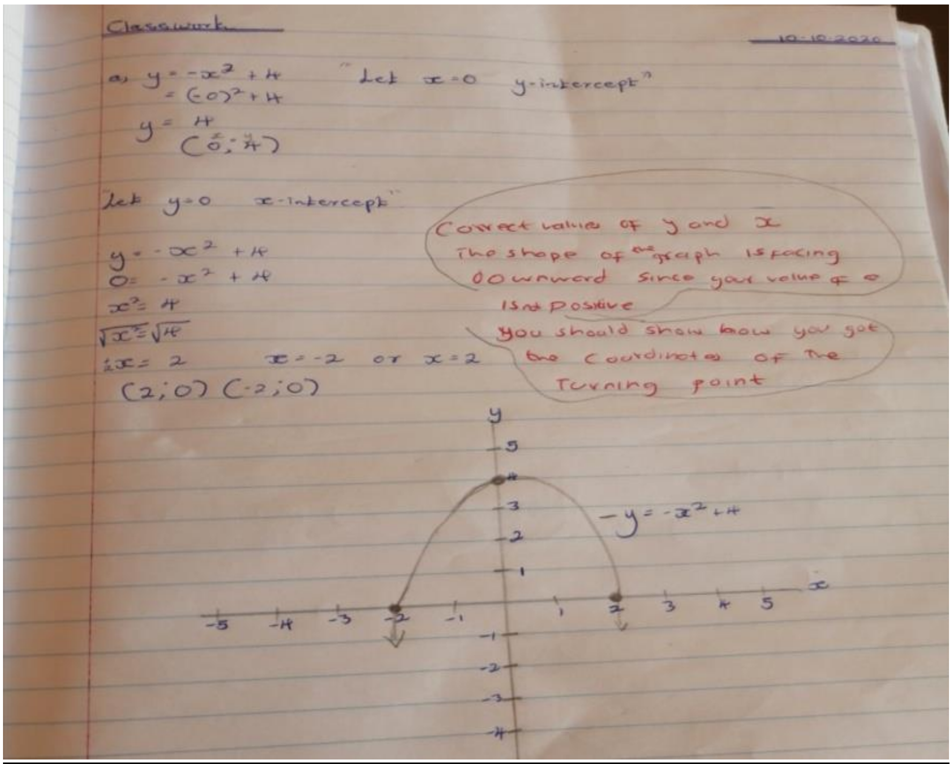


Figure 14: Benad and Kholo's final solution to the graph of $y = -x^2 + 4$ with teacher's comments

Since all of the groups had completed the first question of sketching the graph of $y = -x^2 + 4$, I instructed the learners to proceed with question two about sketching the graph of $y = \frac{1}{2}x^2 + 4$ individually. I intended to provide each learner comment-only feedback on their work so that they may work on the comments on their sketches of $y = \frac{1}{2}x^2 + 4$. However, time has elapsed and the period ended before I could comment on everyone's work. Even those on whose work I had commented had not yet responded to the comments. Some were still attempting to figure out how to correct their mistakes. I then told the learners that they would act on the comments in the next teaching episode. Fortunately, they had finished the question. I handed them reflective journals to fill out, and I collected their books to finish writing the comments on each learner's book so that they may answer in the next teaching episode.

Below are some of the pictures of the graph of $y = \frac{1}{2}x^2 + 4$ and the teacher's comment-only feedback in learners' books:

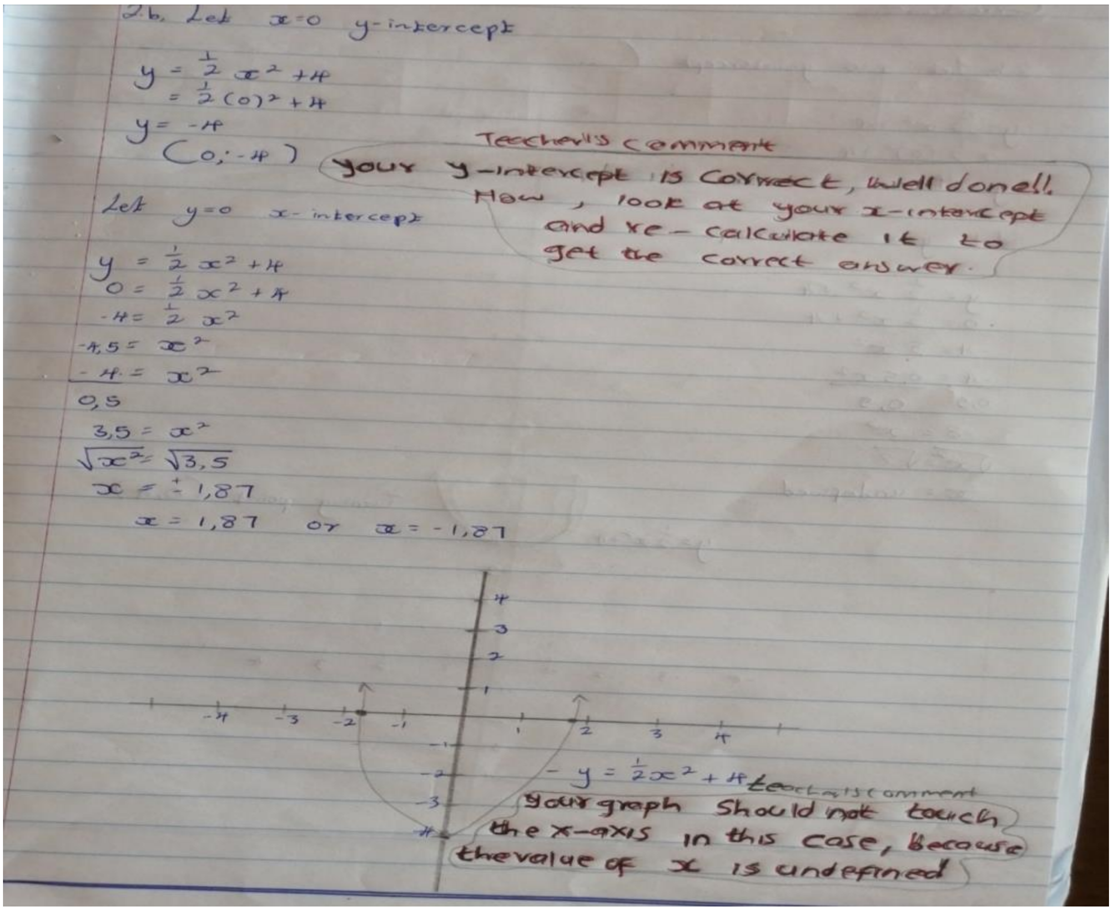


Figure 15: Benad's solutions to the graph of $y = \frac{1}{2}x^2 + 4$ with teacher's comments

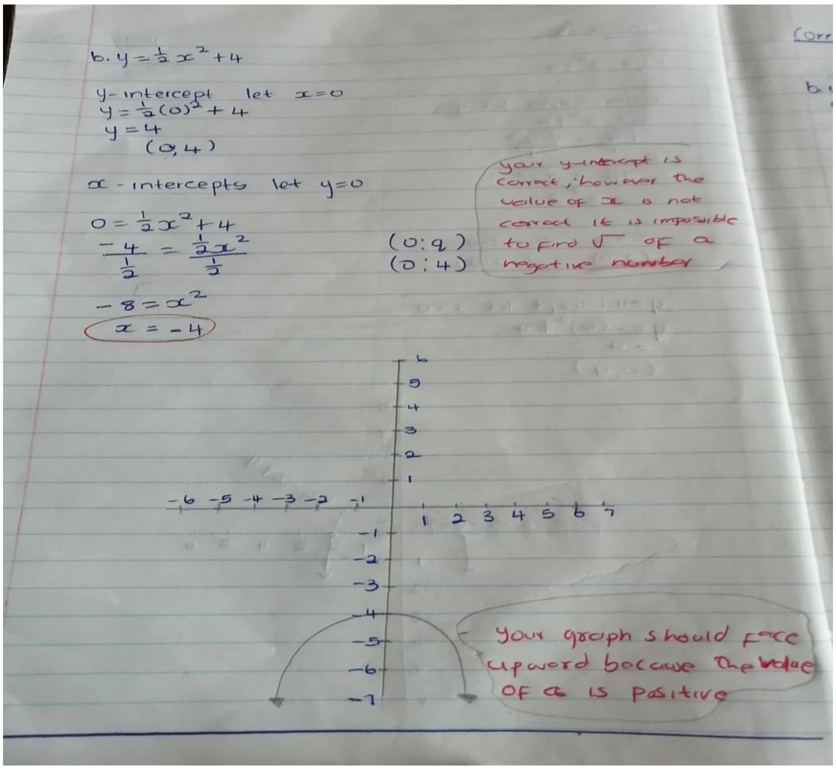


Figure 16: Kholo's solutions to the graph of $y = \frac{1}{2}x^2 + 4$ with teacher's comments

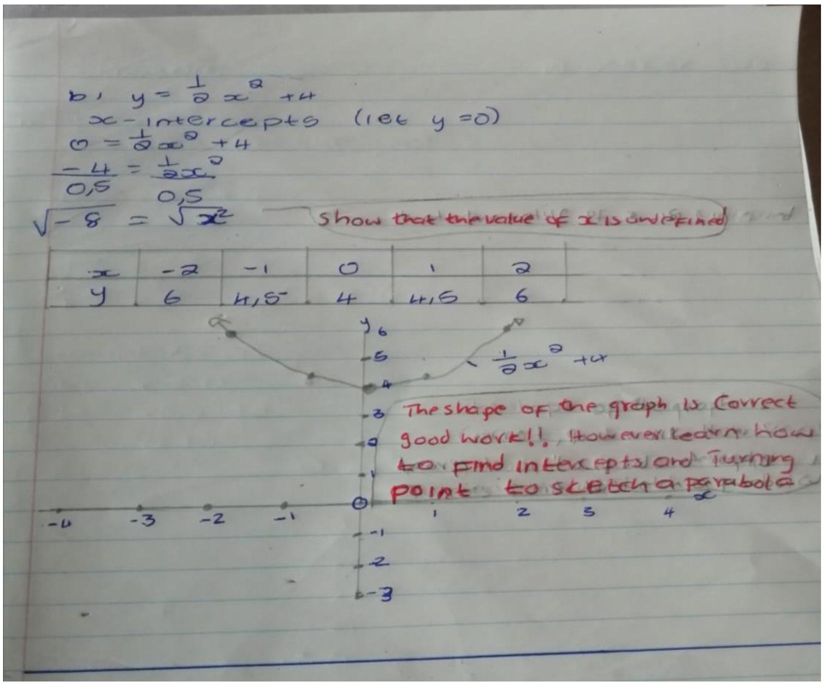


Figure 17: Adelaide's solution to the graph of $y = \frac{1}{2}x^2 + 4$ teacher's comments

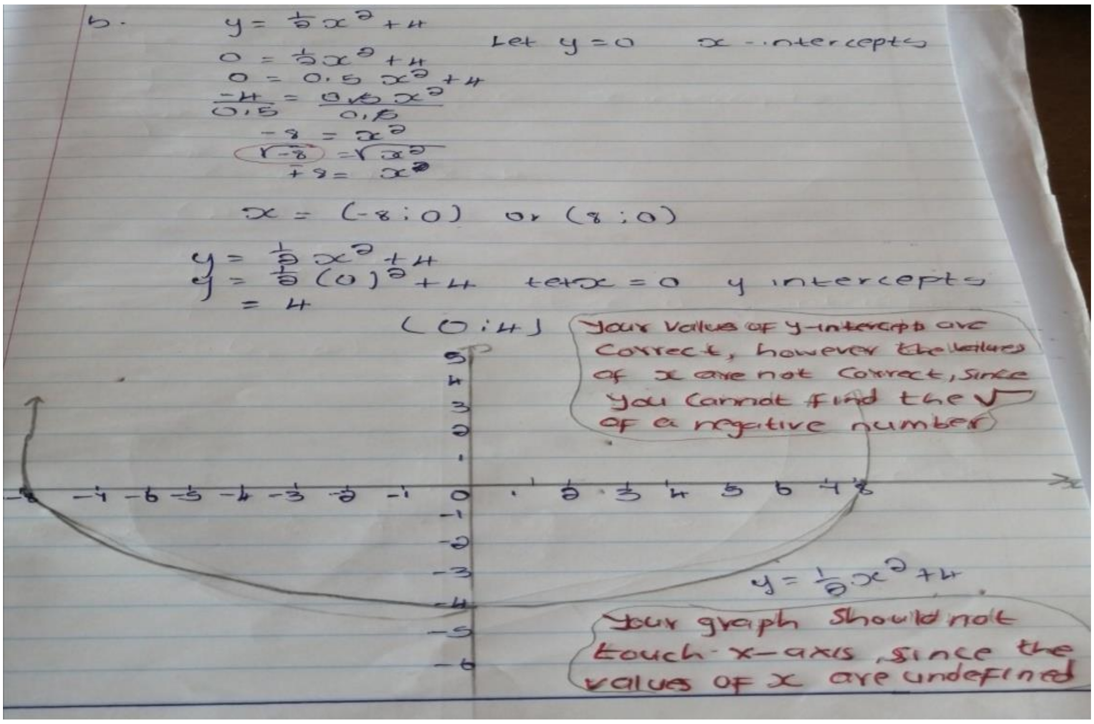


Figure 18: Lesego's solution to the graph of $y = \frac{1}{2}x^2 + 4$ with teacher's comments

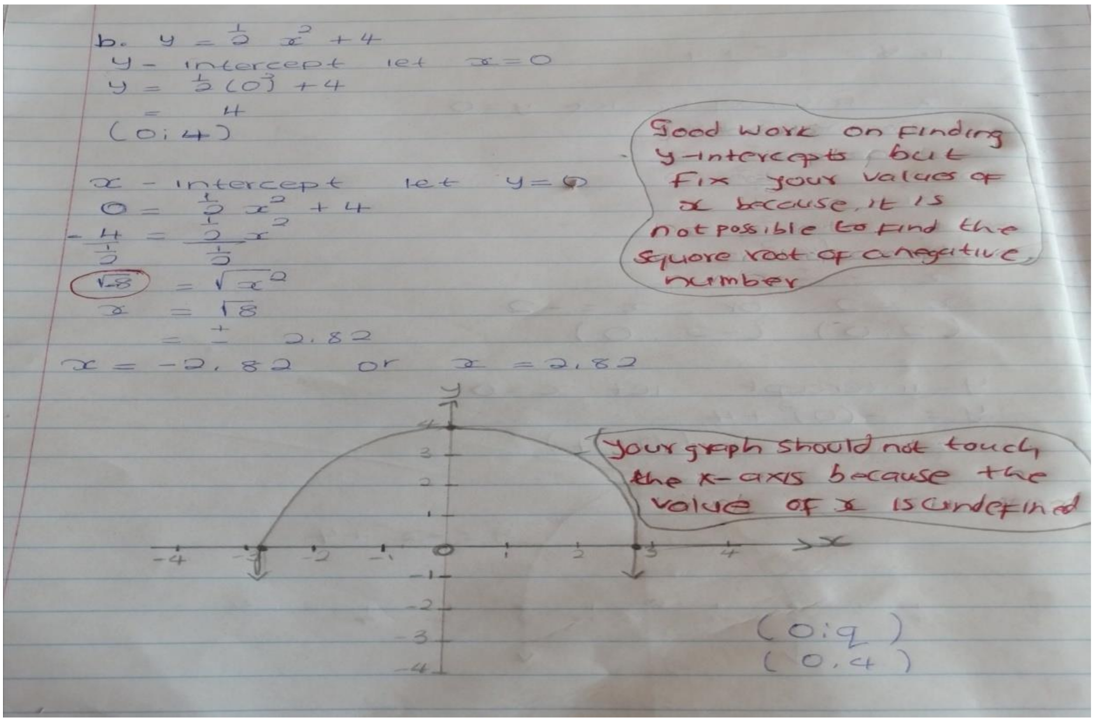


Figure 19: Monica's solution to the graph of $y = \frac{1}{2}x^2 + 4$ with teacher's comments

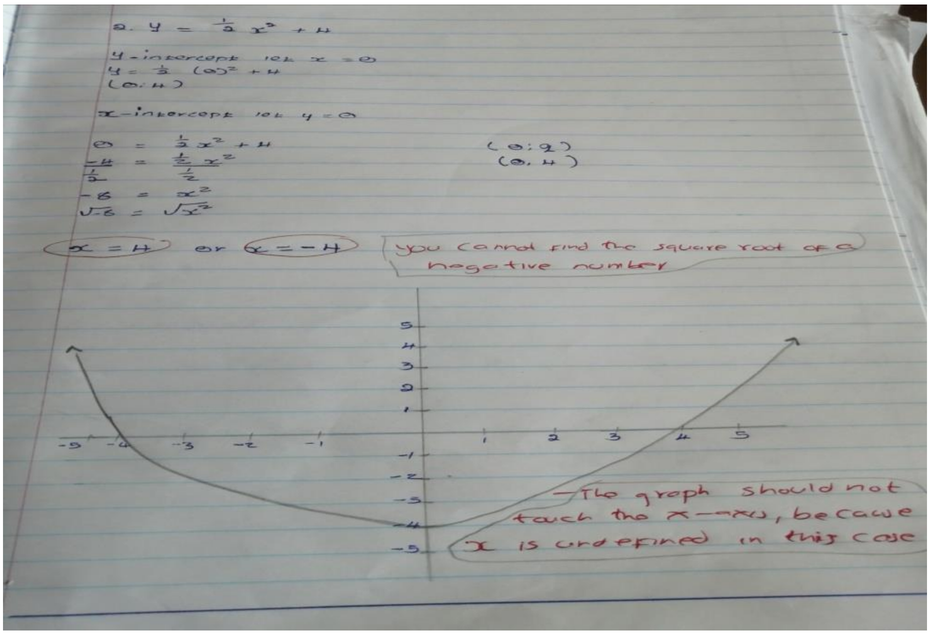


Figure 20: Pharcily's solution to the graph of $y = \frac{1}{2}x^2 + 4$ with teacher's comments

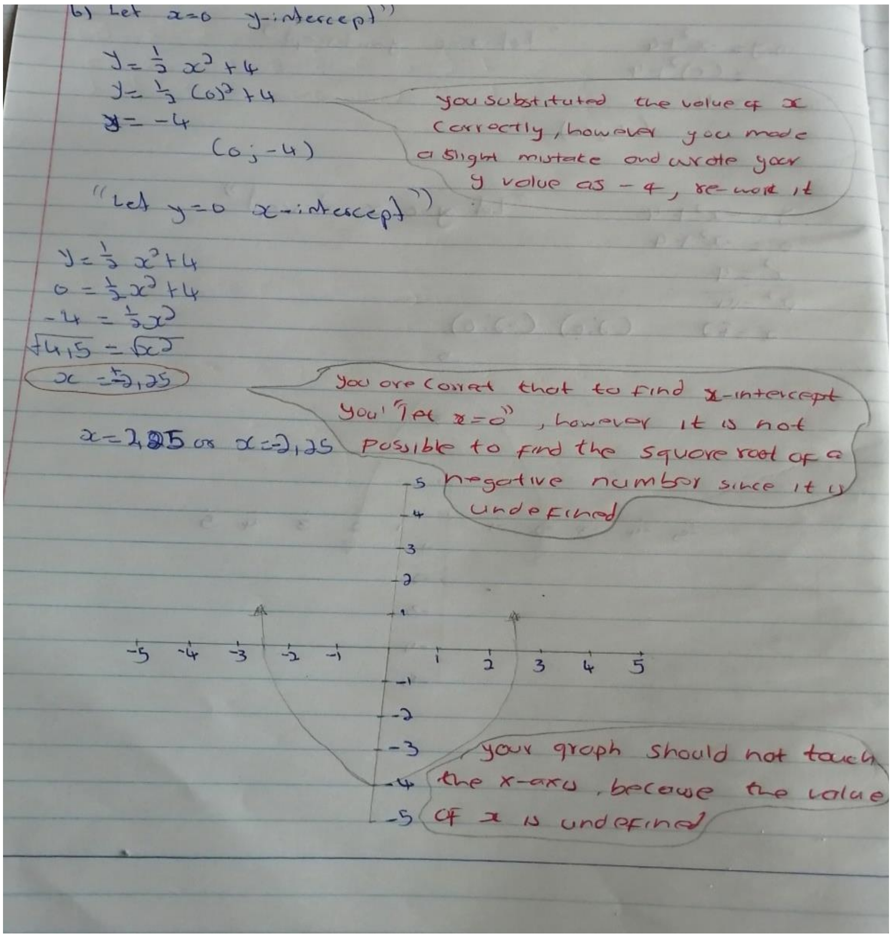


Figure 21: Prince's solution to the graph of $y = \frac{1}{2}x^2 + 4$ with teacher's comments

Analysis of teaching episode 2

I began the lesson by discussing the learning objectives with learners, telling them about the day's learning objectives, which were to draw the parabolic function using the intercepts and the turning point. The learners were now familiar with the shapes of parabolic functions, and they saw instances of intercepts and turning points in figures 10.1 and 10.2 of their Siyavula textbook, and they appeared to be following what was required of them. I asked them effective questions in order to reveal their prior knowledge of calculating intercepts. Learners were able to recall their prior knowledge on how to find x and y-intercepts in a straight-line graph. My questions encouraged them to have a classroom discussion about intercepts in a straight-line graph and how a Cartesian plane works, which is the basis for finding intercepts in a parabolic function.

I allowed learners to work in pairs to go through an example of sketching a parabolic function using the intercepts method from their Siyavula textbooks. They collaborated on the parabolic function example using their prior knowledge of functions and our classroom discussions before they commenced with the example. As a result, it was easier for them to know what needed to be done to get the x and y-intercepts of a parabolic function. It was an intriguing scenario to witness learners learning with their classmates how to calculate intercepts in a parabolic function, which they were seeing for the first time.

Following the example, learners were given a mathematical learning activity in figure 11 to complete in class in order for me to assess their mathematical understanding. The evidence of learning was apparent in question 1 about sketching the graph of $y = -x^2 + 4$. Learners acted as resources for one another through peer feedback and assessment; while others were struggling, their peers ensured that they explained and were on the same page as others. The majority of them were able to determine the intercepts, turning points, and sketch the parabolic function for the first question $y = -x^2 + 4$.

The problem was on the second question, which I was grading using comment-only feedback as part of the five key strategies of assessment for learning. I discovered that the majority of learners were unable to sketch the graph of $y = \frac{1}{2}x^2 + 4$ because they could not find the correct values of x-intercepts that were undefined. They seemed to lack conceptual understanding of working with square roots. The comment–only feedback I wrote in their books indicated that their values of x on question number two of sketching the graph of $y = \frac{1}{2}x^2 + 4$ were not correct. This made learners to be aware of their mistakes and to think of ways to correct them. Some learners were able to do self-reflection. However, due to time constraints, they could not respond to the comments on the same day.

At the end of the teaching episode, learners completed their reflective journals, and the majority of them indicated that they were struggling with question number two, which corroborated what I observed during the observation and through their written work. The use of self-assessment encouraged them to take ownership of their learning because the majority of them noted in their reflective journals what should happen next in their learning. Some learners indicated that they need to learn how to find the values of x in any given parabolic function, including the graph of $y = \frac{1}{2}x^2 + 4$.

Note-taking –teacher's reflective journal (attached in Appendix B)

In this teaching episode, I went on to enact the five key strategies of assessment for learning in order to encourage learners to participate fully in the lesson. I then made certain that I gave them enough time to respond to the questions that I provided before jumping in to answer them myself.

We determined that the use of peer feedback was helpful after analysing the episode with my observer; learners learn best from each other, as Black and William maintain (2009). Specifically, on question one about sketching the graph of $y = -x^2 + 4$, learners were able to get the correct answers by working as resources for one another. Even the low achieving learners were able to learn from their peers with ease. On the question of sketching the graph of $y = \frac{1}{2}x^2 + 4$, that is when I discovered that learners could not draw the graph because they did not comprehend concepts like square roots and basic algebra. I discovered that there was a lot of work to be done before they could respond to the comments that I wrote in their books on the second question of sketching the graph of $y = \frac{1}{2}x^2 + 4$.

What I expected did not occur. My planning was such that learners would achieve all the learning objectives for the lesson, which included sketching the parabolic functions in figure 11 correctly. This did not happen, since learners lacked conceptual understanding of some concepts, such as dealing with square roots, which was critical in the sketching of parabolic functions.

I then chose to conduct a second teaching experiment in order to address and bridge the gaps in knowledge that learners had, which is one of the main purposes of assessment for learning (William & Thompson, 2007). Numerous studies on the enactment of assessment for learning have been conducted (Enrst, 2015; Chapman, 2017; Andika, Sari, Ningsih, Masniladevi &Helsa, 2019; Almujtahid, Hasih & Mardiyana, 2018; Wylie & Lyon, 2015, Johnson et al., 2019; Chapman, 2017; Rakoczy, Pinger, Hochweber, Klieme, Schütze & Blesser, 2009). It is contended that one of the purposes of assessment for learning is to help teachers modify their teaching strategies in order to bridge learning gaps among learners and to elicit evidence of their understanding. Steffe and Thompson (2000) and studies on the teaching experiment methodology by Harja (2013), Muthelo (2010) and Mabotja (2017) indicated that in the teaching experiment design, due to unforeseen circumstances, teacher-researchers may have to change their initial learning paths and create new ones on the spot, which I found appropriate in this scenario.

<u>Analysis of teaching experiment 1</u>

This analysis of the teaching experiment focused on learners' mathematical understanding arising from the two teaching episodes. However, I examined my enactment of the five key strategies of assessment for learning as contributing elements to learners' mathematical understanding. The five key strategies are (i) clarifying, understanding and sharing learning intentions; (ii) engineering effective classroom discussion, tasks and activities that elicit evidence of learning; (iii) providing feedback that moves learners forward; (iv) activating learners as resources for one another; and (v) activating learners as owners of their own learning (William & Thompson, 2007).

(i) Clarifying, understanding and sharing learning intentions

As I enacted the strategy of clarifying, understanding and sharing learning intentions with learners in both teaching episodes, they seemed to follow what I was sharing with them. They were able to determine the learning objectives and success criteria for the shape of the parabola as well as how to sketch parabolic functions using the table and intercepts method. This strategy motivated them to look for examples of good work on parabolic functions in their textbooks. Throughout the teaching experiment, when I implemented this strategy, learners' interest in the lesson was aroused compared to when I used to teach without being intentional about implementing the key strategies for assessment for learning.

(ii) Engineering effective classroom discussion, tasks and activities that elicit evidence of learning.

Learners' curiosity was piqued when they learned about the learning objectives for the lessons in both teaching episodes. When I designed effective classroom discussion, tasks and activities that elicit evidence of learning, learners exhibited interest in the questions and activities I offered since they were related to the lesson's learning objectives. This second strategy of assessment for learning encouraged them to be active participants in the lessons and to understand the concepts taught during the lessons. I was able to identify what their mathematical understanding of parabolic functions was. In this teaching experiment, two mathematical learning activities were given to learners. In teaching episode 1, they were given an investigation in learning activity 1 in Figure 4 where they had to determine the effect of "a" in parabolic function $y = ax^2 + q$. They had to complete the given table, and sketch the parabolic graphs to deduce the effect of "a" in $y = ax^2 + q$ .The activity was intriguing to learners as they were actively engaging with their peers. I was able to uncover their misconceptions. According to Mabotja (2017), a teaching experiment is a living methodology designed to explore what learners say and do while engaging in mathematical activities. Hence, the enactment of this strategy propelled me to uncover that the majority of learners made the same mistake when substituting -2 into $y = x^2$ and getting -4 instead of 4. For me this showed evidence of a lack of conceptual understanding of working with negative numbers in quadratic equations.

However, because they were working in groups on the learning activity, they were able to help each other as peers. For example, in Benad's group, learners assisted each other during the activity discussion and helped those who were struggling to deal with negative numbers to rectify their mistakes, and they finally sketched the parabolic functions in figure 5 correctly. On the other hand, when I noticed that Kholo's group had difficulty in correctly sketching the four parabolic functions, I went to their group to intervene. My intention was to intervene by probing the learners in the group in order for them to recognise their mistakes and devise solutions to correct them. My attempt to help them in recognising and correcting the mistakes they made when sketching their parabolic functions in figure 6 was successful to a certain level. I asked them what happened to their graphs of y_1 and y_2, which did not look like parabolic functions, they became aware that their graphs were incorrect but they could not identify where they went wrong in order to fix their mistakes. Since my observer and I wanted to create a constructivism-learning environment in which learners were active participants, we invited learners from other groups who had correctly sketched all parabolic functions to assist Kholo's group. In this scenario, we asked Benad to help learners in Kholo's group. This act resulted in positive outcomes because Kholo and her group mates were able to correct their mistakes and learn how to work with negative numbers from their peers.

The second teaching episode's mathematical learning activity 2 required learners to sketch parabolic functions with intercepts and turning points. Learners performed well on question one of sketching the graph of $y = -x^2 + 4$, which made me believe that learners could have acquired a conceptual understanding of sketching parabolic functions. On the contrary, with question two of sketching $y = \frac{1}{2}x^2 + 4$, all the learners displayed a lack of conceptual understanding in finding x values in this type of parabolic function. They lacked conceptual understanding since they were unable to connect the dots between the various concepts needed in sketching a parabolic function, hence they were unable to draw any form of parabolic function. This strategy of designing effective classroom discussion, tasks and activities that elicit evidence of learning enabled me to determine what learners understood and what they still lacked in both teaching episodes. I was able to find their misconceptions, evidence of learning, and stimulation of new understandings in the ideas given in this teaching experiment using this strategy (Johnson et al., 2019).

(iii) Providing feedback that moves learners forward

After determining the learners' misconceptions and mistakes, I employed strategy number 3 of providing feedback that moved learners forward in addition to strategy number 2 of engineering effective questioning that elicited evidence of learning. I provided comment-only feedback, both verbal and written, which helped learners to understand both the concepts taught during classroom discussions and the mathematical learning activities that they participated in throughout the teaching experiment. In teaching episode 1, for example, I verbally commented on Benad's group solution for correctly sketching the parabolic functions to let them know they could proceed on to the next question. In teaching episode 2, I further commented on Adelaide and Monica‘s solution for their sketch of $y = -x^2 + 4$ by writing “correct values of x and y-intercepts, you should also show how you got your turning point” on their solution in figure 12. This comment prompted them to search for the turning point, and they eventually sketched the correct shape of the graph. On the same issue, in other groups where learners sketched the same graph of $y = -x^2 + 4$, I used comment–only feedback verbally confirming that their graphs were correct since the majority of the groups managed to sketch the graph of $y = -x^2 + 4$ with ease.

In the question of sketching, the graph of $y = \frac{1}{2}x^2 + 4$, I provided comment-only feedback on each learner’s work to encourage him or her to identify their mistakes because the majority of them could not sketch the graph correctly. The task–focused comments I made in learners books such as the one I made on Pharcily‘s work in figure 19 “your graph should not touch the x-axis because the value of x is undefined in this case” were to stimulate their understanding in order for them to correct their mistakes. According to White and Lyon (2015), learners are more responsive to task-focused comments. According to Black and William (2018), comment-only feedback is more helpful because learners prefer to disregard comments when scores are given. However, in this teaching experiment, learners did not have the opportunity to react to the remarks I made in their books in Figures 15 to 21 for their sketches of $y = \frac{1}{2}x^2 + 4$ due to time. I told them that they will comment in the next teaching experiment.

(iv) Activating learners as resources for one another

In this teaching experiment, the verbal comment-only feedback I gave to learners, as well as the questions I asked when I enacted strategy number 3 of providing feedback that moved learners forward, propelled them to work together as a team. Learners were able to grasp concepts that they had previously been unable to understand thanks to peer input. The fourth strategy, activating learners as resources for one another, appeared to have made a significant contribution to their mathematical understanding. This was demonstrated in teaching episode 1 when they responded to learning activity 1 of deducing the effect of "a" by first sketching the parabolic functions. Learners in groups such as Benad's group were able to correctly sketch all of the graphs by working together and correcting each other's mistakes.

Peer feedback helped the majority of learners in Learning Activity 1 who were trying to work with negative numbers in a quadratic equation. In this teaching experiment, I discovered that learners were able to easily correct their mistakes when taught by their peers. They appeared to be at ease, accepting criticisms from their peers over me. For example, Kholo's group was struggling with learning activity 1 of sketching the parabolic function since they could not work with negative numbers until we invited Benad from one of the groups who could. Learners in Kholo's group were able to correct their mistakes and sketched their graphs correctly. In teaching episode 2 learning activity 2, learners also worked very well as resources for one another when sketching the graph of $y = -x^2 + 4$ on their own.

(v) Activating learners as owners of their own learning

Learners completed their reflective journals in Appendix A at the end of both teaching episodes. They were able to undertake self-reflection by completing reflective journals, indicating what they learned well, how they learned, what they struggled with, and what they still needed to learn. The majority of them stated that while they struggled with negative numbers, they were able to understand them through peer learning. They also noted that they had difficulty working with functions where the value of x is undefined, and that they want to understand how to sketch the graph with undefined values of x. Using this strategy allowed me to assess their understanding of mathematics from their perspective.

My reflections on teaching experiment 1

As a result of this teaching experiment, I have discovered that the strategies can be used interchangeably. As part of strategy number two, I found myself asking learners questions to stimulate their comprehension and to uncover their misconceptions while on strategy three of offering feedback that moves learners forward, and when I enacted strategy number one at the start of the lesson by establishing learning intents and sharing learning objectives. As I shared the learning intentions and success criteria, I found myself using strategy number three, asking learners questions and fostering classroom discussions to incorporate learners' active participation. I used strategy number four, activating learners as resources for one another, throughout the experiment since one of the main goals of my research was to close the gap between not employing the final two strategies effectively while enacting the five key strategies of assessment for learning. I assigned a variety of activities for learners to complete in pairs or groups of three in order to stimulate peer feedback.

Learners' mathematical understanding was evident in some of the activities in this teaching experiment, such as those in episode one, when they had to sketch the parabolic functions using the table method to determine the effect of "a." When other groups, such as Benad's group, were able to correctly sketch all four parabolic functions, I concluded that they had a conceptual understanding of parabolic functions until they could not deduce the effect of "a." I reached the conclusion that they just had procedural fluency in using the table method to sketch the graphs, and lacked conceptual understanding because they could not make the connections between concepts.

In teaching episode two, the majority of learners could not answer question two about sketching the parabolic function $y = \frac{1}{2}x^2 + 4$. My observer and I tried to figure out what was wrong. We discovered that learners did not know how to calculate the values of x when it was undefined. When working with square roots, there was a lack of conceptual understanding. Hence, the majority of learners failed to sketch the graph of $y = \frac{1}{2}x^2 + 4$ in episode number two. This was a clear indication that learners did not have the conceptual understanding in which Andamon and Tan (2018) defined conceptual understanding as a deep understanding and foundation behind working with algorithms performed in mathematics. In this teaching experiment, learners lacked the conceptual understanding of calculating the x intercepts in a quadratic equation that affected their sketch of $y = \frac{1}{2}x^2 + 4$.

In teaching experiment 2 and going forward, I intend to increase learners' conceptual understanding and procedural fluency of the concepts I teach. The emphasis will be on square root revision, parabolic functions, even when there are no defined values of x, and hyperbolic functions. I have found that effective questioning and collaboration help learners learn more. In teaching experiment 2, I intend to use more effective questioning during classroom discussions, and to give learners an opportunity to work in groups/pairs to develop conceptual understanding and procedural fluency with the aforementioned concepts. Furthermore, when learners have a conceptual understanding of sketching parabolic functions, they can create connections between various concepts involved in the sketching of the functions, such as square roots and intercepts. In addition, as stated by Kilpatrick et al. (2001) in his 'theory of mathematical proficiencies,' after learners have mastered conceptual understanding, procedural fluency will come naturally.

4.4 TEACHING EXPERIMENT 2: PARABOLIC AND HYPERBOLIC FUNCTIONS

Background of Teaching Experiment 2

The second teaching experiment consists of two teaching episodes. The focus of the first teaching episode was on the revision of square roots, with emphasis on parabolic functions, notably the sketching of parabolic functions with undefined values of x, such as the graph of $y = \frac{1}{2}x^2 + 4$. The second teaching episode focused on sketching hyperbolic functions. According to Muthelo (2010), what learners say and do in their quest to understand their mathematical reality is critical in the teaching experiment methodology. Thus, the objective of this teaching experiment was to provide learners with more opportunities to collaborate with their classmates in order for me to account for their mathematical understanding. The objective was met as I enacted the five key strategies of assessment for learning during the teaching episodes.

4.4.1 Teaching Episode 1: Square Roots Revision and Parabolic Function

Teaching episode 1 background

In this teaching episode, I began by revising square roots with learners so that they might fix their mistakes in sketching the parabolic function $y = \frac{1}{2}x^2 + 4$ from the teaching experiment one. The sole purpose was not to provide learners with the correct answers to the graph of $y = \frac{1}{2}x^2 + 4$. As part of assessment for learning, comment-only feedback plays a vital role in assisting learners to correct their mistakes (Nkealah, 2019). Hence, I wanted learners to use the comments I wrote in their books to correct the mistakes they made on their sketches of $y = \frac{1}{2}x^2 + 4$. Furthermore, I employed more effective questioning, classroom discussions, and peer assessment to develop learners' conceptual understanding of working with square roots, which impacted their understanding of sketching the parabolic functions even in scenarios where there were no defined values of x.

This is what transpired in the classroom:

Teacher: The learning objective for today's lesson is that we are going to continue working with parabolic functions with undefined values of x such as the graph of $y = \frac{1}{2}x^2 + 4$ you sketched in the previous lesson, together with the revision on calculating square roots. First of all, read the comments I wrote in your books in your sketches of $y = \frac{1}{2}x^2 + 4$ and try to respond to the comments by fixing your mistakes.

Teacher: Lesego read your comment aloud.

Lesego: "Your value of y is correct, however, the values of x are incorrect because you cannot find the square root of a negative number, thus your graph should not touch the x-axis because the value of x is undefined".

Teacher: Do you have any idea on how to correct your solution based on the comment I made?

Lesego: Eish, no ma'am. I have no idea but let me try to correct my mistake.

Makgabo: I got the same comments too.

Pharcily: My comments are the same as Lesego and Makgabo.

Teacher: The majority of you made the same mistake on the question. Who is willing to share her corrective method?

Benad: Ma'am, I do not understand. I am failing to correct my mistake.

Teacher: It seems like you all do not have an idea of fixing this. However, as I was marking your work, I noticed that one of you managed to sketch the graph correctly even though they used a different method, but the shape of the graph is correct.

Adelaide: It is me ma'am, and you commented by saying: "The shape of the graph is correct, however, you should learn how to sketch the graph using intercepts and a turning point and intercepts even when x values are undefined."

Teacher: Show the rest of the class how the shape of the graph with undefined values of x should look like.

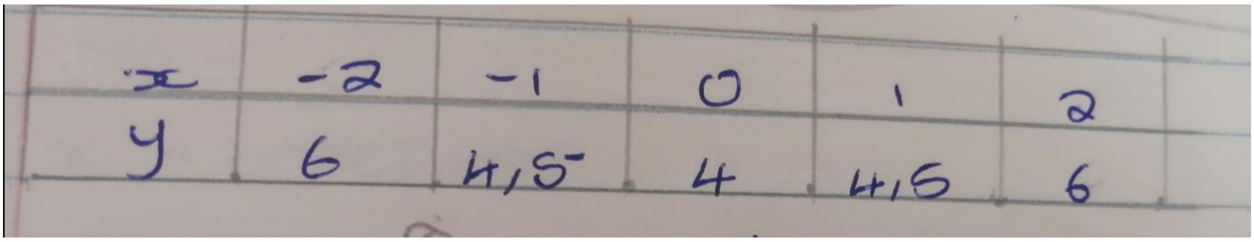
Adelaide: This is how my graph looks like (she showed learners the graph in her book the picture is on episode 2 Figure 17 above). I used the table method.

Prince: How did you use it?

Adelaide: The same way we completed tables in the previous lessons, like this. We were given the equation of $= \frac{1}{2}x^2 + 4$, I created a table from -2 to 2. Then I substituted -2 into the equation $y = \frac{1}{2}(-2)^2 + 4 = 6$ and I…

Kholo: Oh! I see now I remember and for the second value you substituted $y = \frac{1}{2}(-1)^2 + 4 = 4{,}5$.

Adelaide: Yes and here is my completed table below, I used the points to sketch the graph.



x	-2	-1	0	1	2
y	6	4,5	4	4,5	6

Figure 22: Completed table for Adelaide for the graph of $y = \frac{1}{2}x^2 + 4$

Other learners followed how Adelaide sketched her graph of $y = \frac{1}{2}x^2 + 4$. Even though she used a different method, I was satisfied that other learners managed to see how the graph with undefined values of x looked like from their peers. After the peer feedback, learners were aware of how the graph of $y = \frac{1}{2}x^2 + 4$ should look like. It was nothing like what they have sketched in their books. I engaged learners on the mistakes they made when sketching their graphs of $y = \frac{1}{2}x^2 + 4$ by revising the calculation of square roots as this was one of the main reasons they could not sketch the graph correctly. Through questioning and classroom discussions, I led learners to discover that the square roots of negative numbers are undefined. I further made them realise how undefined values of x impacted the sketching of a parabolic function in order to develop their conceptual understanding of working with square roots, enabling them to sketch any type of parabolic functions.

This is how the questioning and the classroom discussion went:

Teacher: As you are now aware of the mistakes you made when sketching the graph of $y = \frac{1}{2}x^2 + 4$, let us do revision about finding the square roots.

Teacher: Use your calculator to find the square root of 4.

Monica: The answer is 2.

Teacher: Find the square of -4.

Pharcilly: It is not possible; we cannot find the answer on the calculator.

Teacher: What do you see on your calculators?

Learners: ‘Maths error’. (Shown in figure 22.1 below)

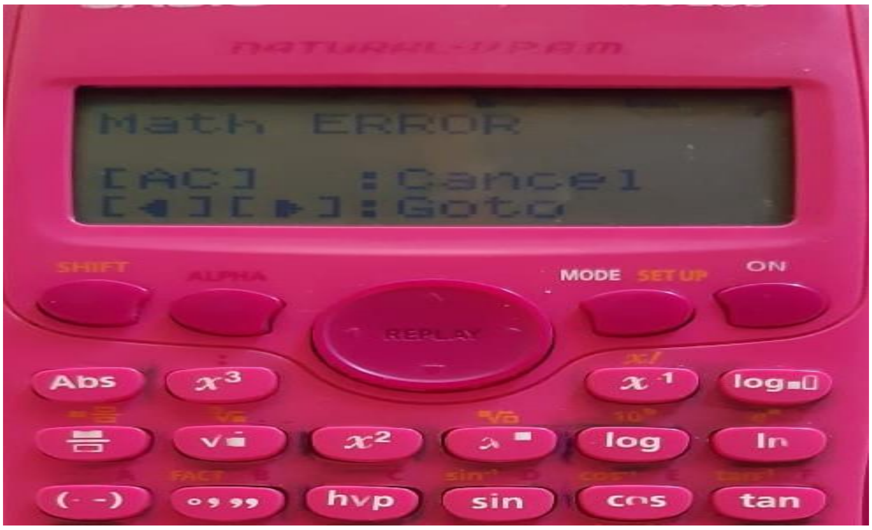


Figure 22.1: ‘Maths error’ appearing on the calculator

Teacher: Find the square of 49 and−49, what do you see on your calculators?

Adelaide: The square root of 49 is 7 but the square root of -49 gives us ‘maths error’ on the calculator.

Teacher: Find the square of -25 and 25, what do you see on your calculators?

Prince: The square root of 25 is 5 but maths error appears on the calculator when we try to find the square root of -25.

Teacher: After the calculations that you have made above, what did you notice?

Monica: The square root of a negative number gives us ‘maths error’.

Benad: Square roots of negative numbers are impossible.

Teacher: You are both correct. Since we are working with real numbers only, the square root of a negative number is undefined, meaning it is not available in simple English.

Kholo: So, what does it mean?

Adelaide: Oh, that is why we do not have x-intercepts on the graph.

Teacher: True, whenever you see a ‘maths error’ on your calculator, you should write undefined. The value of x will not be defined on the graph, meaning your graph is not supposed to touch the x-axis. Lesego, does my comment on your solution for sketching the graph of $y = \frac{1}{2}x^2 + 4$ make sense now?

Lesego: Yes ma'am, it makes sense. I see why the graph was not supposed to touch the x-axis because we do not have it.

Teacher: Yes, now all of you go through the comments that I made in your books and correct your mistakes in groups of three.

Adelaide's group discussion on correcting their graph of $y = \frac{1}{2}x^2 + 4$

Monica: Okay guys, let us look at our solutions and comments from ma'am. She commented by saying, "Your values of y-intercept is correct but fix your x values because it is not possible to find the square root of a negative number."

Benad: Let us compare our values of y, mine is 4.

Adelaide: Let me see, Monica got 4 and I got 4 as well.

Benad: We all substituted x with 0 and $y = \frac{1}{2}(0)^2 + 4 = 4$.

Adelaide: Then let us fix the value of x .

Monica: Substitute y by 0.

Benad: Transpose 4 and it becomes -4.

Adelaide: In my book, ma'am commented by saying, "The shape of your sketch is correct, however, learn how to find x-intercepts and turning point to sketch a parabola." So, it means I should find the values of x using the intercepts method.

Benad: I got the same comment as Monica to re-calculate the x-intercepts by finding square roots on both sides of the equation. So, let us do it.

Adelaide: Yes, since we have -4 on the left, it means the value of x will be undefined.

Monica: We should write undefined and our graph will not touch the x-axis.

Benad: Let me draw the graph.

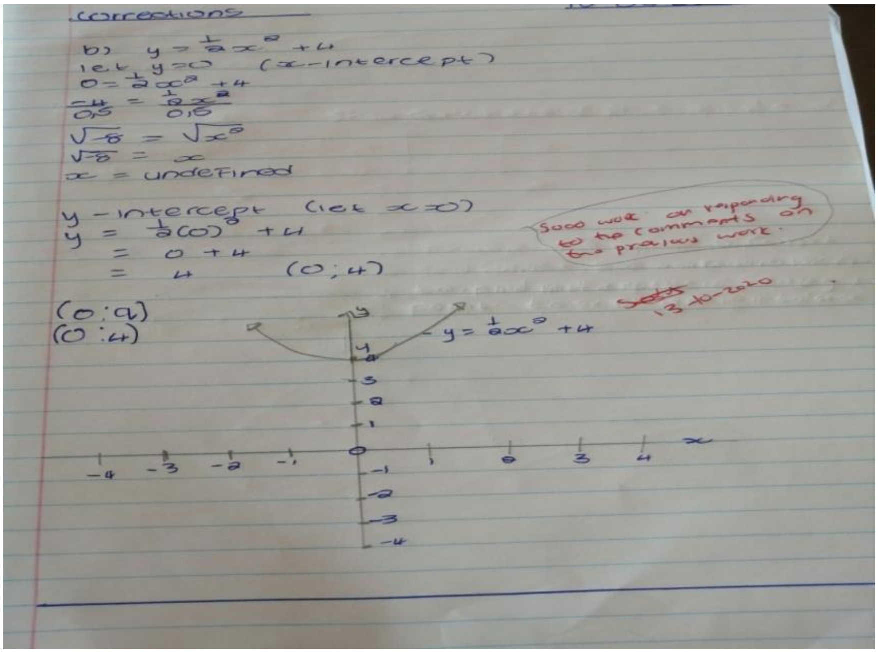


Figure 23: Adelaide's corrections of $y = \frac{1}{2}x^2 + 4$ with teacher's comments.

After the revision that I did with learners on calculating square roots, they were able to use the comments I made in their books to correct the mistakes they made while sketching the graph of $y = \frac{1}{2}x^2 + 4$, as seen in the excerpt above. As I moved around the classroom, I noticed that the learners were developing a conceptual understanding of working with square roots and sketching parabolic functions. This was evident in Adelaide's group, as seen by their discussion in the excerpt above, as well as their corrected sketch of $y = \frac{1}{2}x^2 + 4$ in figure 23. Other groups were also able to correct their mistakes. Some of the solutions from other groups are shown in figures 24 to 25 because they all managed to correct their mistakes and sketch the graph of $y = \frac{1}{2}x^2 + 4$ correctly.

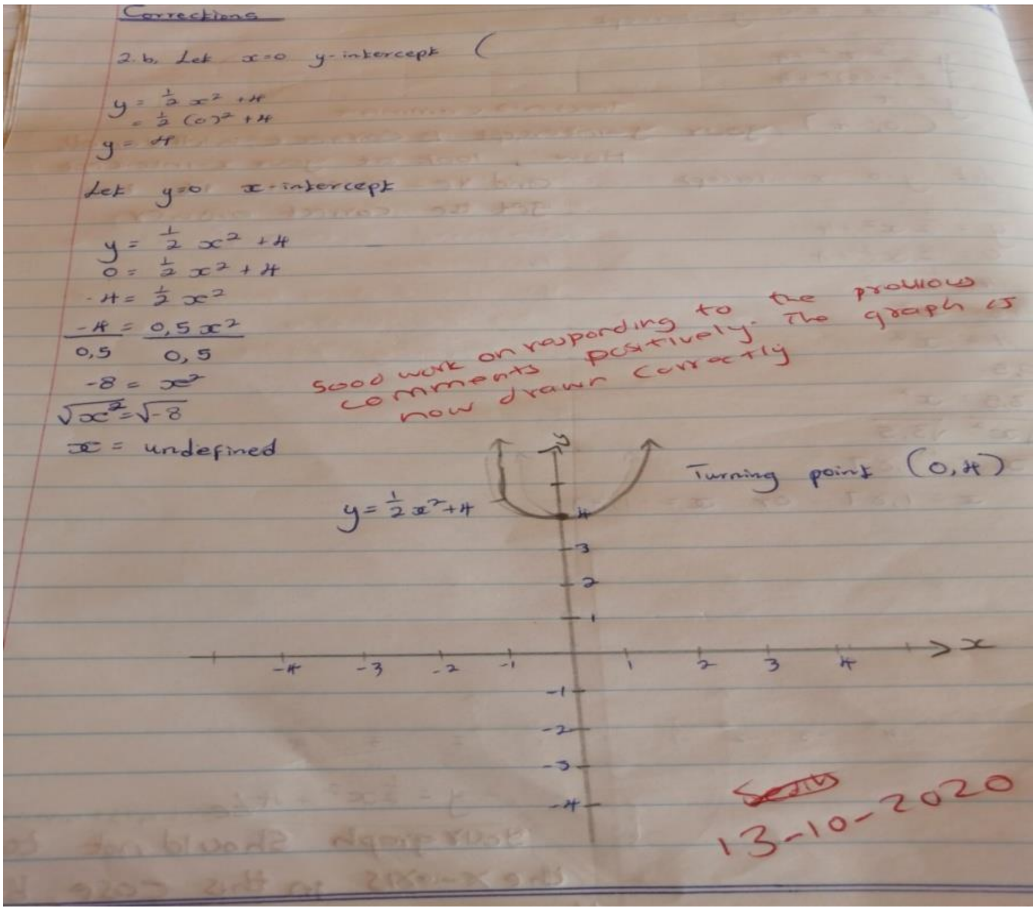


Figure 24: Kholo's group corrections for the graph $y = \frac{1}{2}x^2 + 4$ with teacher's comments

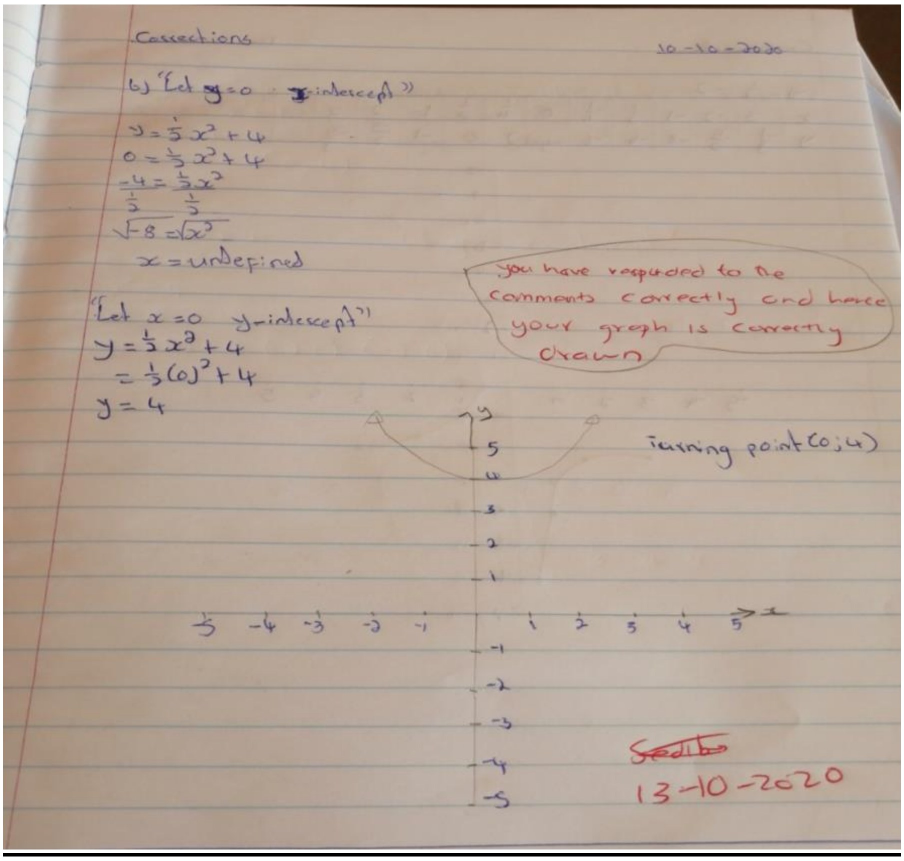


Figure 25: Prince's group corrections for the graph $y = \frac{1}{2}x^2 + 4$ with teacher's comments.

Since the majority of the learners were able to respond to my comments on their graph sketches of $y = \frac{1}{2}x^2 + 4$, in some groups such as Kholo's, I decided to add an extra question with undefined values of x to see if the learners could sketch it. I gave them the graph of $y = x^2 + 4$, and to my satisfaction, the group managed to draw the graph correctly in figure 26 below, which showed some level of procedural fluency in the sketching of parabolic functions because they were able to sketch different types of parabolic functions efficiently.

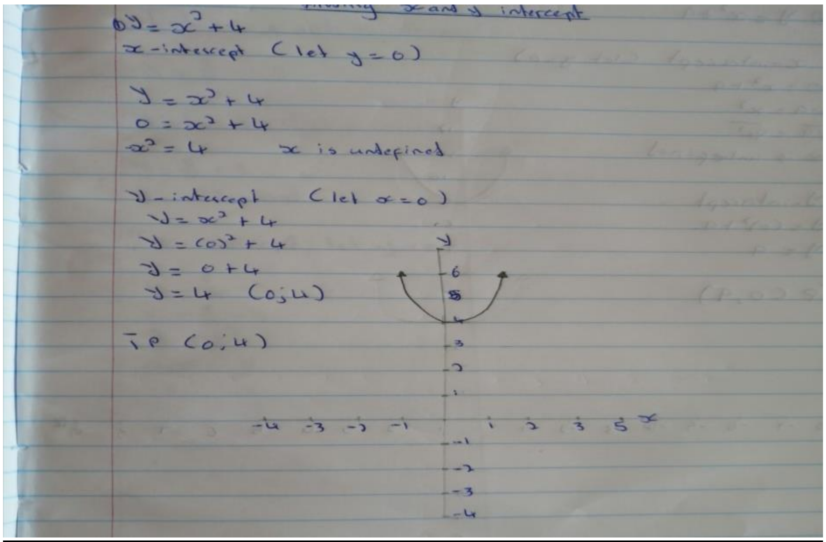


Figure 26: Kholo's group solution to $y = x^2 + 4$

The time for the mathematics period was elapsing and I told the learners to complete their reflective journals for the day. As they finished, I noted that the majority of them were satisfied with the day's learning objectives. However, what piqued my interest was what I read in Prince's reflective journal, in which he stated that he and his group challenged themselves by trying out a similar question that they created of sketching the graph of $y = x^2 + 9$ in figure 27 below on their own, which they managed to do correctly. This demonstrated conceptual understanding, procedural fluency and learners who took responsibility for their own learning.

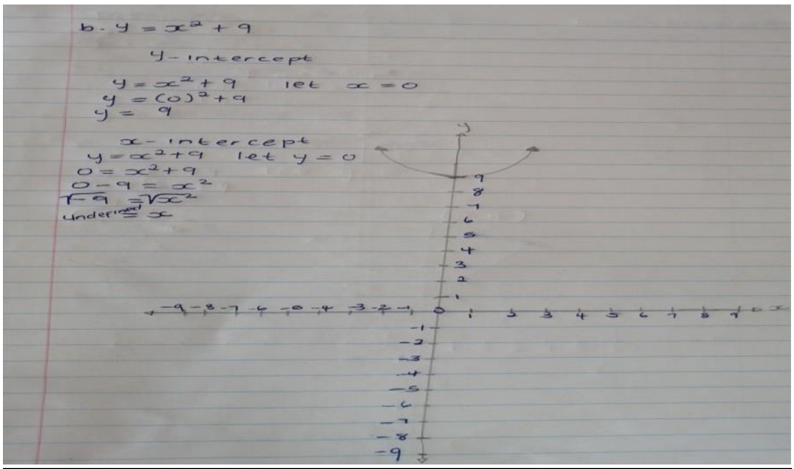


Figure 27: Prince's group solution to $y = x^2 + 9$

Analysis teaching episode 1

I shared the lesson's learning objectives, which included emphasis on sketching parabolic functions with undefined values of x and a revision of calculating square roots so that learners may correct their mistakes when sketching the graph of $= \frac{1}{2}x^2 + 4$. I further provided learners feedback to their incorrectly sketched graphs of $= \frac{1}{2}x^2 + 4$. I asked them to go over the comments I made on their sketches, which guided them to correct the mistakes they made. As they read my comments on their incorrect sketches of $y = \frac{1}{2}x^2 + 4$ for the first time the majority of them indicated that they knew what was required of them based on the comments I made in their books, but they were unable to correct their mistakes and had no idea how the shape of the graph with undefined values of x should look like. Lesego and Benad, for example, noted that they were unsure how to correct their mistakes after reading my comment.

Despite the fact that one of the learners, Adelaide, used a different method to sketch the graph, she was able to draw the correct shape of the graph of $y = \frac{1}{2}x^2 + 4$. I then requested her to demonstrate to her peers how the graph with undefined values of x should look like so that learners may see examples of good and poor work. However, learners were unable to rectify their mistakes while sketching the graph with undefined values of x using the intercepts method. During the classroom discussions, I noted that learners were unable to identify x intercepts with undefined values of x because they lacked the conceptual understanding of working with square roots. Hence, they did not know how to correct their mistakes based on the comments I made on their graphs of $y = \frac{1}{2}x^2 + 4$. The revision lesson on working with square roots was then facilitated through questions and classroom discussions. Learners were actively involved in the lesson and had a conceptual understanding of working with square roots because of this strategy. I was able to uncover their misconceptions, and their new understanding of working with square roots was stimulated since they could now understand that the function with undefined x values cannot touch the x-axis. Their new understandings of square roots helped them to conceptually understand the sketching of intercepts. This prompted them to gain a conceptual understanding of sketching parabolic functions since they could now relate the connections between concepts required in sketching a parabolic function, such as the square roots and intercepts.

Furthermore, learners worked in groups to correct the mistakes they made on sketching the graph of $y = \frac{1}{2}x^2 + 4$. They acted as resources for one another, and they were able to respond constructively to the remarks I made in their books while also sketching the graph of $y = \frac{1}{2}x^2 + 4$ correctly. Learners in other groups, such as Kholo's, were now exhibiting some procedural fluency in sketching parabolic functions. This became clear when I chose to add a sketching question to the mix for $y = x^2 + 4$, when the group had to draw the parabolic function and successfully sketched the graph.

Learners expressed satisfaction with the lesson in their reflective journals since they were able to solve square roots and sketch various forms of parabolic functions. To add to this point, when I read Prince's reflective journal, he stated that he and his group challenged themselves by attempting to answer a similar question that they formulated: $y = x^2 + 9$, which was correct in figure 27 above. Some learners expressed a need for extra practice on sketching parabolic functions, while others indicated a need to learn how to sketch hyperbolic functions, which is the next section of sketching functions. This indicated that they were taking charge of their own learning. What I read in their reflective journals again resonated with what I observed during the video recording.

Note-taking –teacher's reflective journal for episode 1

In this teaching episode, I enacted the five key strategies of assessment for learning to invite learners to participate fully in the lesson. I discovered how interconnected these five key strategies of assessment for learning may be. William and Thompson's (2007) framework explained the interactive nature of the five key strategies of assessment for learning, which I highly observed in this teaching episode. In the strategy of clarifying and sharing learning intentions and success criteria with learners, I found myself having to include peer feedback, engineering effective questioning, and activating learners as owners of their own learning. This intertwining of the five key strategies, in my experience, made the lesson more meaningful to the learners, and spurred their involvement in the lesson, causing them to take ownership of their own learning.

In addition to the foregoing, learners took ownership of their own learning to a higher level, which enhanced their conceptual understanding of sketching parabolic functions. It became evident when Prince's group decided to challenge itself by sketching the graph of $y = x^2 + 9$ with undefined values of x correctly without being directed by the teacher. Furthermore, some sort of procedural fluency was emerging because as I walked around the class, I gave some groups that were done with their corrections of sketching the graph of $y = \frac{1}{2}x^2 + 4$ a similar question of sketching the graph of $y = x^2 + 4$, which they managed to sketch correctly**.** I have realised that teaching learners for conceptual understanding is critical in the building of mathematical understanding in learners. The use of the five key strategies of assessments for learning enhanced learners' conceptual understanding of the concepts taught, which led to learners to acquire procedural fluency.

The second teaching episode will focus on sketching hyperbolic functions, with greater emphasis on strategy number four of activating learners as instructional resources for one another. I will provide learners with more opportunities to collaborate and constructively learn concepts to maximise their mathematical understanding. According to Dagar and Yadav (2016), one of the learning strategies that teachers can utilise to create a constructivist learning environment is for learners to participate cooperatively with one another.

4.4.2 Teaching Episode 2: Hyperbolic Functions

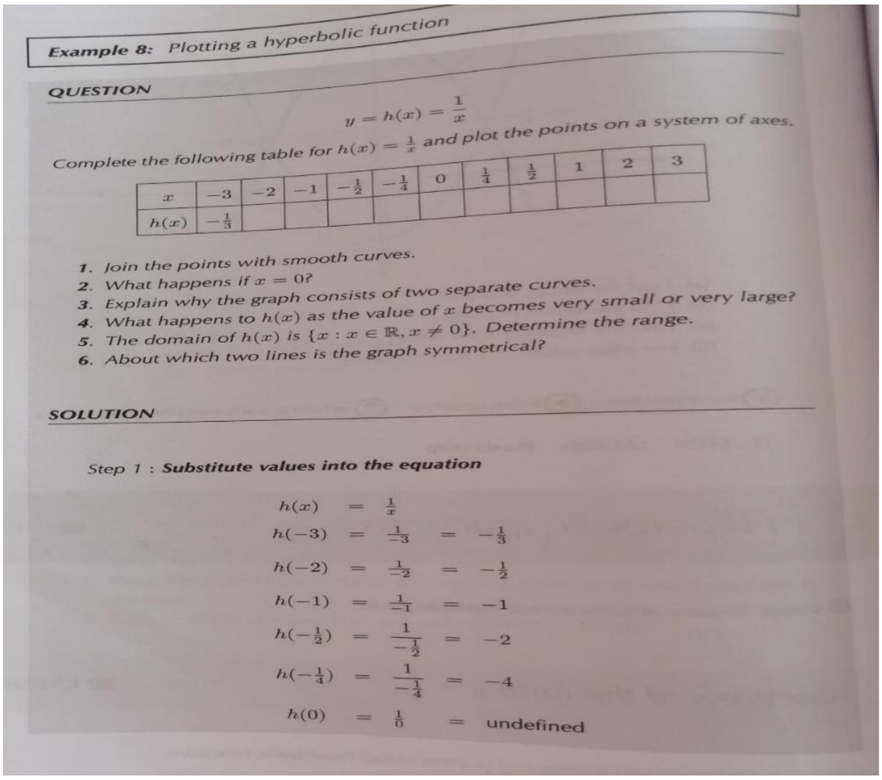
Background of the Teaching Episode 2

This teaching episode concentrated on hyperbolic functions. My observer and I decided that since learners were now grasping the concept of plotting functions, we would give them the opportunity to work in pairs to learn how to sketch hyperbolic functions in this teaching episode. My facilitation during this teaching episode, although not excluding the five key strategies of assessment for learning, was also informed by the constructivism learning theory. I wanted learners to be active participants in their learning and construction of hyperbolic function knowledge.

Mabotja (2017) maintains that in the constructivism paradigm, which was used to frame this study, knowledge is formed through interactions between learners and teachers. Learners worked in pairs on hyperbolic functions examples from their Siyavula textbook, which involved sketching the functions and calculating the intercepts with minimal assistance from the teacher. After completing the examples, learners were given one mathematical learning activity with two questions to work on.

This is what transpired in the classroom:

Teacher: The learning objectives for today will be on plotting a hyperbolic function using the table, and determining the values of x and y-intercept in a hyperbolic function. In your Siyavula textbooks, you will find examples in figures 28 and 29 below of how hyperbolic functions should look like and how they are drawn.
Teacher: In pairs, complete the following learning activity 1 in figure 30 on hyperbolic functions.



Example 8: *Plotting a hyperbolic function*

QUESTION

$$y = h(x) = \frac{1}{x}$$

Complete the following table for $h(x) = \frac{1}{x}$ and plot the points on a system of axes.

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
$h(x)$	$-\frac{1}{3}$										

1. Join the points with smooth curves.
2. What happens if $x = 0$?
3. Explain why the graph consists of two separate curves.
4. What happens to $h(x)$ as the value of x becomes very small or very large?
5. The domain of $h(x)$ is $\{x : x \in \mathbb{R}, x \neq 0\}$. Determine the range.
6. About which two lines is the graph symmetrical?

SOLUTION

Step 1 : **Substitute values into the equation**

$$\begin{aligned}
h(x) &= \tfrac{1}{x} \\
h(-3) &= \tfrac{1}{-3} = -\tfrac{1}{3} \\
h(-2) &= \tfrac{1}{-2} = -\tfrac{1}{2} \\
h(-1) &= \tfrac{1}{-1} = -1 \\
h(-\tfrac{1}{2}) &= \frac{1}{-\frac{1}{2}} = -2 \\
h(-\tfrac{1}{4}) &= \frac{1}{-\frac{1}{4}} = -4 \\
h(0) &= \tfrac{1}{0} = \text{undefined}
\end{aligned}$$

Figure 28: Example of plotting hyperbolic function from Siyavula textbook

$$h(\tfrac{1}{4}) = \frac{1}{\frac{1}{4}} = 4$$
$$h(\tfrac{1}{2}) = \frac{1}{\frac{1}{2}} = 2$$
$$h(1) = \tfrac{1}{1} = 1$$
$$h(2) = \tfrac{1}{2} = \tfrac{1}{2}$$
$$h(3) = \tfrac{1}{3} = \tfrac{1}{3}$$

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
$h(x)$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	-4	undefined	4	2	1	$\frac{1}{2}$	$\frac{1}{3}$

Step 2 : ***Plot the points and join with two smooth curves***

From the table we get the following points: $(-3; -\frac{1}{3})$, $(-2; -\frac{1}{2})$, $(-1; -1)$, $(-\frac{1}{2}; -2)$, $(-\frac{1}{4}; -4)$, $(\frac{1}{4}; 4)$, $(\frac{1}{2}; 2)$, $(1; 1)$, $(2; \frac{1}{2})$, $(3; \frac{1}{3})$.

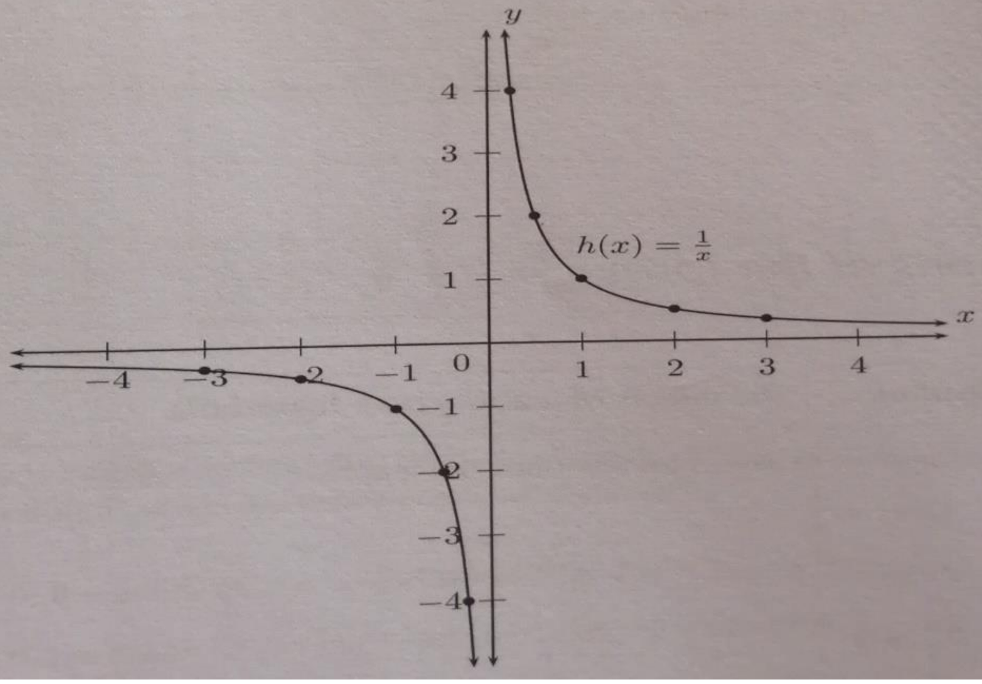


Figure 29: Example of a plotted hyperbolic function from Siyavula textbook

Learning activity 1: Hyperbolic functions

1. Complete the following table and sketch the hyperbolic functions on the same set of axes.

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
$y_{1=}\frac{1}{x}-2$											
$y_2=\frac{1}{x}-1$											
$y_3=\frac{1}{x}$											
$y_4=\frac{1}{x}+1$											
$y_{5=}\frac{1}{x}+2$											

2. Find the values of x and y intercepts in the following functions

$$y_1=\frac{1}{x}-2$$
$$y_2=\frac{1}{x}$$

NB: For question 2 go through example 6 on page 139 to see the example of how to calculate the intercepts in a hyperbolic function.

$$y_1=\frac{1}{x}-2$$

Figure 30: Learning Activity 1 on hyperbolic functions

Conversation between Benad and Monica and their solutions in figures 27 and 28 below.

Monica: We are given equations from $y_1\ to\ y_5$. Let us copy the table in our books.
Benad: And now let us complete the table using a calculator just like the way we were completing the table for parabolic functions.

Monica: Yes, let us start with the graph of $y_1=\frac{1}{x}-2$.

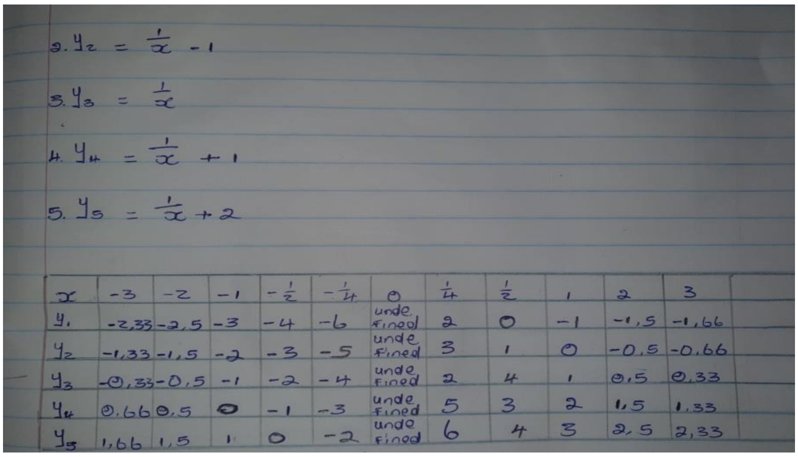
Benad: When I substitute the value of $x = -3$ into $y_1 = \frac{1}{-3} - 2$, the answer is $-2{,}33$.

Monica: And when I substitute the value of $x = -2$ into $y_1 = \frac{1}{-3} - 2$, the answer is $-2{,}5$.

The pair proceeded to use a calculator to complete the table without difficulty until they reached the point when they needed to divide by zero.

Monica: When I substitute the value of $x = 0$ into $y_1 = \frac{1}{0} - 2$, the answer is…oh, it is 'maths error'.

Benad: Then it means we should write undefined, we learnt from parabolic functions that when we see 'maths error' on the calculator, we must know that the value is undefined.
Monica: But this time it is the value of y, not the value of x.
Benad: But we are getting 'maths error' on the calculator. What are we going to write in our table?
Monica: I think you have a point. I agree with you. Let us write undefined because we cannot find the value of y on the calculator.
I noticed in the excerpt above how the pair was able to leverage their prior knowledge of parabolic functions to work with hyperbolic functions, and to calculate the values of y using the given hyperbolic equations to complete the table. The pair completed their table with graphs from learning activity 1 (see figure 31 below).


2. $y_2 = \frac{1}{x} - 1$

3. $y_3 = \frac{1}{x}$

4. $y_4 = \frac{1}{x} + 1$

5. $y_5 = \frac{1}{x} + 2$

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
y_1	-2,33	-2,5	-3	-4	-6	undefined	2	0	-1	-1,5	-1,66
y_2	-1,33	-1,5	-2	-3	-5	undefined	3	1	0	-0.5	-0.66
y_3	-0,33	-0,5	-1	-2	-4	undefined	2	4	1	0,5	0,33
y_4	0.66	0.5	0	-1	-3	undefined	5	3	2	1,5	1.33
y_5	1,66	1,5	1	0	-2	undefined	6	4	3	2,5	2,33

Figure 31: The completed table of Benad and Monica for learning activity 1

Following the completion of their table in figure 31, the pair began plotting the points on the Cartesian plane. However, their concern was how the graphs would look like because it was the first time they had to plot hyperbolic functions. The good news is that even though they were a little hesitant, they proceeded with sketching the graphs, and the final solution with the plotted hyperbolic graphs is shown in figure 32 below.

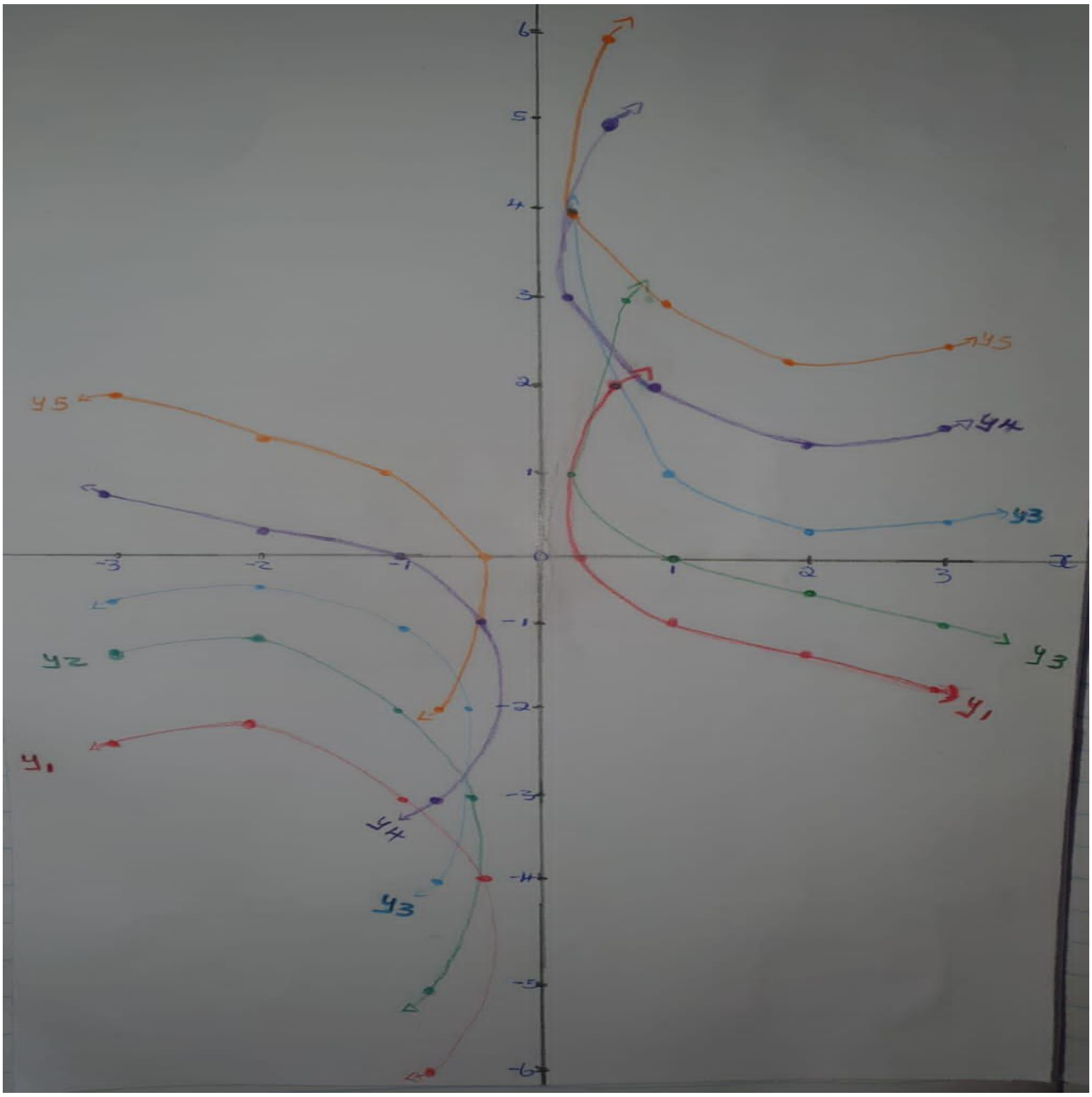
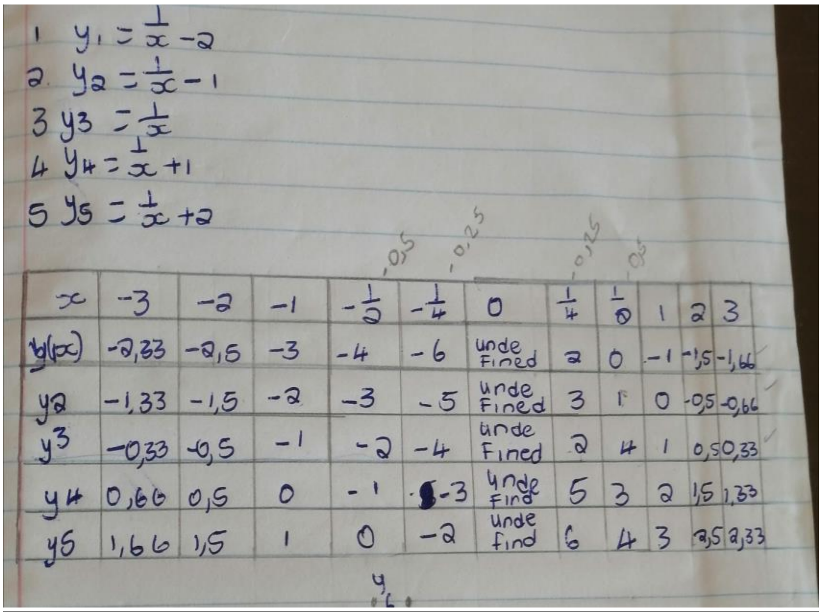


Figure 32: The completed graphs of hyperbolic functions for Benad and Monica

Benad and his peer demonstrated some procedural fluency in the sketching of hyperbolic functions because they were able to draw all of the graphs in figure 32 above correctly on their own. My observer and I continued to assist learners working on learning activity 1 in figure 30. I noticed that Lesego and Pharcilly were also conversant with completing the table, and only paused a bit when they saw 'maths error' in their calculators for division by zero, but they were able to understand that it meant undefined. Their completed table is shown below in figures 33 and 34.



1 $y_1 = \frac{1}{x} - 2$
2 $y_2 = \frac{1}{x} - 1$
3 $y_3 = \frac{1}{x}$
4 $y_4 = \frac{1}{x} + 1$
5 $y_5 = \frac{1}{x} + 2$

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{8}$	1	2	3
y(x)	-2,33	-2,5	-3	-4	-6	undefined	2	0	-1	-1,5	-1,66
y2	-1,33	-1,5	-2	-3	-5	undefined	3	1	0	-0,5	-0,66
y3	-0,33	-0,5	-1	-2	-4	undefined	2	4	1	0,5	0,33
y4	0,66	0,5	0	-1	-3	undefind	5	3	2	1,5	1,33
y5	1,66	1,5	1	0	-2	undefind	6	4	3	2,5	2,33

Figure 33: The completed table of Lesego and Pharcily for learning activity 1

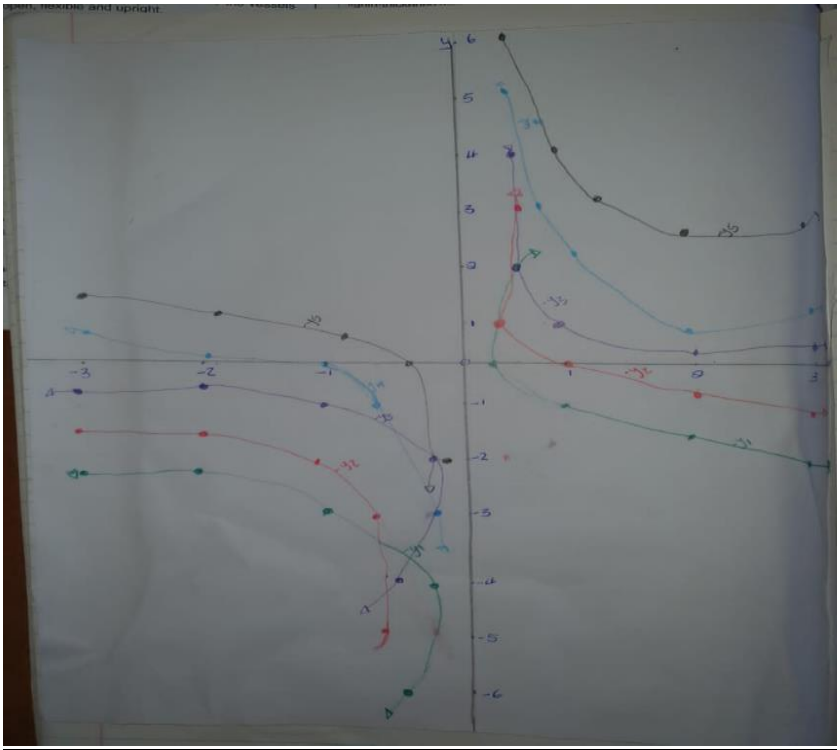
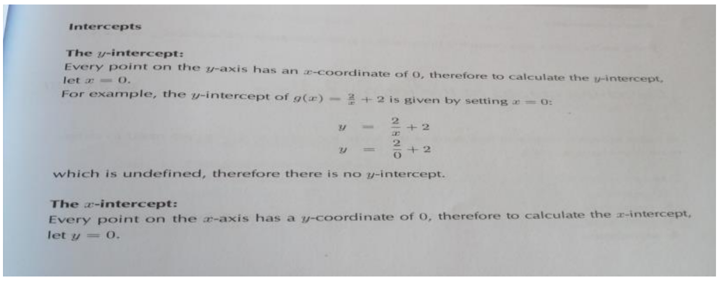


Figure 34: The completed graphs of hyperbolic functions for Lesego and Pharcily

As my observer and I proceeded to move around the class, we determined that all of the groups knew that when the y value is undefined, it implies that it does not exist, cannot be plotted on the Cartesian plane, and the graph will not touch the y-axis at that point. This was evident because all of the learners were able to accurately sketch all of the hyperbolic functions, indicating that learners have procedural fluency in sketching hyperbolic functions. They were able to efficiently and correctly complete the mathematical procedures for sketching all of the hyperbolic functions. We then instructed them to go on to question 2, which required them to work in pairs to find the values of x and y-intercept in a hyperbolic function.

Monica's group conversation before answering question 2 in learning activity 1 in figure 30 above

Matle: Ok guys, how are we going to find x and y-intercept in a hyperbolic function?
Benad: I think we will use the knowledge from the parabolic and straight-line functions.
Monica: Yes. You are right. Remember when we find y-intercept, we let x to be zero and vice versa.
Matle: Okay, let us start with the example of finding x and y-intercept on page 139 of our textbook.
Example on page 139: Determine x and y-intercepts in the following hyperbola $y = \frac{2}{x} + 2$.

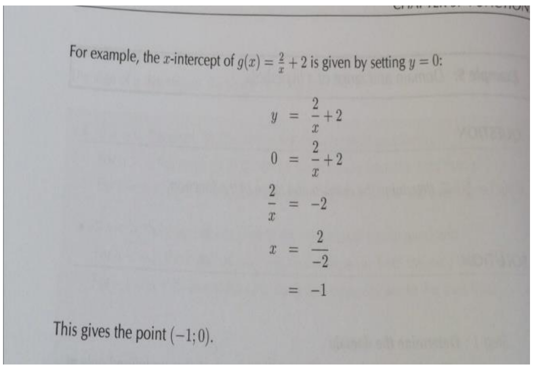

Intercepts

The y-intercept:
Every point on the y-axis has an x-coordinate of 0, therefore to calculate the y-intercept, let $x = 0$.
For example, the y-intercept of $g(x) = \frac{2}{x} + 2$ is given by setting $x = 0$:

$$y = \frac{2}{x} + 2$$
$$y = \frac{2}{0} + 2$$

which is undefined, therefore there is no y-intercept.

The x-intercept:
Every point on the x-axis has a y-coordinate of 0, therefore to calculate the x-intercept, let $y = 0$.


For example, the x-intercept of $g(x) = \frac{2}{x} + 2$ is given by setting $y = 0$:

$$y = \frac{2}{x} + 2$$
$$0 = \frac{2}{x} + 2$$
$$\frac{2}{x} = -2$$
$$x = \frac{2}{-2}$$
$$= -1$$

This gives the point $(-1; 0)$.

Figure 35: An example from page 139 of the Siyavula textbook

Monica: To find the y-intercept we substitute x by 0.

Matle: Then we replace x by 0, $y = \frac{2}{0} + 2$.

Monica: 2 over zero plus 2 the answer will be undefined.

Teacher: Why is that?

Matle: Because we have divided 2 by zero and we got ‘maths error’ on the calculator.

Monica: Okay, then for x-intercept we let y to be 0.

Matle: Then $0 = \frac{2}{x} + 2$.

Monicca: And then now we transpose 2 and it becomes negative and 2 over x remains positive $-2 = \frac{2}{x}$.

Benad: Why is it positive?

Monica: Because we only transposed 2, so it is the one that changes the sign.

Matle: And now we cross-multiply and the value of x becomes -1.

Benad: I still do not understand the positive step.

Teacher: Let me give you an example. If you are given $0 = -x - 3$, what will be the value of x?

Benad: I will transpose -3 and have $3 = -x$.

Monica: Then what will be the value of x?

Benad: $x = 3$ because I will divide by -1, oh I see now.

Benad, Monica, and Matle were able to understand how to get the values of x and y intercepts from the extract above by connecting their prior knowledge with the example in figure 35 above. While Monica's group was able to discover the y and x-intercepts with little assistance from me, other learners in the group, such as Benad, were stuck when doing cross multiplication. To help them understand, I guided them by asking questions and providing examples. This was also true for other groups, who were able to figure out how to find the values of x and y intercepts in a hyperbolic function. Following the example, learners moved on to solve question 2 in figure 30 above, which required them to determine the values of x and y intercepts in hyperbolic functions. Learners correctly identified the intercepts in figures 36–38. Below are examples of the groups' solutions, along with the teacher’s comments.

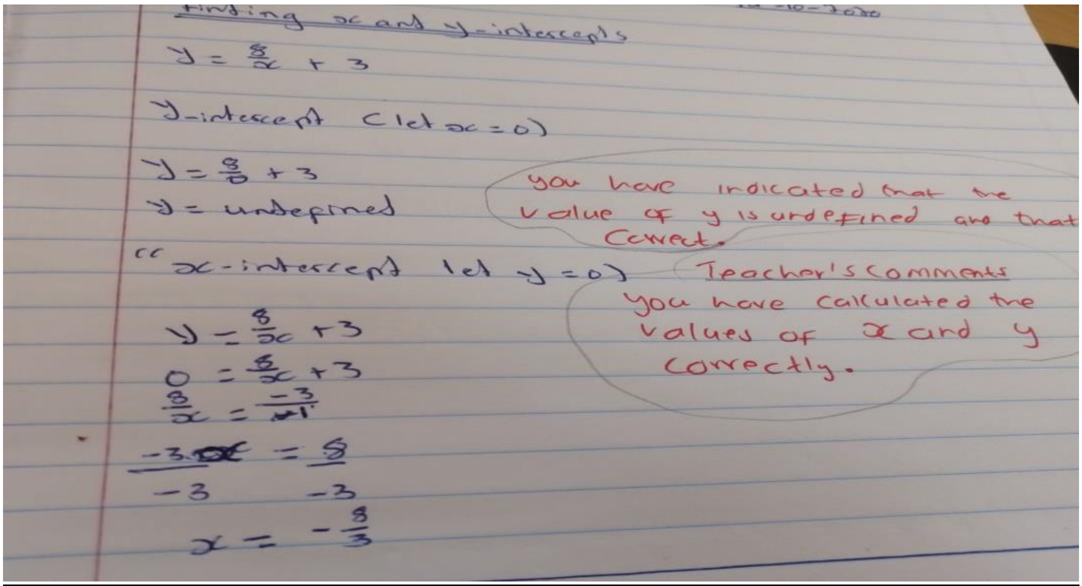


Figure 36: Prince's solution to learning activity 2 with teacher's comments.

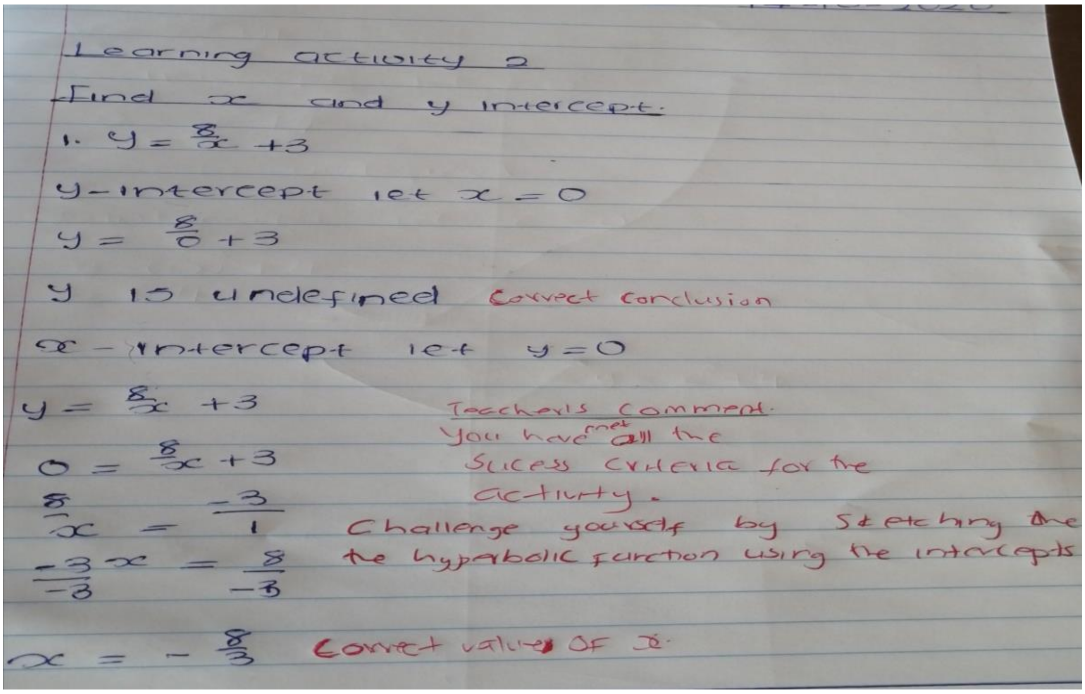


Figure 37: Monica's solution to learning activity 2 with teacher's comments.

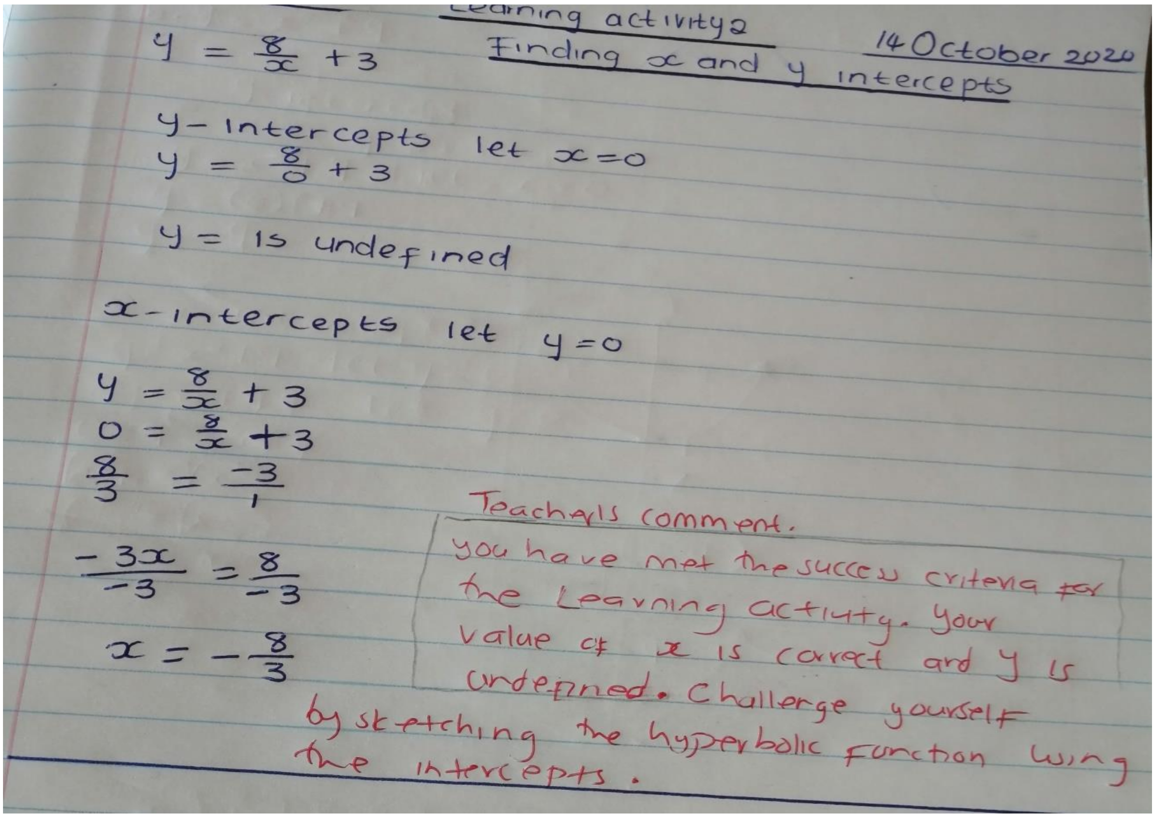


Figure 38: Kholo's solution to learning activity 2

<u>Analysis teaching episode 2</u>

I shared the day's learning objectives with the learners, indicating that they would be plotting hyperbolic functions using the table method, and determine the values of the x and y-intercepts in a hyperbolic function. I also directed them to the Siyavula textbook to see good examples of hyperbolic functions, and how the shape of the hyperbolic function appears in figure 35 above. Learners seemed to understand what was required when sketching a hyperbolic function, determining the x and y-intercepts. I assigned them learning activity 1 that consisted of two questions, the first of which required them to draw five different hyperbolic functions on the same set of axes to elicit evidence of learning. The learning activity provoked some vigorous discussion among the learners. Since the learners worked in pairs, they were able to help each other and ask each other questions that helped them sketch the correct hyperbolic function sketches. The questions they asked each other while conversing in pairs assisted them in uncovering their prior knowledge of sketching functions. For example, when discussing, Benad and Monica remembered that when they get an undefined answer for the value of x, it indicates the graph will not touch the x-axis. They were able to apply their prior knowledge of undefined values of x to undefined values of y, and concluded that for undefined values of y, their graph will not touch the y-axis.

As I walked around the classroom, I noticed that the majority of learners were able to complete the table, but they were perplexed when plotting their points on the Cartesian plane since they discovered numerous undefined values when completing the table. To my delight, they continued with the learning activity, acting as instructional resources for one another while I facilitated and provided limited assistance. They eventually plotted all five hyperbolic functions accurately. I offered them verbal feedback to compliment them on correctly sketching all of the hyperbolic functions in question 1. After that, learners moved on to question 2, where they had to calculate the x and y intercepts. When each group finished their work, I would write comment-only feedback on their answers, following the strategy of providing learners with feedback that moved them forward. In cases where they made minor mistakes, I utilised task-focused comments to encourage them to correct these mistakes. Furthermore, in cases where they were able to discover the right values of the x and y-intercepts of a hyperbolic function, I provided task-focused comments. In Monica's solution to question 2 of finding the intercepts in a hyperbolic function in figure 34 above, for example, I wrote, "challenge yourself by sketching the hyperbolic function using the intercepts."

As time elapsed, learners completed their reflective journals (Appendix A) at the end of the lesson. As I went through them, I discovered that the majority of them indicated that they were learning well from their peers and were looking forward to the next lesson.

Note-taking –teacher's reflective journal

In this episode, I enacted the five key strategies of assessment for learning, with particular emphasis on strategy number four, activating learners as resources for one another. I saw how effective it was in boosting conceptual understanding of mathematics concepts among learners in the previous episode. According to Black and William (2018), learners are more receptive to critiques and remarks from their peers, and it is easier for them to be corrected by their peers than by their teachers. What I have seen is that using peer feedback encouraged them to achieve procedural fluency of functions because they were able to correctly sketch all five hyperbolic functions. Peer feedback supplements self-assessment. It allows learners to assess and be assessed by their peers, which is how they were able to identify the x and y-intercepts of a hyperbolic function on their own. This asserts that after learners have conceptually understood certain concepts, it is easier for them to achieve procedural fluency, which is a crucial aspect of learning mathematics.

Analysis of teaching experiment 2

The analysis of this teaching experiment was on learners' mathematical understanding arising from the two teaching episodes. I however looked at my enactment of the five key strategies of assessment for learning as factors contributing to learners' mathematical understanding: (i) clarifying, understanding and sharing learning intentions; (ii) engineering effective classroom discussion, tasks and activities that elicit evidence of learning; (iii) providing feedback that moves learners forward; (iv) activating learners as resources for one another; and (v) activating learners as owners of their own learning.

(i) Clarifying, understanding and sharing learning intentions

In this teaching experiment, I executed this strategy in collaboration with other strategies. In both teaching episodes, I shared the learning objectives as well as the success criteria for the lessons with the learners to captivate their interest. In teaching episode one, Adelaide showed learners an example of good work on a parabolic function with undefined values of x when she showed the class how the graph of $y = \frac{1}{2}x^2 + 4$ should look like. Learners saw the examples of poor work by seeing and reading the comments I made in their books on their sketches of $y = \frac{1}{2}x^2 + 4$ since their graphs were incorrectly sketched. In teaching episode two, I showed learners how a proper shape of a hyperbolic function should look like from their Siyavula textbooks. Furthermore, I instructed them to work in pairs to go through the examples of working with hyperbolic functions in their textbooks. This strategy of clarifying and sharing learning objectives with the learners played a vital role in developing their mathematical understanding because the strategy captured their attention into the lesson. Moreover, involving learners in this manner allows them to acquire selfregulating qualities, propelling them to become owners of their learning (Peters & Kitsantas, 2009).

In this teaching experiment, I discovered that the five key strategies of assessment for learning go hand in hand. As I executed the first strategy of assessment for learning, I had already incorporated practically all of the other strategies into the lesson. When I asked learners effective questions in teaching episode one to clarify the success criteria of sketching the parabolic function with undefined values of x, I was enacting strategy number two. This strategy was evident in teaching episode one, for example, when I asked Lesego, "Do you have any idea on how to correct your solution based on the comment I made?" after reading my remark on her erroneous graph of $y = \frac{1}{2}x^2 + 4$. Strategy number four of activating learners as instructional resources for one another was also included when Adelaide showed her classmates the shape of the graph of $y = \frac{1}{2}x^2 + 4$ which was correct. This strategy of activating learners as resources for one another was even more evident as she was explaining to the rest of the class the method she used to sketch the graph.

(ii) Engineering effective classroom discussion, tasks and activities that elicit evidence of learning

Learners were now aware of the learning objectives and success criteria of the concepts taught in this teaching experiment, which were parabolic functions with undefined values of x and hyperbolic functions. In both teaching episodes, this strategy seemed to be very effective because the effective questioning, classroom discussions, and mathematical activities that learners engaged in fostered a conducive learning environment whereby they participated willingly in the classroom discussions and collaborated with their peers who propelled them to uncover their misconceptions, prior knowledge and stimulated new understandings. In this teaching experiment, conceptual understanding and procedural fluency were evident in the sketching of functions.

In teaching episode one, learners seemed to develop the conceptual understanding of plotting the intercepts of a parabolic function after I have revised the calculation of square roots with them and what it means when they have undefined values. I have realised that learners started to understand the sketching of graphs conceptually because they even managed to correct their mistakes of sketching the parabolic functions with undefined values of x such as the graph of $y = \frac{1}{2}x^2 + 4$. The majority of them, like Prince's group, went so far as to look up additional comparable questions in the textbook to see if they would get them right, which they did. Some of the learners demonstrated procedural fluency in sketching parabolic functions.

When asked to sketch hyperbolic functions in teaching episode two, learners demonstrated significant improvement in sketching graphs. Furthermore, they demonstrated procedural fluency when I assigned them a mathematics learning activity, including five hyperbolic functions to sketch on the same set of axes. According to Johnson et al. (2019), when the classroom setting fosters a climate in which learners engage in positive classroom discussions, they feel more at ease sharing ideas with their classmates and constructing knowledge together. An observation by Johnson et al. (2019) resulted in great improvement in this teaching experiment, with learners displaying procedural fluency in graph sketching.

The relationship between conceptual understanding and procedural fluency was observed in this teaching experiment. Kilpatrick et al. (2001) contend that when learners understand certain concepts conceptually, procedural fluency develops naturally. This was observed in this teaching experiment when learners were able to sketch parabolic and hyperbolic functions in collaboration with their peers while I was mostly acting as a facilitator. A bit of strategic competence was also evident in few learners such as Adelaide, who managed to sketch the graph of $y = \frac{1}{2}x^2 + 4$ using a different strategy to plot the graph. She used the table method over the intercepts method that they were supposed to use.

(iii) Providing feedback that moves learners forward

The effective questioning, mathematical activities and classroom discussions prompted me to provide learners with either verbal or comment-only feedback in learners' books throughout the teaching experiment. In all of the mathematical activities written for this teaching experiment, I gave learners comment-only feedback, with no scores – and this resulted in learners responding to the comments by trying to correct their mistakes. Black and William (1998) concur that when scores are given, learners prefer to ignore the remarks. Therefore in the absence of scores, learners were taking responsibility for their learning and asked for solutions from their peers as well. While the learners were working in pairs, I would provide them with verbal feedback through questions, which made them aware of their mistakes and engaged their thinking. According to Sondergeld et al. (2010), feedback to learners can be provided by both peers and teachers to motivate learners to make improvements on their work. In this teaching experiment, particularly in teaching episode one, Adelaide was able to provide feedback to her peers during the process of sketching the graph of $= \frac{1}{2}x^2 + 4$, together with the feedback that I gave, the learners were able to correct their work.

As I enacted this strategy, I then again ascertained how the key strategies of assessment for learning are very much intertwined. The feedback I gave learners during classroom discussion incorporated effective questioning to stimulate new understandings in learners and to uncover their misconceptions. The comment-only feedback propelled learners to ascertain their mistakes and to correct them. Moreover, procedural fluency of parabolic functions with undefined values of x and hyperbolic functions was evident, and emanated from their conceptual understanding of sketching graphs.

(iv) Activating learners as resources for one another

Despite the fact that this teaching experiment focused on learners working in pairs, particularly in teaching episode 2, the feedback that learners received from myself as the teacher encouraged them to collaborate. Their understanding of concepts and their relationships improved dramatically as they worked as resources for one another. In teaching episode one, as Adelaide demonstrated to the rest of the class how the shape of the graph with undefined values of x should look like, it was well-received by the learners. I also noticed that the fact that their classmate was able to correctly sketch a graph that they found difficult boosted their confidence. Learners learn best from their peers. I find that they are more willing to accept criticisms from their peers than from me as their teacher. This was evident in teaching episode 1 when they were correcting their mistakes while sketching the graph of $y = \frac{1}{2}x^2 + 4$. This strategy played a role in building their conceptual understanding of parabolic functions.

Learners working together as a team played an important role in the learning of mathematics, as demonstrated in teaching episode 2 when they worked in pairs to complete question 1 of sketching hyperbolic functions in learning activity 1 without the teacher's assistance. I simply directed them to the pages of their Siyavula textbook where they could find the shape of hyperbolic functions. After that, they were able to work in pairs and perform all of the hyperbolic functions correctly on their own.

Furthermore, the majority of learners were able to correctly answer question 2 of learning activity 1, which required them to calculate the values of intercepts in hyperbolic functions. They were able to answer all of the questions correctly because they practised calculating the intercepts in hyperbolic functions from their Siyavula textbooks with little or no assistance from the teacher. As a result, I assisted the groups that were struggling, such as Monica's, with a bit of questioning to stimulate their new understanding for them to uncover their mistakes. Then, using effective questioning, they identified their own mistakes and proceeded to obtain the correct answers.

As a result of this teaching experiment, learners demonstrated conceptual understanding of graph sketching, as well as procedural fluency in the sense that they were able to sketch all of the hyperbolic functions correctly and calculate the intercepts with the assistance of their peers. As a result, strategic competence was evident in a few learners as they sought out various strategies to solve the mathematical activities. Learners were now taking charge of their own learning. Black and William (2018) concede that when learners provide feedback to their peers, they become more reflective of their own work.

(v) Activating learners as owners of their own learning

I discovered that as learners completed their reflective journals, they developed a sense of ownership of their own learning. In teaching episode one, I saw learners asking themselves extra questions about parabolic functions with undefined values of x, which assured me that they had mastered how to draw the parabolic functions conceptually. In addition, the majority of their reflective journals indicated that they understood how to draw the functions using tables and intercepts, which they learned through peer assessment. The majority of learners demonstrated conceptual understanding and procedural fluency, and eagerly anticipated the next lesson. As a result, one of the learners mentioned that he would like to learn how to sketch functions using a calculator.

My reflections on teaching experiment 2

What I observed in teaching experiment 1 happened again in teaching experiment 2. Since they are interactive, the five key strategies of assessment for learning work in tandem. As I enacted the first key strategy of assessment for learning, I had already incorporated almost all of the other strategies into the lesson. The fourth strategy, activating learners as resources for one another, was critical in propelling them to acquire their mathematical understanding. The strategy stimulated their conceptual understanding and procedural fluency in sketching parabolic and hyperbolic functions in this teaching experiment.

The next teaching experiment will focus on using the five key strategies of assessment for learning to foster strategic competence in sketching functions among the majority of learners. My observer and I determined that the strategy of activating learners as instructional resources for one another and activating them as owners of their own learning produced the most positive results. Therefore, we decided to give the two strategies more attention in the next teaching experiment while keeping in mind that the five key strategies of assessment for learning are interactive.

4.5 TEACHING EXPERIMENT 3: STRATEGIC COMPETENCE ON SKETCHING FUNCTIONS

Teaching Experiment 3 background

The focus of this teaching experiment was to determine learners' strategic competence in graph sketching, as the majority of them had acquired conceptual understanding and procedural fluency in graph sketching. I gave learners four different types of functions to sketch using their preferred methods in this teaching experiment. My planning called for them to use different strategies for each type of function or even devise their own strategies to sketch the functions. The functions given to the learners included the straight line, hyperbolic and parabolic functions that they had learned in previous teaching experiments, as well as the exponential graph, which was new to them. So, incorporating the exponential graph that they had not done previously was also important for me to assess their strategic competence and to amplify strategies four and five of activating learners as resources for one another and activating them as owners of their own learning.

4.5.1 Episode 1: Consolidation Lesson on Functions

This is what transpired in the classroom:

Teacher: The objective for today is sketching the four different functions in learning activity 1 consisting of a straight line, parabola, hyperbola and exponential graph using any method that you prefer. You are going to work in pairs and later share the method you used with the rest of the class. Be sure to go through your textbook to remind yourselves of the correct shapes of the graphs, including the new graph that is the exponential graph.

Teacher: Before you proceed with the learning activity, let us recap on the different types of methods you know in the sketching of graphs.

Prince: We can use the table method.

Teacher: How is it done?

Adelaide: You use the equation given to you to find the ordered pairs and sketch the graph.

Teacher: Does it apply to all the graphs?

Kholo: Yes, ma'am. I can draw all the graphs using it.

Teacher: What other methods can we use to sketch the graph?

Lesego: For example ma'am, if I want to sketch a parabolic function I can find the intercepts and the turning point.

Prince: Even with the straight line, we can use the intercepts methods.

Teacher: Interesting, right now complete the following learning activity 1, you are not limited to the methods that we have used before, new strategies are acceptable.

Learning activity 1

Sketch the following graphs using any method of your choice.

1. $f(x) = -x + 1$

2. $f(x) = x^2 - 4$

3. $f(x) = \frac{-4}{x} + 7$ with intercepts and asymptotes

4. $f(x) = 3^x$

Success criteria: Plotting each graph correctly with any preferred method

Figure 39: Learning activity 1

Learners began working on the activity while I walked around the classroom observing and exploring what they were saying and doing as they wrote their responses to the activity. According to Steffe and Thompson (2000), one of the most important aspects of the teaching experiment methodology is to explore what learners say or write as they engage in mathematical activities. The extracts below represent what they were writing and saying during their interactions. The learners were active participants in the lesson, and I was the facilitator.

Below are the learners' solutions and discussions for the graph of $f(x) = -x + 1$

Figure 40 shows Benad's solution in which he used the table method to sketch the graph of $f(x) = -x + 1$. However, he did not correctly sketch the graph, and I quickly commented on both his completed table and the sketch itself. I intended to get him to think about how he could fix his mistakes because the purpose of comment-only feedback is to get learners to think about how they can fix their mistakes, especially when the comments are task-focused, as stated in William and Thompson's (2007) framework.

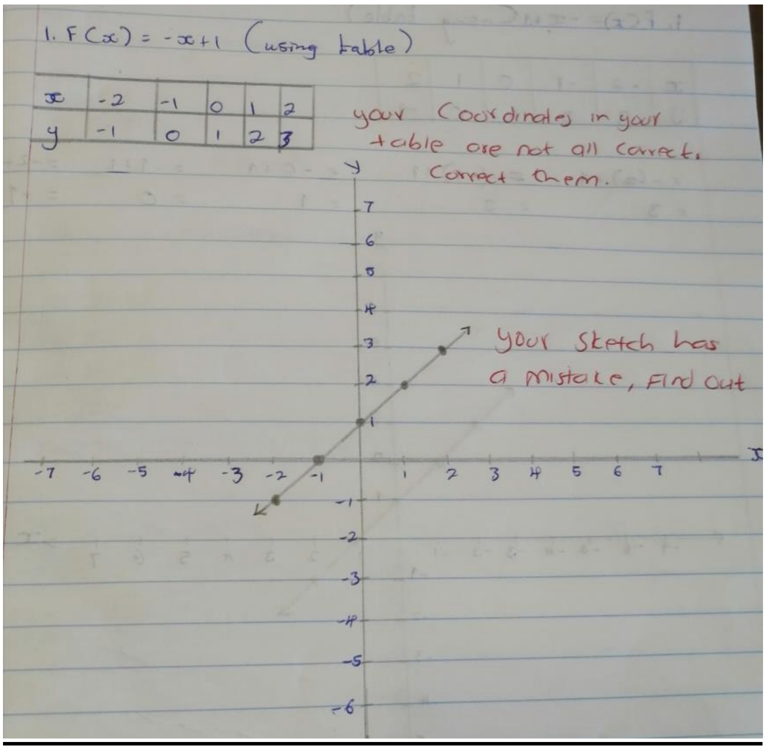


Figure 40: Benad's solution to $f(x) = -x + 1$ with the teacher s comments.

When I looked at Kholo's solution, I noticed that she used the same table method as Benad to sketch the graph of $f(x) = -x + 1$. I have confirmed that she correctly sketched the graph. I commented verbally on her solution, indicating that it was correct and that she should share what she did with the rest of the class. My goal here was to have Benad and other learners who used the table method to sketch the graph of $f(x) = -x + 1$ to self-reflect on their own solutions.

<u>*A discussion between Kholo and the class for the solution to $f(x) = -x + 1$*</u>

Kholo: This is how I completed my table, my table starts from -3 to 3

x	-3	-2	-1	0	1	2	3
$f(x) = -x + 1$	4						

Kholo: Where I see x I substitute with -3.

Lesego: What is $f(x)$?

Monicca: $f(x)$ is the same as y

Kholo: $y = -(-3) + 1$ and the answer is 4

Adelaide: Why do we write -3 in a bracket?

Kholo: Because we are working with negative numbers.

Lesego: We have to include the bracket; otherwise, we will not find the correct answers.

Prince: Let us confirm our answers with the calculator.

Lesego: Oh, you are right. I am getting 4 after substituting with -3.

Kholo: I have completed the whole table using this method and I sketched my graph using the points.

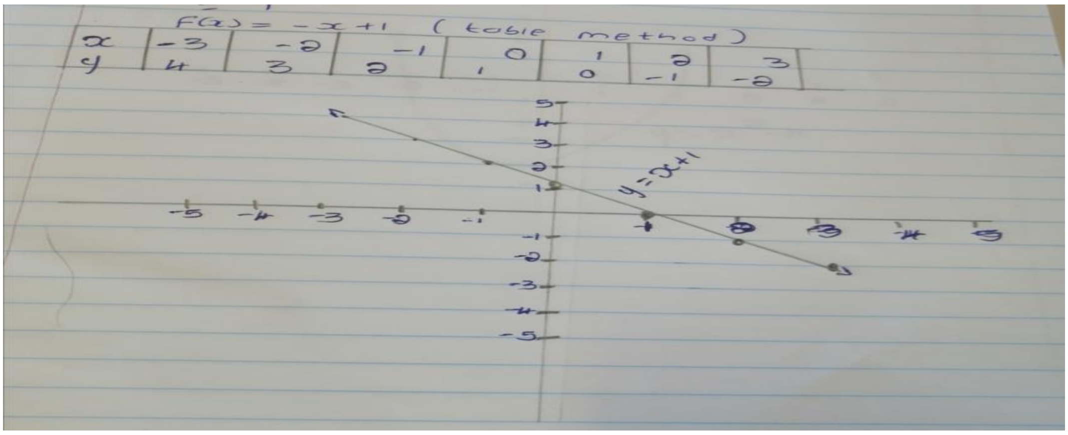


Figure 41: Kholo's final solution to $f(x) = -x + 1$

Kholo's presentation of her solution to the graph $f(x) = -x + 1$ assisted other learners like Benad to self-reflect and to correct their mistakes. Below is the corrected version of Benad's work in figure 42.

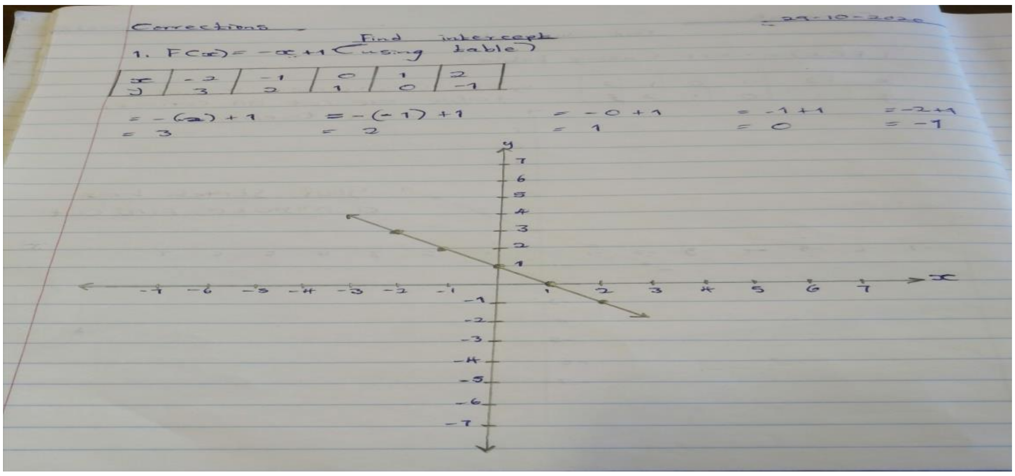


Figure 42: Benad's correction for $f(x) = -x + 1$

On the other hand, as I observed what learners were doing, I noticed that Prince drew the graph using the calculator method for $f(x) = -x + 1$. Below in figure 43 is his sketch.

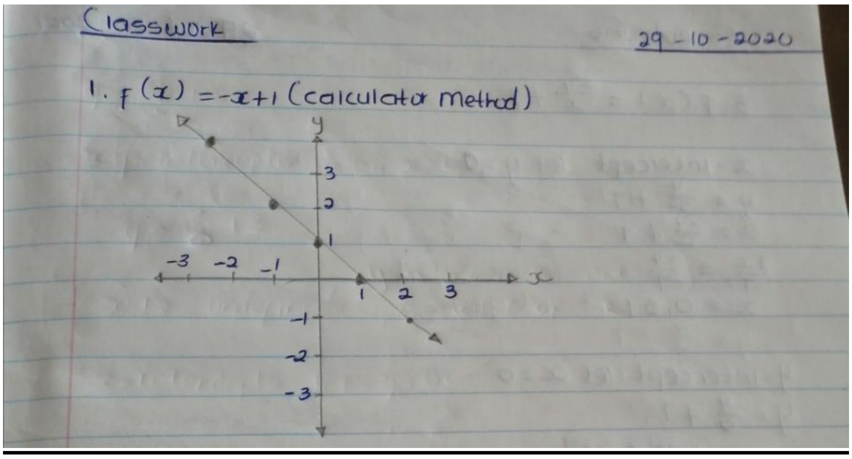


Figure 43: Prince’s solution to $f(x) = -x + 1$

When I looked at Prince's solution in figure 43, I noticed that it was correct. So I asked him to present it to the rest of the class. This was interesting because he taught himself the calculator strategy, which I had not yet taught the rest of the learners. He thoroughly explained all of the steps, and the learners showed strong interest in using the calculator.

Prince: Press mode.
Learners: Followed.
Prince: Choose table.
Prince: Enter the function as it is on the calculator $f(x) = -x + 1$.
Adelaide: I am not getting the correct answers.
Prince: You should press an equal sign after each step.
Pharcily: For start, what do we enter?
Prince: You look at where your values of x start in this case they start at -3 and end at 3.
Makgabo: I see two columns on my calculator.
Prince: Yes, there is x and $f(x)$ plot the graph using those values x is the value of x and $f(x)$ is the value of y.
Prince: I used the coordinates on the calculator (in figure 43.1 below) and sketched my graph.

Figure 43.1: calculator coordinates for the graph of $f(x) = -x + 1$

Furthermore, Makgabo used the intercepts method to answer the question, but faced some challenges while presenting her solution on the board. However, her peers assisted and managed to sketch the graph of $f(x) = -x + 1$ in figure 44 below using the intercepts method.

A discussion between Makgabo and the class

Makgabo: I calculated y-intercept first and you let $= o$, and substitute x in the equation $f(x) = -x + 1$ which gave me $y = -0 + 1$.
Adelaide: Why are you not writing your zero inside the bracket?
Monica: Because the value of x is not a negative number.
Makgabo: Then $y = 1$ and my coordinate for $y - intercept$ is (0; 1).
Makgabo: 0 is the x value and 1 it is the y value.
Class: We understand.
Makgabo: then for $x - intercept$ we let $y = o$.
Makgabo: $0 = -x + 1$ and then $x = -1$.
Kholo: The value of x cannot be -1.
Makgabo: Why? It is -1 I took 1 to the left and changed the sign.
Pharcily: You are right, but are you aware that now the equation will be $-1 = -x$.
Prince: Yes, Pharcily is correct and we are going to divide both sides by negative 1 and the answer will be $x = 1$.
Makgabo: Oh, I see and now the $x - intercept$ will be (1; 0).

Adelaide: Yes, you can now sketch the graph.

Makgabo: Okay, then I am going to sketch my graph using my coordinates for y-intercept (0; 1) and coordinate for x-intercept (1; 0) and plot the graph.

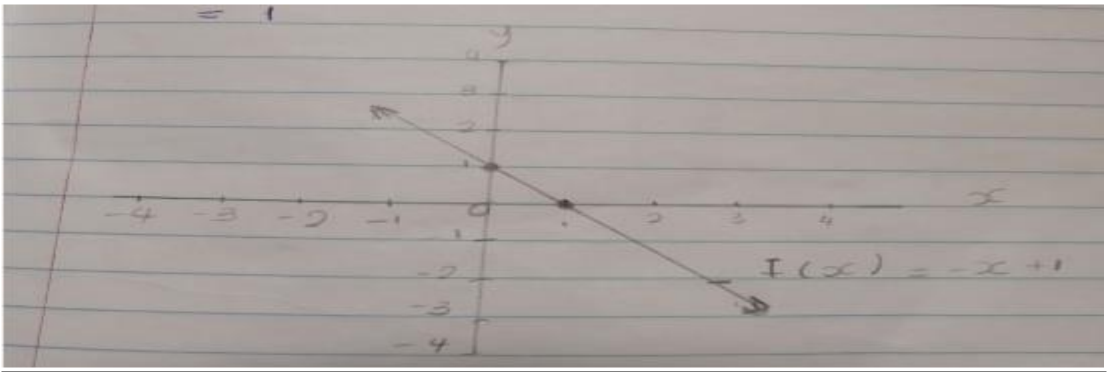


Figure 44: Makgabo's final solution to $f(x) = -x + 1$

From the extracts above, learners used three different methods to sketch the graph of $(x) = -x + 1$. Through peer and self-assessment, learners managed to sketch the graph correctly in their three different methods. As a result, the strategic competence was evident in this question of sketching the graph of $f(x) = -x + 1$**.** Since learners used various strategies to solve the question, one of them, Prince, used a strategy that we had not yet done in class of using a calculator, demonstrating the formulation of new strategies. What I found more interesting was that Prince was able to share step by step with the rest of the class how he used the calculator method, and they seemed interested and understood how he used the method.

As learners proceeded to answer the learning activity, they were now answering the second question of sketching the graph of $f(x) = x^2 - 4$, which was number two on the learning activity. I have noticed that Adelaide sketched the graph in figure 45 below correctly using the intercepts and turning point to sketch the parabolic function. I then requested that she presents her solution to the rest of the class.

<u>**Conversation between Adelaide and the class for the graph of** $f(x) = x^2 - 4$</u>

Adelaide: For intercept, I let $x = 0$ and $y - intercept$ is $(0; -4)$.

Adelaide: And to find x-intercept I let $y = o$ and $x - intercepts$ are $-2\ and\ 2$.

Adelaide: My turning point is $(0; -4)$.

Lesego: How did you find it?

Adelaide: -4 is from the formula.

Kholo: I do not understand.

Adelaide: $y = -4$ let us continue.

Lesego: Where did you get -4?

Monica: We cannot proceed and leave others behind, guys where your confusion is?

Kholo: The -4 of the formula mentioned by Adelaide.

Prince: It is the value of q, from the standard form of a parabola.

Adelaide: Standard form of a parabola is $y = ax^2 - q$.

Makgabo: No, the formula does not have $-q$.

Monica: Let us check the formula in the book.

Prince: The standard form is $y = ax^2 + q$.

Adelaide: Okay, I see so in this is the formula of turning point $(0; q)$, we take the value of q from the equation of the function but still my turning point will be $(0; -4)$ because of the given function of $f(x) = x^2 - 4$ **,** our q is -4.

Makgabo: It is clear now, I understand.

Adelaide: Then I can now sketch the graph since I have everything I need.

Kholo: And the shape of the graph will be a smiling one, right?

Adelaide: Yes, because the value of “a” is positive the graph will look upwards.

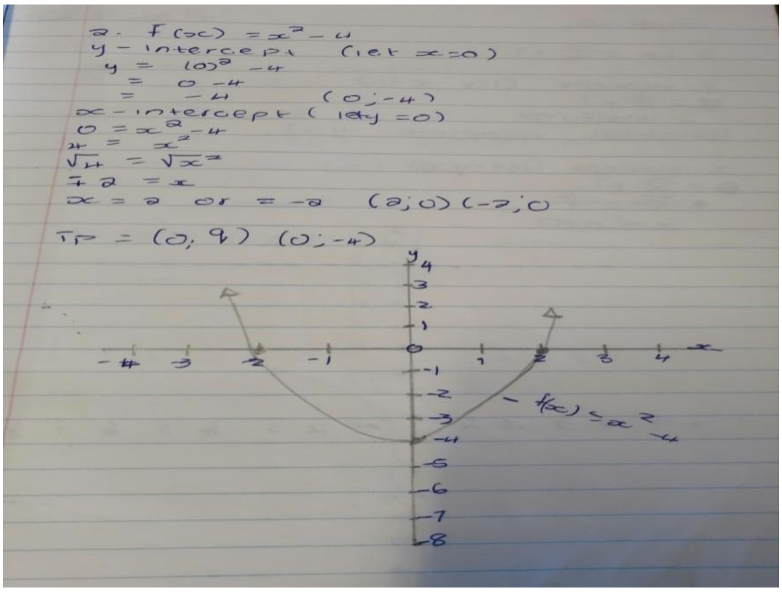


Figure 45: Adelaide's solution to the graph of $f(x) = x^2 - 4$

Following Adelaide's presentation, I went on to observe what the learners were doing. What caught my attention was Prince's solution for the graph of $f(x) = x^2 - 4$. He used the calculator method again to answer the question, and he managed to correctly sketch the parabolic function in figure 46 below. When I asked him to present his solution to the rest of the class, he used the same calculator strategy and steps he used to solve the straight-line graph $f(x) = -x + 1$ in number 1 and the majority of learners followed what he was doing.

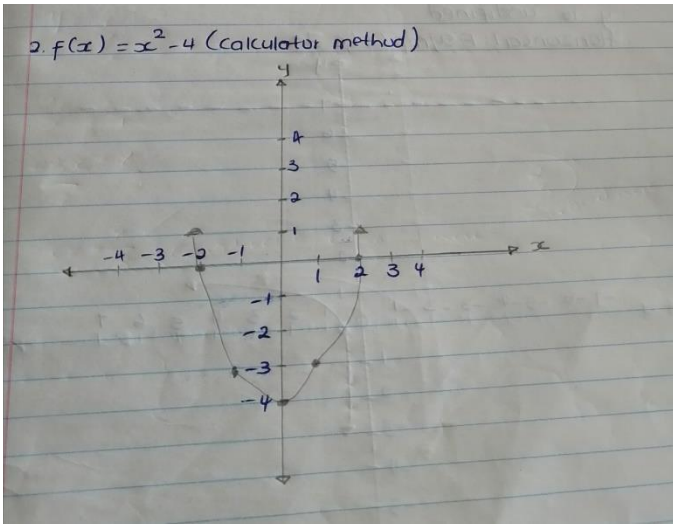


Figure 46: Prince's solution to the graph of $f(x) = x^2 - 4$

In sketching the parabolic function $f(x) = x^2 - 4$, the majority of learners used the intercepts method, the table method while a few used the calculator method. However, all of the learners were able to correctly sketch the parabolic function, demonstrating procedural fluency in sketching the parabolic function.

Furthermore, learners moved to graph number three of sketching the hyperbolic function $f(x) = \frac{-4}{x} + 7$ with intercepts and asymptotes. As I walked around the classroom looking at what they were writing, I realised that the majority of them could not draw the hyperbolic function using the intercepts and asymptotes. Half of them used the table and calculator method, which resulted in the incorrect shape of the graph, while others used the intercepts method but were still unable to sketch the graph properly. Figures 47 to 48 show a few of their incorrect solutions, along with task-focused comments from me to help them correct their mistakes.

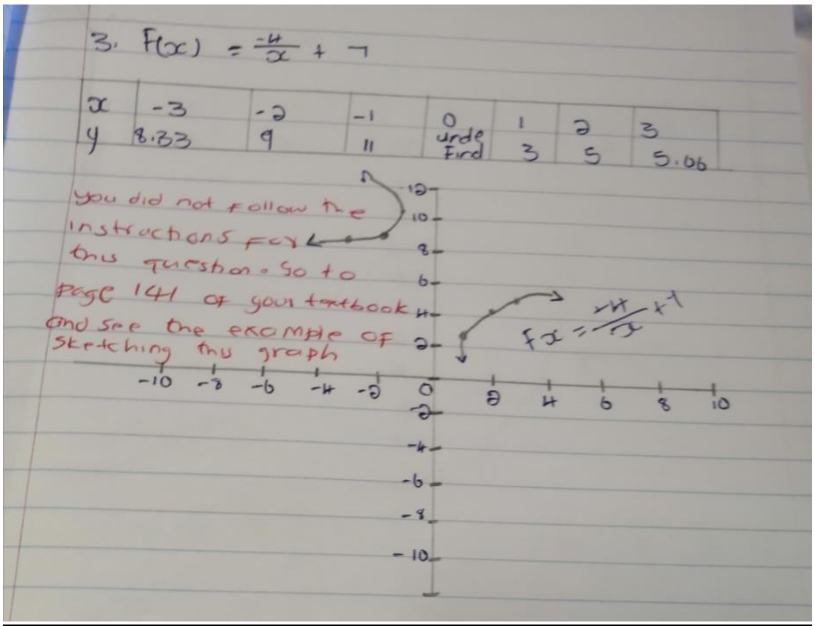


Figure 47: Lesego's solution with teacher's comments

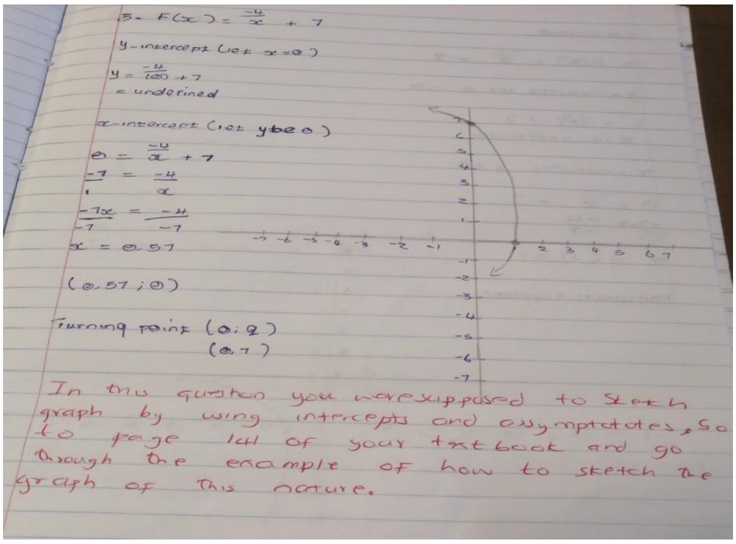


Figure 48: Pharcily's solution with teacher's comments

Lesego, Pharcily, and other learners to whom I provided the comment-only feedback on the question of graph sketching of $(x) = \frac{-4}{x} + 7$, which included going to page 141 of their Siyavula textbook and practising sketching a hyperbolic function with intercepts and asymptotes responded to the comments and were able to correctly sketch the hyperbolic function in figure 49, which is one of the corrected graph sketches of $f(x) = \frac{-4}{x} + 7$.

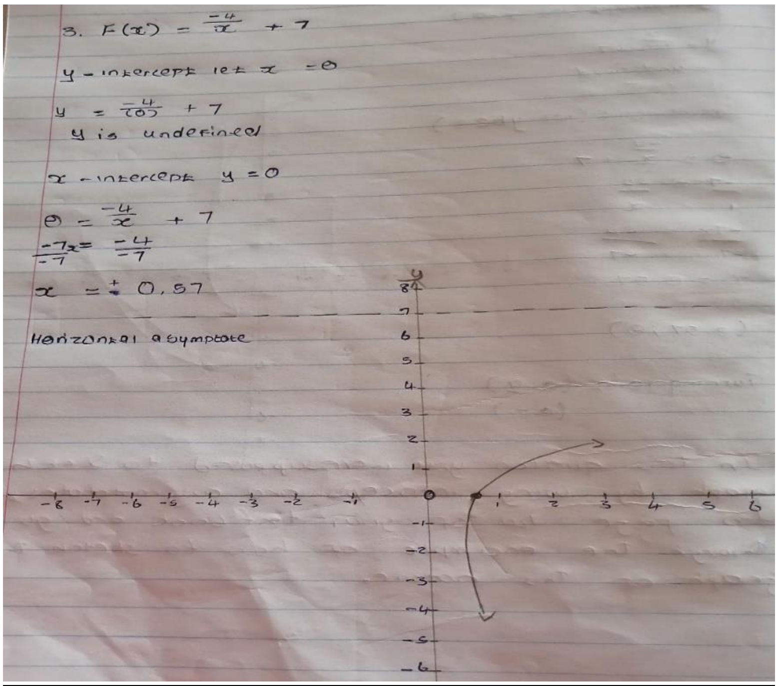


Figure 49: Corrections for Pharcily to the graph of $f(x) = \frac{-4}{x} + 7$

Through the comments and by following an example in their Siyavula textbook, all of the learners were able to sketch the correct hyperbolic function. Furthermore, learners collaborated with their peers, despite the fact that it was time-consuming and frustrating because by this moment, I expected every learner to be able to sketch a hyperbolic function with ease. Despite this, learners were able to correct their mistakes.

Moreover, learners moved to the last function of sketching the exponential graph of $f(x) = 3^x$**,** which was a new graph since they did not do it in the previous lessons. I was curious to see how they would apply their prior knowledge of graph sketching to the exponential graph. As I moved around the classroom facilitating the lesson, I noticed that Monica's exponential shape of $f(x) = 3^x$ was not correct. I got closer to figure out what went wrong, and I commented on her graph so she could rectify her mistakes. Figure 50 shows her graph of $f(x) = 3^x$with my task-focused comments**.**

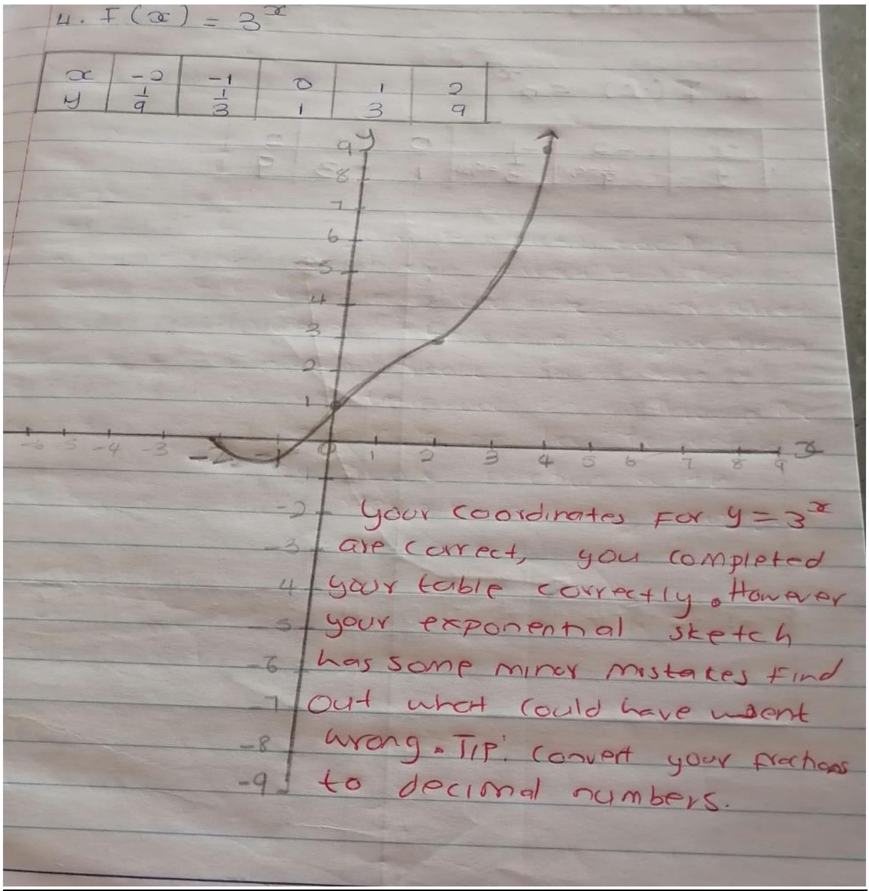


Figure 50: Solution for Monica using table method with teacher's comments for $f(x) = 3^x$

Monica was able to correct the graph on her own after reading the comments and following my advice, and the corrected graph of $f(x) = 3^x$ in figure 51 is shown below.

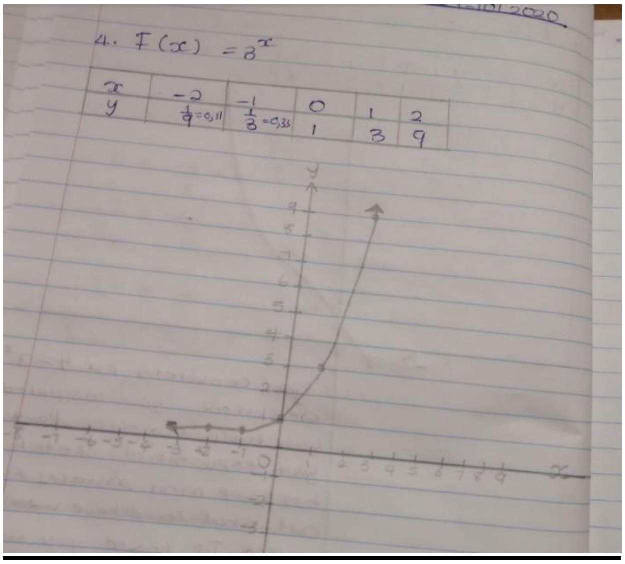


Figure 51: Corrections for Monica using table method for the graph of $f(x) = 3^x$

At the same time, Prince and Adelaide used different methods to sketch the graph of $f(x) = 3^x$ and managed to sketch it correctly. In addition, I noted these learners' strategic competence and procedural fluency with the sketching of functions. Figure 52 shows Prince's calculator method, while Figure 53 shows Adelaide's table method.

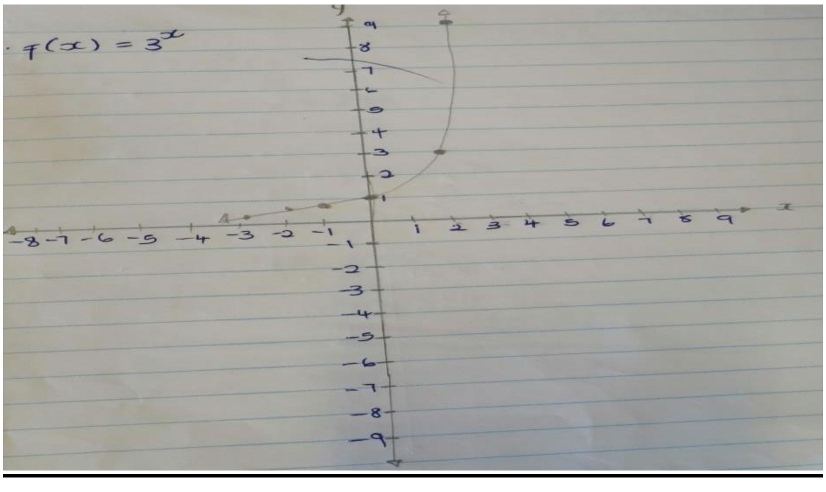


Figure 52: Prince’s solution to the graph of $f(x) = 3^x$

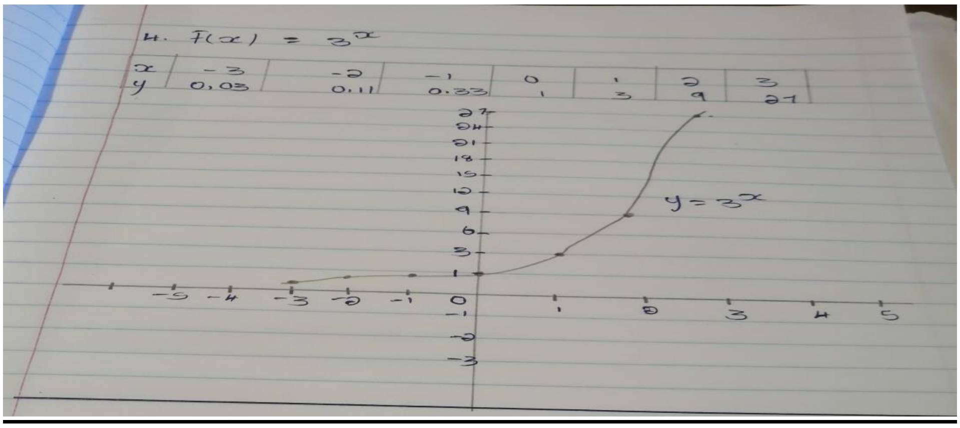


Figure 53: Adelaide's solution to the graph of $f(x) = 3^x$

Even though it was their first time to see the graph, the majority of learners were able to correctly sketch the exponential graph using the table and calculator method. Pharcily, on the other hand, attempted to sketch the graph of $f(x) = 3^x$ using the intercepts method but her solution was incorrect. Below in figure 54 is her solution and my comment on her work.

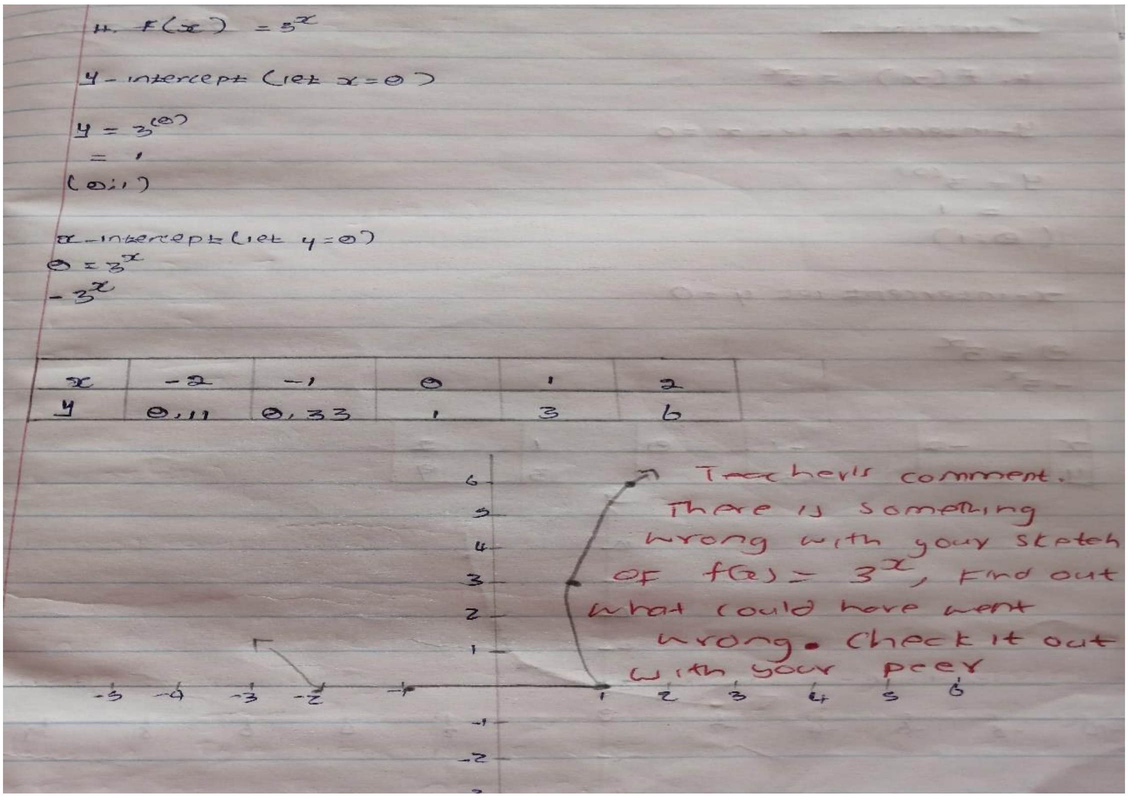


Figure 54: Pharcily's solution to the graph of $f(x) = 3^x$ with the teacher's comment

In the comment, I wrote that "There is something wrong with your sketch of $f(x) = 3^x$, find out what could have gone wrong. Check it out with your peer." After reading the comment, Pharcilly discussed her work with Adelaide. Below is their conversation.

Adelaide: Which method did you use?

Pharcily: I calculated x and $y - intercept$.

Adelaide: Okay, I see for y-intercept you let $x = 0$.

Pharcily: Yes and the answer was 1, that is why I wrote $(0; 1)$.

Adelaide: You are right, so what happened to your $x - intercept$?

Pharcily: I let y to be 0 but I could not proceed and I decided to follow the table method.

Adelaide: Even though you started at $-2\ to\ 2$ in the table, your sketch was supposed to be correct. Let us complete the table together from the beginning while comparing your solutions with mine.

Pharcily: Oh, I see my mistake. I was multiplying instead of finding exponents. For example, when I substituted 2 into $f(x) = 3^x$ instead of finding 2^3 which is 8 I worked it as a multiplication $2 \times 3 = 6$.

Adelaide: Oh, I see complete your table correctly then the shape of your graph will be okay.

Pharcily: Thank you, I get it.

Pharcily was able to figure out where she went wrong in sketching the graph of $f(x) = 3^x$. Thanks to the teacher's comments and peer assessment by Adelaide.

ANALYSIS OF TEACHING EXPERIMENT 3

The analysis of the teaching experiment focused on learners' mathematical understanding as a result of the two teaching episodes. However, I explored my enactment of the five key strategies of assessment for learning as factors contributing to learners' mathematical understanding - (i) clarifying, understanding and sharing learning intentions; (ii) engineering effective classroom discussion, tasks and activities that elicit evidence of learning; (iii) providing feedback that moves learners forward; (iv) activating learners as resources for one another; and (v) activating learners as owners of their own learning.

(i) Clarifying, understanding and sharing learning intentions

In this strategy, I shared the day's learning objective with the learners, which was to sketch various types of functions such as parabolic, straight line, hyperbolic and exponential graphs. I clarified that learners were allowed to use any method of their choice when sketching the graphs, and were not limited to the table or intercepts method they had previously used. I then proceeded to show learners examples of both good and poor work on sketching of functions. I further referred the learners to examples of correct different shapes of graphs from their previous work as well as pages in their Siyavula textbooks to see how the shape of a straight-line graph, hyperbola graph and exponential graph, which was a new graph in this episode, should look like. Learners appeared to do what was expected of them and became aware of the required success criteria.

(ii) Engineering effective classroom discussion, tasks and activities that elicit evidence of learning.

Since learners were familiar with the day's learning objectives of sketching various types of functions, they eagerly participated when I engineered effective questioning to assist in uncovering prior knowledge before they could begin writing the learning activity for the day. The questions I posed to the learners influenced their willingness to participate in the classroom discussions. Black and William (1998) concede that the quality of questions and activities influences learners' willingness to participate in classroom activities and to collaborate with their peers. I also gave them a learning activity with four functions to draw in pairs or individually: a straight line, parabola, hyperbola and exponential graph. Learners were not required to use a specific method to sketch their graphs; instead, they were encouraged to use the methods or strategies with which they were familiar. My objective was to develop strategic competence in sketching of functions in my learners. As a result, I encouraged them to use the strategies I taught them in previous lessons, as well as to formulate their own strategies. In this teaching episode, I used mathematical activities to ascertain evidence of learning.

I have noticed that when it came to sketching straight-line graphs, learners used a variety of strategies based on their preferences. Kholo and Benad sketched the graph using the table method. Their solutions, however, differed, and Kholo, whose solution was correct, presented it to the rest of the class. In addition, while Kholo was presenting, Benad was able to self-reflect and correct a mistake in his solution. In addition, Makgabo also presented her solution to the graph of $f(x) = x + 1$ to the rest of the class since she opted to sketch the graph by calculating the intercepts, a different approach to what Benad and Kholo used to draw the same graph. Even though she made a few minor mistakes while presenting, she was able to correct them through classroom discussions with her peers and finally sketched the graph correctly.

As I continued to check what the learners were writing, I noticed something more interesting in Prince's group. Prince used a different strategy that I had not shared with them in previous lessons; he used the calculator method and was able to sketch the graph of $f(x) = x + 1$ correctly. I allowed him to share his strategy with the rest of the class, which stimulated new understandings in the sketching of straight-line graphs using the calculator method. The activation of learners as resources for one another was evident in this strategy, and because learners were assisting one another, the strategies remained intertwined.

The mathematical understanding achieved in this strategy was that learners had conceptual understanding of sketching graphs. The strategic competence was evident because learners were able to use different methods to solve the same questions to the point where the likes of Prince came up with their own strategies that seemed to work perfectly. According to Kilpatrick et al. (2001), learners demonstrate strategic competence skills when they can make sound decisions about which strategies to use or construct their own strategies to solve mathematical activities and problems. The same thing happened with the other two graphs, the parabolic and the exponential. Learners employed various strategies to answer the learning activity, and were able to get the correct answers. Their procedural fluency skills were also strengthened, and some of them were able to correctly draw all of the graphs given to them.

Few learners such as Pharcily and Monica were unable to correctly sketch the exponential graph of $f(x) = 3^x$. This revealed a lack of new understanding for them. Furthermore, the majority of them struggled to get the correct answers when sketching the hyperbolic function with intercepts and asymptotes. They were unable to draw the hyperbolic function using the intercepts and asymptotes, uncovering their lack of proficiency with the method, as most of them were stuck sketching the hyperbolic function using the table, which was ineffective in this learning activity.

(iii) Providing feedback that moves learners forward

I guided learners who did not correctly sketch their graphs when answering questions from learning activity 1 by providing written comment-only feedback in their books or verbally commenting on their work to encourage them to see their mistakes and seek solutions. As learners were at the centre of the lesson in this teaching experiment, they also gave each other feedback that moved others forward through peer assessment. I gave learners who could not sketch some of the graphs correctly comment–only feedback. For instance, Monica could not draw the correct exponential graph of $f(x) = 3^x$. My comments on her graph prompted her to correct her mistakes, and she eventually sketched the correct graph.

The comments I made to Pharcily inspired her to collaborate with her peer Adelaide in identifying her mistakes and finally sketching the exponential graph correctly. Benad's work for the graph received only comment-only feedback of $f(x) = x + 1$. Together with classroom discussion, he was able to correct his mistakes. Furthermore, because it was task-focused, the feedback I gave to learners who could not sketch the hyperbolic functions using intercepts and asymptotes moved them forward. I referred them to page 141 of their textbook for an example that assisted them in correcting their mistakes. According to Hattie and Kimberley (2007), feedback is essential for learners to take ownership of their own learning.

Again, the fourth strategy of activating learners as resources for one another was evident in this strategy, and we saw them being able to move forward and correct their mistakes as they gave each other feedback. For example, Pharcily and Adelaide were able to collaborate in responding to my comments on Pharcily's solution. Throughout this strategy, conceptual understanding, procedural fluency and strategic competence of the sketching the graphs were evident in learners.

(iv) Activating learners as resources for one another

Learners used the feedback I provided to collaborate with their peers to correct their mistakes. This was possible because the planning of this teaching experiment allowed for more opportunities for learners to collaborate with one another. They did an excellent job of serving as resources to one another. In the second question of sketching the graph of $f(x) = x^2 - 4$, Adelaide used the intercepts and the turning point to sketch the graph. However, some learners did not understand the concept of the turning point, and we saw Prince clarify the process by referring them to the standard form of the parabolic function. When Makgabo made some minor mistakes in the sketching of the straight-line graph, Lesego and other learners helped her by providing peer feedback. As a result of peer assessment and learners acting as resources for one another, new understandings were stimulated, and learners were able to identify their mistakes.

Under this strategy, we observed Prince demonstrating to the rest of the class how to sketch graphs, including exponential graphs, without the assistance of the teacher, indicating high levels of strategic competence. This strategy appeared to produce excellent results in all groups. Learners were able to do self-reflection while observing their peers present their solutions, which fostered their conceptual understanding and procedural fluency of sketching functions involuntarily. The intertwining of strategies of assessment for learning has resurfaced; there is a high correlation between strategy number four of activating learners as resources for one another and strategy number five of activating learners as owners of their own learning. According to Johnson et al. (2019), when learners present their solutions as part of peer feedback, they get a chance to self-reflect on their own work and see where they went wrong and how they should have done the correct work.

(v) Activating learners as owners of their own learning

The majority of learners were able to do self-reflection on their own work as they worked in groups, demonstrating a close relationship between peer and self-assessment. My observer and I devised a lesson plan for these teaching episodes that included opportunities for learners to take charge of the lesson while the teacher served as a facilitator. According to William and Thompson’s (2007) framework, teachers should move away from being the sole source of knowledge; and instead, act as facilitators, as I did, allowing learners to become active participants in the lesson. As a teacher-researcher, I only shared the lesson's learning objectives, facilitated the recapping of prior knowledge through questioning, and assigned the learners to work in pairs to determine their level of strategic competence. Otherwise, I proceeded to facilitate their discussions and presentations.

Learners' completion of reflective journals played an important role in propelling them to take ownership of their own learning because the journals provided them with the opportunity to write about what they were struggling with, the value of what they were learning, and what else they still needed to learn. The majority of learners in this teaching episode indicated that they were able to sketch all of the graphs given to them using their preferred strategies. In the sections where they had to indicate what they were still struggling with, some of them wrote that they were struggling with the sketching of an exponential graph, which was the first time they saw the graph, but that they eventually managed to sketch the graphs with the assistance of their peers. Others stated that they had difficulty with hyperbolic functions, and that by reading the teacher's comments, they were able to correct their mistakes. The developed strategic competence in function sketching reinforced the procedural fluency in function sketching.

Note-taking teacher’s reflective journal

In this teaching episode, I enacted the five key strategies of assessment for learning. The focus was on strategy four of activating learners as resources for one another, and strategy five of activating learners as owners of their own learning to assess my learners' strategic competence in function sketching. What I have discovered is that strategies four and five are closely related because learners indicated that working as resources for one another helped them learn how to sketch graphs they have never seen before, such as the exponential graph in this teaching episode. Furthermore, when learners used peer assessment as part of strategy four of activating learners as resources for one another, other learners were able to do self-reflection on their work and to take initiative to correct their mistakes. On the other hand, I discovered that the fact that I was teaching learners for conceptual understanding in previous lessons was critical in the development of procedural fluency and strategic competence.

What I expected to happen did happen to some extent. Some learners were able to demonstrate their strategic competence by selecting strategies that worked for them, and Prince was able to formulate his own strategy. In the same breath, some learners were still struggling with some of the functions, and they had to do corrections through peer assessment. While this yielded positive results in helping learners, it can be time-consuming, and time is rarely available in most cases.

4. 6 REFLECTING ON THE THREE TEACHING EXPERIMENTS

Although I was able to implement the five key strategies of assessment for learning throughout the three teaching experiments, it was time-consuming. For this reason, some of the teaching episodes could not be completed during mathematics period as per school time table, and the episodes would be completed during afternoon study. However, collecting data during the covid-19 pandemic exacerbated the situation because Grade 10 learners were not coming to school on a daily basis. Hence, some of the teaching episodes were completed in two days while others over some weekends. This slowed down the teaching and learning, which impacted the rate in which learners acquired their mathematical understanding of the concepts taught guided by Kilpatrick et al.'s (2001) first three strands of mathematical proficiency: conceptual understanding, procedural fluency and strategic competence.

The analysis of the three teaching experiments focused on learners' mathematical understanding arising from all the teaching episodes. However, I explored my enactment of the five key strategies of assessment for learning as factors contributing to learners' mathematical understanding. The strategies are: (i) clarifying, understanding and sharing learning intentions; (ii) engineering effective classroom discussion, tasks and activities that elicit evidence of learning; (iii) providing feedback that moves learners forward; (iv) activating learners as resources for one another; and (v) activating learners as owners of their own learning.

(i) Clarifying, understanding and sharing learning intentions

This strategy was important in propelling me to share the learning objectives and success criteria of each teaching episode with the learners. The strategy was well enacted in all the three teaching experiments, and as I enacted it, I saw learners becoming more interested in the lesson, which intrigued their interest in the learning of mathematics. By so doing, this strategy positioned learners to be part of the lessons, which paved a way for them to grasp the conceptual understanding, procedural fluency and the strategic competence of the concepts to be taught in a particular lesson.

(ii) Engineering effective classroom discussion, tasks and activities that elicit evidence of learning

According to Black and William (2018), engineering effective questioning propels learners to participate willingly during classroom discussions. In the three teaching experiments, my observer and I created the main questions to ask learners as we made our lesson plans for a particular teaching episode to create meaningful classroom discussions. However, this did not deter us from asking extra questions during the lesson based on the learners' responses and challenges on a particular concept. The effective questioning and the tasks given to learners to elicit evidence of their learning throughout the teaching experiments encouraged them to participate willingly. One of the most important roles of effective questioning during classroom discussions was in revealing learners' misconceptions and stimulating their mathematical understanding. Through this strategy of assessment for learning, learners were able to gradually acquire the conceptual understanding, procedural fluency and strategic competence of sketching parabolic functions in the three teaching experiments.

(iii) Providing feedback that moves learners forward

In all three teaching experiments, I gave learners comment-only feedback without grades – to encourage them to work on the comments and correct their mistakes. According to Black and William (1998), when learners are given comments to work on alongside scores, they tend to ignore the comments and focus solely on the scores, which hinders learning. According to Sondergeld et al. (2010), teachers should facilitate feedback that is linked to the learning objectives in order to amplify classroom discussions and establish meaningful learning and conceptual understandings. Looking back on the three teaching experiments, this type of feedback was critical in learners gaining conceptual understanding of working with functions. They became self-regulated and attempted to correct their mistakes because of comment-only feedback, whether verbal or written. The majority of the time, learners worked on the comments with their peers. This strategy was more effective in promoting their learning and conceptual understanding.

(iv) Activating learners as resources for one another

Allowing learners to collaborate with their peers and act as resources for one another resulted in massive gains in mathematical understanding across the three teaching experiments. According to Johnson et al. (2019), activating learners as resources for one another is critical in the promotion of learning because when learners give each other feedback, they are also doing self-reflection on their own work. When learners worked in pairs or groups in the three teaching experiments, they learned the most from one another. This is supported by Reinholz (2016), who indicated that peer assessment plays a pivotal role in improving conceptual understanding and self-assessment skills. Furthermore, learners readily accepted criticisms from their peers, allowing them to develop conceptual understanding, procedural fluency and strategic competence in the sketching of functions. One of the major outcomes of using this strategy of learners acting as resources for one another is that they were able to learn new concepts together with minimal help from the teacher and, in some cases, without any help at all. Although this was evident in all of the teaching experiments, it was especially evident in teaching experiment 2 when learners learned how to sketch hyperbolic functions and were able to use their prior knowledge and examples from their Siyavula textbook to learn something new on their own.

(v) Activating learners as owners of their own learning

According to Johnson et al. (2019), allowing learners to take ownership of their learning propels them to become self-regulated learners who acquire strategies to set learning goals for themselves, self-evaluation and time management. I required learners to complete reflective journals at the end of each teaching episode in the three teaching experiments, which provided them with opportunities for self-reflection and evaluation of their own learning. They wrote in their reflective journals about what went well in the lesson, what they struggled with, and what they need to learn in the next one. This is supported by Reinholz (2016), who indicates that self-assessment provides learners with the opportunity to adjust and improve their practice in order to close the gap between where they are and where they need to go in their learning process. Furthermore, my planning was such that instead of being the primary knowledge dispenser, I allowed learners to be active participants in the lessons, allowing them to take ownership of their own learning. For example, in teaching experiment 3, learners were in complete control of the lesson, while I acted as a facilitator, allowing the majority of them to acquire strategic competence skills in sketching functions. The strategy of involving learners in their own learning improved their conceptual understanding and procedural fluency with sketching functions.

4.7 CHAPTER SUMMARY

The chapter presented the data gathered from teaching experiment one to teaching experiment three through learning activities and classroom interactions between learners and the teacher. In addition, the section presented the analysis of each teaching experiment in the previous chapter. The next chapter discusses the major findings of the study as well as the recommendations.

CHAPTER 5: CONCLUSION AND RECOMMENDATION

5.1. INTRODUCTION

The study investigated the enactment of the five key strategies of assessment for learning in my mathematics classroom to account for learners' mathematical understanding. According to the literature reviewed in this study, while the five key strategies are touted to be beneficial to teaching and learning, there is insufficient information on how mathematics teachers should enact these strategies of assessment for learning in their mathematics lessons.

Numerous studies lauded the importance of using the five key strategies of assessment for learning in classrooms and outlined some challenges, including a lack of understanding of how teachers should enact the strategies of assessment for learning in their mathematics classrooms. The studies include Enrst (2015), Chapman (2017), Andika, Sari, Ningsih, Masniladevi and Helsa (2019), Almujtahid, Hasih and Mardiyana (2018), Wylie and Lyon (2015), Johnson et al. (2019), Chapman (2017), and Rakoczy, Pinger, Hochweber, Klieme, Schütze and Besser (2019). In studies where these strategies of assessment for learning were enacted, teachers did not fully enact strategies four and five of activating learners as resources for one another and activating learners as owners of their own learning, both of which have the primary goal of inviting learners to participate.

According to Ramsey and Duffy (2016), despite the fact that there is ample evidence relating to the effectiveness of assessment for learning in supporting learners' learning, the majority of teachers do not enact the key strategies of assessment for learning in their classrooms. Johnson et al. (2019) also claimed that the research base is limited in terms of how teachers should enact assessment for learning key strategies in their classrooms. The purpose of assessment for learning is to expose learners' thinking so that teachers can work on their (learners') existing conceptualisations and incorporate these ideas into teaching and learning so that they have a thorough understanding of concepts (Heritage, 2007). Studies such as Johnson et al. (2019), Furtak et al. (2016) and Kingston and Nash (2011) concur that when teachers use the five key strategies of assessment of learning as a central part of teaching and learning, it has a significant impact on learners' understanding of learning outcomes.

The purpose of this study was to document my enactment of the five key strategies of assessment for learning in my mathematics classroom to account for learners' mathematical understanding. The answers to the following research questions were critical to achieving the purpose of the study:

- What are teaching strategies that allow for meaningful enactment of assessment for learning?
- What mathematical understanding is accounted for during the enactment of assessment for learning?

Therefore, this chapter presents the main research findings, as well as the limitations and recommendations of the study.

5.2. SUMMARY OF THE MAIN FINDINGS OF THE STUDY

I detailed the summary of the findings after a rigorous data analysis in chapter four based on what happened during data analysis in this section. The above-mentioned research questions guided the findings. The first presentation of the findings focused on teaching strategies that enabled meaningful enactment of assessment for learning. The second part of the findings described the mathematical understanding that I took into account during the enactment of assessment for learning.

5.2.1. What are teaching strategies that allow for meaningful enactment of assessment for learning?

In this section, I described teaching strategies that enabled meaningful enactment of assessment for learning in the mathematics classroom using some of the scenarios from Chapter 4 to support my findings. The teaching strategies included developing lesson plans that detailed how the five key strategies of assessment for learning would be enacted in the classroom. Second, task-focused feedback is preferred over numerical scores. Third, through peer feedback and self-assessment, learners are invited to participate and to take ownership of their own learning. Finally, at the end of each lesson, the teacher must complete reflective journals.

5.2.2. Creating lesson plans that comprise how the five key strategies of assessment of learning will be incorporated in the lesson

Figure 55 depicts one of the lesson plans I developed with my observer prior to the start of each teaching episode. The lesson plan below includes the main components of how I enacted the five key strategies of assessment for learning for a specific lesson. The purpose of the lesson plan was to assist me to be intentional about how I will enact the five key strategies of assessment for learning in my mathematics classrooms. The lesson plan allowed me to frame key questions to ask learners during the lesson in relation to the topic to be taught in order to uncover learners' prior knowledge and misconceptions, and to stimulate new understandings (William & Thompson, 2007).

Thus, including questions to ask learners in the classroom in the lesson plan prompted us to frame questions that are both intriguing and relevant to the learning objectives. According to Black and William (1988), the questions posed in classroom discussions should pique learners' interest in participating in the lesson. The learning activities for learners to write in order to elicit evidence of learning during teaching and learning were included in the lesson plan. However, I was not restricted from developing additional follow-up questions and activities during the lesson based on the learners' level of mathematical understanding and responses to the questions posed, or answers to the learning activities. Assessment for learning, according to Black and William (2018), plays a role in assisting teachers to adjust their teaching strategies in order to address learners' learning gaps.

Lesson Plan: 08-10-2020

<table>
<tr><td colspan="3">Teacher: SEDIBENG KM

TEACHING EXPERIMENT 1</td></tr>
<tr><td>Subject: Mathematics</td><td>Grade: 10</td><td>Teaching Episode 1</td></tr>
<tr><td>Lesson duration:</td><td colspan="2">30 minutes</td></tr>
<tr><td>TOPIC:</td><td colspan="2">FUNCTIONS</td></tr>
<tr><td>LEARNING OBJECTIVES AND SUCCESS CRITERIA</td><td colspan="2">Sketching a graph using a table, intercepts and the turning point. The success criteria for this episode will be met by taking learners to pages 123-124 of their Siyavula textbook to see examples of parabolic functions.</td></tr>
<tr><td>Knowledge/prior beliefs</td><td colspan="2">Sketching straight line functions</td></tr>
<tr><td>Resources</td><td colspan="2">Siyavula Grade 10 mathematics , CAPS document , Pencil and Ruler</td></tr>
</table>

<table>
<tr><td>Teacher activities</td><td>Learner activities</td></tr>
<tr><td>Sharing learning intentions and success criteria
Recapping on straight line functions through effective questioning
<u>Main questions to be asked to recap on straight line graphs</u> but not limited to follow up questions the teacher may ask based on learners' responses during the lesson
1. How does the shape of a straight-line graph look like?
2. Can someone come and sketch a rough sketch of a straight-line graph on the chalkboard?
3. What is the standard formula for a straight-line graph?
4. What are the important components when sketching a straight-line graph?
To Instruct learners to answer learning activity 1 in figure 2 not limited to follow-up activities in order to build understanding of concepts in learners.
To facilitate the lesson while learners are working in groups. Giving learners comment-only feedback, either written or verbally, which is task-focused to move them forward. Allowing learners to assess each other's work as part of peer assessment and feedback.
To fill the teacher's reflective journal at the end of the lesson (Appendix B).</td><td>Learners to page through pages 123-124 to see examples of parabolic functions.
Learners are to answer questions asked by the teacher to uncover prior knowledge on straight line and to build new understanding on sketching parabolic functions.

Learners are to answer learning activity 1 in figure 2 on parabolic functions in groups. In the activity, they have to complete the table with parabolic functions, plot the graphs on the same set of axis and deduce the effect of "a" in $y = ax^2 + q$
Learners to respond to the comments from the teacher by acting on the them and to respond to questions posed by the teacher to their peers as they strive to answer learning activity 1 in figure 2.

Learners to fill the reflective journals at the end of the lesson. (Appendix A).</td></tr>
</table>

Figure 55: Lesson plan example

5.2.3. Using task- focused feedback grading over numerical scores

During or after the lesson, I should mark learners' books with comment-only feedback. Learners should be given comment-only feedback, which means that I should write task-focused comments in their books so that they can see where they went wrong and work on finding the correct solutions. Several studies support the notion that when learners are given numerical scores with comments, they tend to focus on the scores and ignore the comments (Black & William, 1998; Chapman, 2017; Hattie & Timperley, 2007; Wylie & Lyon, 2015). Ernst (2014) contends that while comment-only feedback has numerous benefits, even for low achievers, it can be time-consuming, which I agree with.

However, in situations where there is no enough time to write comment-only feedback in each of the learners' books, to save time, I may provide comment-only feedback verbally to a group of learners, an individual learner, or the entire classroom. For instance, in the teaching experiment one teaching episode one, learners responded to learning activity 1 in figure 4 by sketching four different types of parabolic functions and deducing the effect of “a” in $y = ax^2 + q$ in groups. I only had 30 minutes for the lesson. So I gave learners verbal comments as I passed by their groups to save time. After seeing Benad's solution for learning activity 1 in figure 5, I commented, "Your table is correctly completed and your graphs are correctly sketched, what is your conclusion about the effect of a? What did you notice as you sketched?” The comment-only feedback I provided to the group moved them forward in the sense that they confirmed that they sketched the graphs correctly. From my comments, they were determined to continue working on determining the effect of “a” in the parabolic general formula of $y = ax^2 + q$. Giving feedback to learners in groups is supported by Chapman (2017), who stated that feedback can be given to an individual learner, a small group, or the entire class.

In addition, when I realised that Kholo's group solution in figure 6 was incorrect, I verbally commented on it to make them aware of their mistakes. I started by saying, “your graph of y_1 is correctly sketched but my question is what happened the graph of y_2 and y_3? They do not look like the parabolic functions.” My intention was to push them to recognise their mistakes. Learners in Kholo's group were able to respond to my comments, and they discovered that something was wrong with their graphs, which I told them to investigate further.

In addition, at the end of teaching experiment 1, I gave each learner written comment-only feedback in their books ranging from figure 15 to figure 21. However, due to time constraints, I was unable to complete the comments in each learner's book during the lesson; I completed the comments after the lesson. In the following lesson, which was in teaching experiment two, where the majority of learners developed conceptual understanding in sketching functions, learners responded to my comments and corrected their mistakes. Figures 23–25 show the sampled corrected work and group discussions. According to Hattie and Timperley (2007) and Kingston and Nash (2011), teachers should ensure that feedback is linked to learning goals in order to grow classroom discourse that establishes conceptual understandings. This was the case in this section, where I gave learners comment-only feedback, which grew classroom discussions and allowed them to correct their mistakes while developing conceptual understanding of parabolic function sketching.

5.2.4. Inviting learners' participation and taking ownership of learning through peer feedback and self-assessment

Several studies, including Johnson et al (2019), Ramsey and Duffy (2016) and Chapman (2017), contend that teachers frequently enacted some of the five key strategies of assessment for learning in their classrooms, primarily the first three strategies of clarifying, understanding and sharing learning intentions; engineering effective classroom discussion, tasks and activities that elicit evidence of learning; and providing feedback that moves learners forward. The challenge is on the last two strategies of assessment for learning where they must invite learner participation, which include strategy number four of activating learners as resources for one another and strategy number five of activating learners as owners of their own learning. The last two strategies were not routinely used by teachers in cases where they were enacted. As a result, in order to bridge the gap, I amplified the last two strategies throughout my teaching experiments.

I should allow learners to serve as resources for one another by giving them the opportunity to collaborate on mathematical activities, assess one another's work, and provide feedback to one another. In this study, I used these strategies in all of the teaching episodes. For example, in teaching experiment one, teaching episode one, when learners completed learning activity 1 in figure 4, they learnt from each other through group discussions. One of the group conversations I sampled was Benad's, where learners were able to help each other through peer feedback and assessment. Even in other groups, learners were highly receptive to peer criticisms, and willingly corrected their mistakes without being offended. This outcome is supported by Black and William (1998).

Peer feedback elevates learners' confidence in their abilities, and pique their interest in the subject (Rakoczy, Pinger, Hochweber, Klieme, Schütze & Besser, 2019). The amount of interest that learners show when they are used as resources for one another is enormous, as evidenced by my most recent teaching experiment - teaching experiment three. The teaching episode was built around learners being active participants in the lesson, with me serving as the facilitator. Learners demonstrated significant improvement in their learning and participation. Using peer feedback in this way is very effective because peer assessment is also beneficial to learners who are assessing or providing feedback to their peers because they do self-reflection on their own work. When Makgabo was giving the class feedback on her work in teaching experiment three, she was able to self-reflect and identify her mistakes while presenting, which helped her write her final solution in figure 44 correctly.

As a mathematics teacher, it is difficult to teach when learners are not self-reflecting and taking ownership of their own learning. In this study, requiring learners to complete reflective journals at the end of each lesson encouraged them to take responsibility for their own learning. According to Chapman (2017), teachers can use various self-assessment practices such as journals, reflections, traffic light cards, and self-evaluation to activate learners to be owners of their own learning. However, in this study, I used reflective journals kept by learners (Appendix A), which proved to be effective.

Reflective journals encouraged learners to be aware of their own weaknesses and strengths, as well as how they learn and plan their next steps. The completion of learners' reflective journals aided me in determining what learners were struggling with. It also piqued my learners' interest in mathematics, and they were always looking forward to the next lesson, fostering a positive relationship between my learners and me. Furthermore, I observed that while recording my teaching experiments, learners became active participants in their mathematical learning, and mathematical understanding developed organically. This strategy of activating learners to be owners of their own learning encouraged them to collaborate, as the majority of them stated in their reflective journals that they learnt best when working with their peers.

In the classroom, peer-assessment and self-assessment practices complement each other. Learners were able to develop mathematical understanding through these two strategies, which in this study meant conceptual understanding, procedural fluency and strategic competence of the concepts taught. What learners wrote in their reflective journals at the end of each lesson informed me on what to include in the next lesson to address their learning gaps. Furthermore, as learners provided verbal or written feedback to one another through mathematical activities, I was able to determine their weaknesses and strengths, which helped me plan my lessons in relation to their learning needs.

5.2.5. Completing teacher's reflective journals at the end of each lesson

It is critical that I complete my reflective journal (Appendix B) at the end of each lesson. The journal allowed me to explain what went wrong and why it happened, which helped me improve my practice. After each teaching episode, I would watch the recording again to take note of anything I might have missed during the class. On days when my observer could not be physically present in the classroom, we would watch the video together and he would give me his objective perspective, which helped me plan for the next teaching episode. Completing teacher reflective journals assisted me to improve my practice while also developing mathematical understanding in my learners.

One of the numerous decisions I made while doing self-reflection was that at the end of teaching episode one in teaching experiment one, my observer indicated that I was not observing proper wait time during questioning as required by William and Thompson's (2007) theory of the five key strategies of assessment for learning. Based on his observations and my viewing of the video after the lesson, I realised that I was too quick to give learners answers without giving them enough time to respond. From then on, I gave learners more time to respond before jumping in to answer for them.

Second, as I was doing self-reflections at the end of teaching episode two in teaching experiment one, I realised that I was not teaching the learners for conceptual understanding, but rather for procedural fluency, which hampered their understanding of parabolic functions. Based on the learners' reflective journals and written work, it was determined that they lacked knowledge of working with square roots, which formed the foundation of sketching parabolic functions. I then decided to deviate slightly from the topic and planned a lesson with the learners to review how to calculate square roots with the goal of developing conceptual understanding of parabolic functions.

5.2.6. What mathematical understanding is accounted for during the enactment of assessment for learning?

The mathematical understanding that I accounted for in this study was based on Kilpatrick et al.'s (2001) first three strands of mathematics proficiencies, namely; conceptual understanding, procedural fluency and strategic competence. Reflecting on my interactions with learners in the classroom, I noticed that the majority of them had a better conceptual understanding and procedural fluency with mathematics concepts. However, only a few of them demonstrated strategic competence. I have included some figures and classroom interactions below to back up my observations of Kilpatrick's (2001) mathematical understanding.

5.2.7. Conceptual understanding

One of the mathematical understandings was conceptual understanding. I accounted for it in teaching experiment two. But when I first started with the teaching experiments, learners lacked conceptual understanding of sketching parabolic functions because they could not relate the concepts and their connections. In teaching experiment one, episode one, learners made a lot of mistakes when working with negative numbers. For example, when Monica substituted -2 into $y_3 = x^2$, she substituted it like this: $y_3 = -2^2$ and got the answer -4 instead of 4. This showed her lack of conceptual understanding in working with negative numbers in equations. The majority of learners made a mistake similar to that of Monica. However, through peer feedback, other learners who understood how to work with negative numbers helped those who were struggling, which aided in their conceptual understanding of working with negative numbers in equations.

According to a study conducted by Bahr and Bossé (2008), learners who memorise procedures or methods without proper understanding are often unsure when or how to use the method. When learners were answering learning activity 2 in figure 11 (where they had to sketch the parabolic functions $y = -x^2 + 4$ and $y = \frac{1}{2}x^2 + 4$.) in teaching experiment one episode two, they did well in the sketching of the graph 1 of sketching $y = -x^2 + 4$, but they were not able to sketch the graph of $y = \frac{1}{2}x^2 + 4$. This indicated to me that they lacked conceptual understanding of all the concepts required in the sketching of parabolic functions, such as working with square roots, because only one learner managed to sketch the correct shape of the graph despite using a different method.

Their challenges were also evident as they completed their reflective journals (Appendix A), which are part of strategy number five of empowering learners to be owners of their own learning. Learners stated that they were unable to work with negative numbers and that they had difficulty sketching parabolic curves with undefined x values. In order to develop conceptual understanding in the learners, I modified my teaching strategies in accordance with assessment for learning in order to remedy and bridge the learning gaps that hindered their sketching of parabolic functions. After addressing the learning gaps, the learners developed conceptual understanding of mathematics in graph sketching.

A conversation between Monica, Adelaide and Benad in teaching episode one of teaching experiment two when they were doing the corrections of sketching the graph of $y = \frac{1}{2}x^2 + 4$ indicated learners' conceptual understanding. In their conversation, they started to make the relations and connections between concepts, and they were able to correct their mistakes when sketching the graph of $y = \frac{1}{2}x^2 + 4$. Moreover, learners were also able to respond to the comments I made in their books based on their wrong sketches of $y = \frac{1}{2}x^2 + 4$, which helped them to sketch the graph correctly in figure 23 as part of their corrections.

According to Kilpatrick et al. (2001), learners with conceptual understanding of mathematics understand mathematical concepts, operations and ideas, as well as the relationships between different concepts. Those who have conceptual understanding understand why a particular idea in mathematics is important and the different contexts in which it is used. As learners corrected the graph in teaching experiment number two, teaching episode number two $y = \frac{1}{2}x^2 + 4$, all of the groups were able to correct their mistakes. In some cases, such as Kholo's group, I gave them additional questions with undefined values of x, the graph of $y = x^2 + 4$, and the group managed to draw the graph correctly.

Learners demonstrated conceptual understanding in the sketching of parabolic functions because they were able to answer various questions even when the value of x was undefined. They comprehended why a particular mathematical concept is important and the various contexts in which it is used. According to the findings of a study conducted by Andamon, Abao and Tan (2018), conceptual understanding was one of the components identified in ensuring performance in the field of mathematics. According to the Department of Education (2018), learners with conceptual understanding are quick to remember and apply a method they have learnt because they acquired facts and methods with understanding, and can easily recreate if forgotten. Furthermore, “a student having conceptual understanding of mathematics knows more than isolated facts and methods” (Mwakapenda, 2004, p.28).

5.2.8. Procedural fluency

According to Kilpatrick et al. (2001), procedural fluency allows learners to learn procedures that can strengthen and build mathematical understanding. I gave learners the opportunity to develop procedural fluency in carrying out mathematical procedures by enacting the five key strategies of assessment for learning. In teaching experiment two, near the end of teaching episode one, learners began to demonstrate some procedural fluency because they had developed conceptual understanding in the sketching of parabolic functions by that point. This was evident in Kholo and Prince's groups, as both groups correctly sketched extra work on parabolic functions with undefined values of x.

According to NCTM (2000), even though conceptual understanding and procedural fluency compete in the classroom, the two strands are interwoven. Procedural fluency is at its peak when learners can use mathematical procedures accurately, flexibly and efficiently. Teaching episode two of teaching experiment two demonstrated a high level of procedural fluency. In this teaching episode, learners were given five hyperbolic functions that they were seeing for the first time, and they were able to correctly sketch all of the hyperbolic functions. In Figures 31 through 34, I sampled some of the solutions learners wrote in their books, and the majority of them were able to correctly sketch the graphs using prior knowledge and examples from their Siyavula Grade 10 textbook of sketching hyperbolic functions. Learners correctly sketched all five hyperbolic functions with no mistakes. From the completion of the tables to the sketching of the graphs, learners collaborated as resources for one another to successfully complete the learning activity.

The accuracy with which learners sketched the hyperbolic functions is supported by the Department of Education (2018), which states that when learners face difficulties in understanding mathematical concepts, they will be unable to develop sufficient procedural fluency. This was evident in teaching experiment one teaching episode one, where the majority of learners, such as Kholo's group in figure 6, were unable to correctly sketch the parabolic functions because they had not yet acquired conceptual understanding, and thus could not demonstrate fluency in their procedures. When learners have a conceptual understanding of certain concepts, procedural fluency comes naturally to them, whereas when learners perform procedures without conceptual understanding, they make many mistakes.

5.2.9. Strategic competence

A learner's mathematical understanding can be proved by demonstrating strategic competence skills in solving mathematical problems and activities. According to Kilpatrick et al. (2001), learners should be able to make rational decisions about which strategies to use or to devise their own strategies for working out mathematical activities and problems. When I marked Adelaide's work for sketching the graph of $y =$

$\frac{1}{2}x^2 + 4$ in this study, a bit of strategic competence became apparent in learning activity 2 of teaching experiment one, teaching episode two - the solution for Adelaide is in figure 17. The learning activity in Figure 11 required learners to sketch the parabolic function by determining the intercepts and turning point. However, when looking at figure 17 with Adelaide's drawing of the graph of $y = \frac{1}{2}x^2 + 4$, her sketch was correct even though she used a different strategy to sketch the graph. This demonstrated some strategic competence in her because she was able to solve mathematical activities using her own strategy.

According to Kilpatrick et al. (2001), there is a supportive relationship between these three strands of mathematical proficiency: strategic competence, procedural fluency and conceptual understanding. This was evident in teaching experiment three, where the majority of learners had mastered their conceptual understanding and procedural fluency in sketching graphs in previous lessons. Some learners began to demonstrate strategic competence in teaching experiment three learning activity 1, in which they worked in groups using various methods to sketch various types of functions in figure 39. The enactment of the five key strategies of assessment for learning in mathematics classrooms was also critical in amplifying learners' development of high levels of strategic competence.

In teaching experiment number three, I used all five key strategies of assessment of assessment for learning, with particular emphasis on strategies four and five. Because I was acting as a facilitator in this episode, and learners were in control of the lesson through peer feedback and self-assessment, the two strategies encouraged learners to take ownership of their own learning. That being said, learners became overly involved in their learning and became aware of the learning strategies that worked best for them, which they used in sketching the functions in Figure 39, demonstrating evidence of strategic competence.

Learners used various strategies to sketch the functions in figure 39, beginning with the first one, a straight-line graph sketch of $f(x) = x + 1$. Kholo used the table method and managed to sketch the correct graph in figure 41. Makgabo used the intercepts method and managed to sketch the correct graph in figure 44, despite making some mistakes while presenting her solution through peer feedback. Learners such as

Prince went so far as to devise strategies that were never used in class; he correctly sketched the graph of $f(x) = x + 1$ using the calculator method in figure 46. Prince presented his solution to the rest of the class, and in the conversation between him and the class after sketching his solution in figure 46, he explained how he used a calculator to sketch the graph.

As learners demonstrated some strategic competence skills, I observed that their procedural fluency was also enhanced. According to Kilpatrick et al. (2001), the strands of mathematical proficiency are interconnected, and that in strategic competence, as learners choose the strategies that work best for them, they develop procedural fluency. I have also realised that in mathematics, the use of strategy number four, activating learners as resources for one another; and strategy number five, activating learners to take ownership of their own learning, play an important role in fostering learners' mathematical understanding. This study took into account conceptual understanding, procedural fluency and strategic competence as mathematical understanding.

5.3. LIMITATION OF THE STUDY

The study has some limitations that should not be overlooked when interpreting the results. The study sampled a small number of participants for reporting purposes using the teaching experiment research design. As a result, the findings of the study may not be generalisable to a larger scale. Furthermore, the data was collected from a single school in a Grade 10 mathematics classroom, which may have influenced the findings. If the study had been conducted in more than one school, it is likely that the research would have come up with more teaching strategies on how to enact the key strategies of assessment for learning to account for learners' mathematical understanding.

5.4. RECOMMENDATIONS

Without disregarding the limitations, the study recommends that proper planning is required before teachers can enact the five key strategies of assessment for learning in their mathematics classrooms, and that creating a learning environment that will foster the enactment of the key strategies of assessment for learning in mathematics classrooms is of paramount importance.

Based on the findings of the study, I have made the following recommendations:

- Prior to their lessons, teachers should create lesson plans outlining how they will enact the key strategies of assessment for learning.
- Teachers should plan and create a conducive learning environment in order to use strategies 4 and 5 more frequently in mathematics classrooms. Since the two strategies encourage maximum participation and stimulate learners to take ownership of their own learning, they promote mathematical understanding among learners.
- One of the most important strategies for developing learner mathematical understanding is to provide task-focused comment-only feedback. Despite the fact that it can be time-consuming, teachers may occasionally provide learners with comment-only verbal feedback in mathematics classrooms to save time.
- Teachers should keep reflective journals at the end of each lesson to self-reflect and continue to improve their practice in enacting the five key strategies of assessment for learning.
- The department of education should take the enactment of the five key strategies of assessment for learning in mathematics classrooms seriously, and develop proper guidelines for teachers to follow, as well as monitor them in the same way that formal assessment tasks are.
- More research should be done on the enactment of the five key strategies of assessment for learning in mathematics classrooms.

5.5. CHAPTER SUMMARY

In the preceding chapter, I outlined the overall findings of the study in relation to the research question that motivated me to conduct this study. In the midst of summarising the findings, I also mentioned some of the difficulties I encountered while enacting the five key strategies of assessment for learning in my mathematics classrooms. The difficult nature of enacting the key strategies of assessment of assessment for learning for effective learning was one of the challenges. Although they are beneficial to mathematics teaching and learning, they can be time-consuming. The secondary school timetable does not provide enough time for teachers to fully enact the key strategies of assessment for learning; the time allotted to mathematics is limited to us completing the curriculum; otherwise, one will fall behind and never complete the curriculum for a particular term or year.

In order to complete this study to the best of my ability, I had to stay with the learners during afternoon studies and some weekends to complete my teaching episodes and enact all the five key strategies optimally to fulfil the purpose of my study. The major findings revealed that strategies four and five, activating learners as resources for one another and activating learners as owners of their own learning, are critical in developing learners' conceptual understanding, procedural fluency and strategic competence in mathematical concepts. I have learnt that making lesson plans that include how the five key strategies of assessment for learning will be enacted during the lesson, as well as creating a learning environment that amplifies strategies four and five of assessment for learning, may aid in the enactment of the key strategies of assessment for learning in mathematics classrooms. In addition, I addressed the limitations of the study and provided recommendations for my enactment of the five key strategies of assessment of learning in the mathematics classroom.

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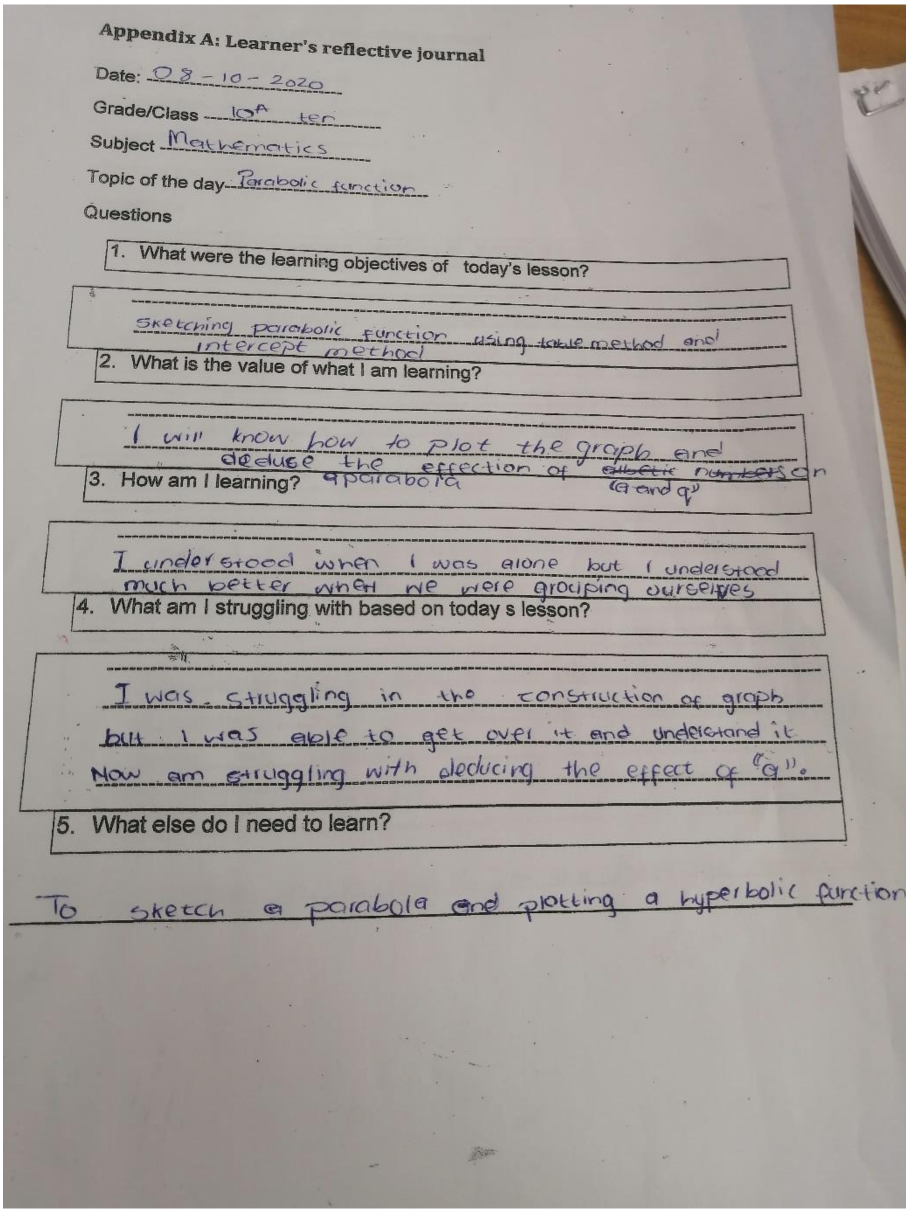
APPENDICES

Appendix A: Learners' Reflective Journals



Appendix A: Learner's reflective journal

Date: 08-10-2020

Grade/Class 10A ten

Subject Mathematics

Topic of the day Parabolic function

Questions

1. What were the learning objectives of today's lesson?

Sketching parabolic function using ~~table~~ method and intercept method

2. What is the value of what I am learning?

I will know how to plot the graph and deduse the effection of ~~albetic numbers~~ on a parabola ("a and q")

3. How am I learning?

I understood when I was alone but I understood much better when we were grouping ourselves

4. What am I struggling with based on today s lesson?

I was struggling in the construction of graph but I was able to get over it and understand it Now am struggling with deducing the effect of "a".

5. What else do I need to learn?

To sketch a parabola and plotting a hyperbolic function

Appendix A: Learner's reflective journal

Date: 08-10-2020

Grade/Class 10A

Subject Mathematics

Topic of the day Parabolic function

Questions

1. What were the learning objectives of today's lesson?

Stetching parabolic function using table method and intercept method.

2. What is the value of what I am learning?

To understand how to solve parabolic functions and how to plot graphs.

3. How am I learning?

When I don't understand anything I ask someone who understands, and that is how I learn.

4. What am I struggling with based on today s lesson?

I'm struggling on how to complete the table based on parabolic functions but someone managed to make me understand.

5. What else do I need to learn?

The effects of a and q on the Parabolic functions.

Appendix A: Learner's reflective journal

Date: 08-10-2020

Grade/Class 10A

Subject mathematics

Topic of the day Parabolic functions.

Questions

1. What were the learning objectives of today's lesson?

Sketching parabolic function using table method and intercept method.

2. What is the value of what I am learning?

To have knowledge and to be successful in life.

3. How am I learning?

I am learning when I'm paired in group and when I am alone.

4. What am I struggling with based on today s lesson?

I was struggling to substitude values into the equation but my group members assisted me.

5. What else do I need to learn?

The effects of a and q on the parabolic functions

Appendix A: Learner's reflective journal

Date: 08-10-2023

Grade/Class 10A

Subject MATHEMATICS

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Sketching a parabolic function (using the table)

2. What is the value of what I am learning?

Is to understand how to Plot the intercepts in the graph

3. How am I learning?

I am learning well

4. What am I struggling with based on today s lesson?

Noting as far

5. What else do I need to learn?

To calculate the all intercepts including turning Point

Appendix A: Learner's reflective journal

Date: 08 October 2020

Grade/Class ten^A (10^A)

Subject Mathematics

Topic of the day: Sketching parabolic function using table method and intercept method.

Questions

1. What were the learning objectives of today's lesson?

Sketching parabolic function using table method and intercept method

2. What is the value of what I am learning?

To have bright future and to be dependent woman

3. How am I learning?

I am listen and understand what the teacher is doing.

4. What am I struggling with based on today s lesson?

I was struggling ~~with~~ on brackets but now i learn how to multiplying with brackets.

5. What else do I need to learn?

finance and growth

Appendix A: Learner's reflective journal

Date: 10-10-2020

Grade/Class 10A

Subject Mathematics

Topic of the day- Sketching a parabola/Function

Questions

1. What were the learning objectives of today's lesson?

Sketching a parabolic Function using x intercept, y-intercept, Turning point

2. What is the value of what I am learning?

To know how to sketch a parabolic Function

3. How am I learning?

I'm learning good with everyone even a teacher

4. What am I struggling with based on today s lesson?

I'm struggling with sketching Function using x-intercept and turning point

5. What else do I need to learn?

To learn how to calculate x-intercepts in any parabolic Function

26

Appendix A: Learner's reflective journal

Date: 10/10/2020

Grade/Class 10A

Subject Mathematics

Topic of the day Function

Questions

1. What were the learning objectives of today's lesson?

Sketching a graph using x intercept, y-intercept and Turning point

2. What is the value of what I am learning?

To sketch a parabola using intercepts

3. How am I learning?

Using the table method

4. What am I struggling with based on today s lesson?

I drew my graph correctly but however i need to know how to ~~find intercept~~ sketch a parabola using intercepts

5. What else do I need to learn?

To be able to sketch a parabola using intercepts

26

Appendix A: Learner's reflective journal

Date: 13/10/2020

Grade/Class 10A

Subject Mathematics

Topic of the day Functions and surds

Questions

1. What were the learning objectives of today's lesson?

Corrections on the graph of $y=\frac{1}{x}$ and revising square roots

2. What is the value of what I am learning?

How to sketch any graph even when x is undefined

3. How am I learning?

Comments from the teaches in the book plus the revision

4. What am I struggling with based on today s lesson?

Nothing, we even gave our selfs extra work ($y=x^2+9$) and we got it correct

5. What else do I need to learn?

Finding the intercept of any parabola graph

26

Appendix A: Learner's reflective journal

Date: 13/10/2020

Grade/Class 10A

Subject Mathematics

Topic of the day-Functions and Surds

Questions

1. What were the learning objectives of today's lesson?

Corrections on the graph of $y = \frac{1}{2}$ and revising square roots

2. What is the value of what I am learning?

How to sketch a parabolic function even when x is undefined

3. How am I learning?

comments from teacher in the book plus revision

4. What am I struggling with based on today s lesson?

nothing, The teacher gave us the extra questions of $y = x^2 + 4$

5. What else do I need to learn?

Finding intercept any parabola graph

26

Appendix A: Learner's reflective journal

Date: 29 October 2020

Grade/Class: 10A

Subject: Mathematics

Topic of the day: Functions

Questions

1. What were the learning objectives of today's lesson?

Sketching graphs using the methods that we understand better

2. What is the value of what I am learning?

To be able to understand how to plot graphs

3. How am I learning?

Using calculator method; i even got the exponential graph that i was seeing for the first time correct

4. What am I struggling with based on today s lesson?

I am struggling on how to plot a hyperbola using the table methods

5. What else do I need to learn?

To be able to plot graphs using any method even the hyperbolic functions

26

Appendix A: Learner's reflective journal

Date: 29-10-2020

Grade/Class 10A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Sketching parabola, straight line, hyperbola and exponential Function

2. What is the value of what I am learning?

To understand how to plot graphs

3. How am I learning?

I was learning using table method and also I did exponential function and It was the First time doing it but I got it right

4. What am I struggling with based on today s lesson?

I'm struggling to plot the graph using calculator, and intercepts

5. What else do I need to learn?

To plot graph using table and exponential Functions

26

Appendix A: Learner's reflective journal

Date: 14/10/2020

Grade/Class 10 A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Drawing a hyperbolic using a table and finding y and x intercept in a hyperbolic function

2. What is the value of what I am learning?

To find x-intercepts and y-intercepts

3. How am I learning?

Working in pairs

4. What am I struggling with based on today s lesson?

Today i was not struggling with anything

5. What else do I need to learn?

Sketching a hyperbolic using x-intercept and y-intercept.

26

Appendix A: Learner's reflective journal

Date: 14-10-2020

Grade/Class 10A

Subject maths

Topic of the day-- Function

Questions

1. What were the learning objectives of today's lesson?

Drawing a hyperbolic using table Find x and y-intercepts in a hyperbolic Function

2. What is the value of what I am learning?

To Find x and y-intercepts

3. How am I learning?

working in pairs

4. What am I struggling with based on today s lesson?

I'm strugging with nothing

5. What else do I need to learn?

sketching hyperbolic using x and y intercept

26

Appendix A: Learner's reflective journal

Date: 29/10/2020

Grade/Class 10A

Subject Mathematic

Topic of the day function

Questions

1. What were the learning objectives of today's lesson?

Sketching Parabola Straight line, hyperbola and exponential function

2. What is the value of what I am learning?

y and x intercept and also ploting graph

3. How am I learning?

By my teacher's strategy

4. What am I struggling with based on today s lesson?

Nothing, because I know how to use x and y intercept

5. What else do I need to learn?

more about hyperbola function

26

Appendix A: Learner's reflective journal

Date: 29-10-2020

Grade/Class 10A

Subject Mathematics

Topic of the day Function

Questions

1. What were the learning objectives of today's lesson?

Sketching graphs using the methods that we understand better.

2. What is the value of what I am learning?

To know how to plot a graph and understand it.

3. How am I learning?

using a table method and I got my exponetial graph correct.

4. What am I struggling with based on today s lesson?

I'm struggling to plot a graph, using calculator.

5. What else do I need to learn?

I need to know how to plot a graph using calculat

Appendix A: Learner's reflective journal

Date: 29 October 2021

Grade/Class 10A

Subject MATHEMATICS

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Sketching graphs using the methods that we understand better.

2. What is the value of what I am learning?

To know how to plot graphs.

3. How am I learning?

peer leaning

4. What am I struggling with based on today s lesson?

im struggling with sketching using intercept

5. What else do I need to learn?

To be able to plot graphs using method even the hypobolic functions.

26

Appendix A: Learner's reflective journal

Date: 29-10-2020

Grade/Class 10^A

Subject Mathematics

Topic of the day Function

Questions

1. What were the learning objectives of today's lesson?

Sketching parabola, straight line, hyperbola and exponential function.

2. What is the value of what I am learning?

To understand how to sketch a graph.

3. How am I learning?

When a teacher is teaching.

4. What am I struggling with based on today s lesson?

I'm struggling to sketch $f(x)=3^x$.

5. What else do I need to learn?

To plot/sketch graph using table and exponential functions.

26

Appendix A: Learner's reflective journal

Date: 14-10-2021

Grade/Class 10A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Drawing a hyperbolic in a hyperbolic functions.

2. What is the value of what I am learning?

Finding x-intercept and y-intercept

3. How am I learning?

Peer learning

4. What am I struggling with based on today s lesson?

Im not struggling with anything today

5. What else do I need to learn?

Sketching hyperbolic using x and y intercept.

26

Appendix A: Learner's reflective journal

Date: 14-10-2020

Grade/Class 10A

Subject Maths

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Drawing a hyperbolic using table and find x and y-intercepts

2. What is the value of what I am learning?

To find x and y-intercept

3. How am I learning?

working with my peer's.

4. What am I struggling with based on today s lesson?

I'm struggiling to plot the graph of hyperbolic

5. What else do I need to learn?

-Need to know how to plot hyperbolic graph

26

Appendix A: Learner's reflective journal

Date: 14/10/2020

Grade/Class 10 A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Drawing a hyperbolic using a table and finding y and x intercept in a hyperbolic function

2. What is the value of what I am learning?

To find x-intercepts and y-intercepts

3. How am I learning?

Working in pairs

4. What am I struggling with based on today s lesson?

Today i was not struggling with anything

5. What else do I need to learn?

Sketching a hyperbolic using x-intercept and y-intercept.

26

Appendix A: Learner's reflective journal

Date: 14-10-2020

Grade/Class 10A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?
Finding x and y intercept of

2. What is the value of what I am learning?
To find x-intercept and y-intercept.

3. How am I learning?
When a teacher is teaching.

4. What am I struggling with based on today s lesson?
As for today everything went well in ~~this~~ today's lesson.

5. What else do I need to learn?

Sketching a hyperbolic using x-intercept and y-intercept.

26

Appendix A: Learner's reflective journal

Date: 14/10/2020

Grade/Class 10A

Subject Mathematic

Topic of the day- function

Questions

1. What were the learning objectives of today's lesson?

Drawing a hyperbolic using a table and finding y and x intercept

2. What is the value of what I am learning?

To find x-intercept and y Intercept

3. How am I learning?

By working with my peer

4. What am I struggling with based on today s lesson?

I am struggling to plot a graph of hyperbolic

5. What else do I need to learn?

More about hyperbolic function.

26

Appendix A: Learner's reflective journal

Date: 13 - 10 - 2020

Grade/Class 10A

Subject Maths

Topic of the day Function and Surds

Questions

1. What were the learning objectives of today's lesson?

- revising from the corrections for the graph of $y \frac{1}{2}x^2 + a$
- Finding the intercepts

2. What is the value of what I am learning?

To know how to sketch the graph with undefined values of x

3. How am I learning?

Working with pairs.

4. What am I struggling with based on today s lesson?

Nothing soo far.
- I know how to find x-intercept in any parabolic function.

5. What else do I need to learn?

Sketching hyperbolic function.

26

Appendix A: Learner's reflective journal

Date: 13-10-2020

Grade/Class 10A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Revising square roots.

2. What is the value of what I am learning?

Being able to sketch the graph with undefined values of x.

3. How am I learning?

When a teacher is teaching.

4. What am I struggling with based on today s lesson?

Nothing, I understand how to find x-intercept in any parabolic function.

5. What else do I need to learn?

I need to learn how to draw a hyperbolic function.

26

Appendix A: Learner's reflective journal

Date: 13/10/2020

Grade/Class 10A

Subject Mathematatic

Topic of the day function

Questions

1. What were the learning objectives of today's lesson?

Sketching a parabolic function

2. What is the value of what I am learning?

Being able to sketch the graph with undefined value of x

3. How am I learning?

By reading comment from my teaching and working with my Peers

4. What am I struggling with based on today's lesson?

nothing, I understand how to find x-intercept in any parabolic function

5. What else do I need to learn?

Drawing hyperbolic function

26

Appendix A: Learner's reflective journal

Date: 13 October 2021

Grade/Class 10A

Subject Mathematics

Topic of the day Functions and surds

Questions

1. What were the learning objectives of today's lesson?

To calculate functions and surds

2. What is the value of what I am learning?

Being to sketch the undefined intercept.

3. How am I learning?

Peer learning

4. What am I struggling with based on today s lesson?

Nothing, I understand how to find x-intercept in any parabolic function

5. What else do I need to learn?

drawing hypabolic ~~Functi~~ function.

26

Appendix A: Learner's reflective journal

Date: 10/10/2020

Grade/Class 10A

Subject Mathematic

Topic of the day function

Questions

1. What were the learning objectives of today's lesson?

Stretching a parabolic function using x intercept, y-intercept and turning point

2. What is the value of what I am learning?

To find x-intercep and y intercept

3. How am I learning?

By peer learning

4. What am I struggling with based on today s lesson?

I am struggling to draw $y = \frac{1}{2}x^2 + 4$

5. What else do I need to learn?

To find x- intercept in all types of parabolic function

26

Appendix A: Learner's reflective journal

Date: 10-10-2018

Grade/Class 10A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Sketching a parabolic function using x-intercept y-intercept and turning point.

2. What is the value of what I am learning?

To find x-intercept and y-intercept.

3. How am I learning?

When the teacher is teaching.

4. What am I struggling with based on today s lesson?

I am failing to sketch $y = \frac{1}{2}x^2 + 4$

5. What else do I need to learn?

To find x-intercept in all types of parabolic function.

26

Appendix A: Learner's reflective journal

Date: 10-10-2020

Grade/Class 10A

Subject Maths

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

Sketching a parabolic function using x-intercept and y-intercept and turning point

2. What is the value of what I am learning?

To find x-intercept and y-intercept and find the turning point.

3. How am I learning?

I'm doing good when the teacher explain to us.

4. What am I struggling with based on today s lesson?

~~I'm strug~~

Sketching the graph of $y = \frac{1}{2}x^2 + 4$.

5. What else do I need to learn?

- I need to learn more about x-intercept and y-intercept.

I need to know how to find the turning point, in all types of parabolic function.

26

Appendix A: Learner's reflective journal

Date: 10/10/2021

Grade/Class 10A

Subject Mathematics

Topic of the day Functions

Questions

1. What were the learning objectives of today's lesson?

sketching a parabolic function using intercept Turning point

2. What is the value of what I am learning?

To find the value of x-intercept and y-intercept

3. How am I learning?

peer learning

4. What am I struggling with based on today s lesson?

The calculate of x-intercept

5. What else do I need to learn?

calculate the point of x-intercept in type of all parabolic fuction

26

Appendix A: Learner's reflective journal

Date: 08/10/2020

Grade/Class 10A

Subject Mathematics

Topic of the day Function

Questions

1. What were the learning objectives of today's lesson?

Sketching a parabolic function using x-intercept y-intercept and a turning point.

2. What is the value of what I am learning?

To know how to sketch parabola.

3. How am I learning?

When the teacher is teaching.

4. What am I struggling with based on today s lesson?

For today's ~~learning~~ lesson I managed to ~~write~~ sketch a the parabolic graph with an understanding. I didn't struggle with anything.

5. What else do I need to learn?

I need to learn more steps on how to sketch a parabolic function.

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Appendix B: Teacher's Reflective Journal

Date: 10 – 10 – 2020

Grade/Class: 10A

Subject: MATHEMATICS

Topic of the day: Functions

Questions

1. What did I expect would happen?

I expected that learners will be able to sketch parabolic functions using intercepts and Turning point without some major challanges.

2. What actually happened?

Learners could not draw the graph of $y = \frac{1}{2}x^2 + 4$, which had undefined values of x however they managed to draw $y = x^2 - 4$ correctly

3. Why did this happen?

I noticed that learners lacked conceptual understanding of working with square roots and basic algebra.

4. What is the next step?

My observer and I, decided that in the next teaching experiment we should start by revising with learners how to calculate square roots in order for them to learn how to sketch the parabolic function conceptually.

27

Date: 13-10-2020

Grade/Class 10A

Subject MATHEMATICS

Topic of the day Functions and revision on square roots

Questions

1. What did I expect would happen?

That learners will be able to recall how to calculate square roots which will in turn help them to acquire conceptual understanding of parabolic f

2. What actually happened?

Learners did understand how to calculate square roots and they were able to respond to the comments I made in their class work bo

3. Why did this happen?

In this episode I involved peer-feedback to a greater level. Self-reflection by reading the comments I made in their books impacted their understanding positively. Finally learners understood how t

4. What is the next step? Sketch Functions Concepts

Moving on to the next type of functions which are hyperbolic functions. The plan is to involve learners in the lesson optimally by the use of peer-feedback and assessment.

27

Date: 14-10-2020

Grade/Class 10A

Subject MATHEMATICS

Topic of the day FUNCTIONS

Questions

1. What did I expect would happen?

That learnas will be able to plot hyperbolic function using the table and determining the values of x and y - intercept.

2. What actually happened?

Learnas worked very well with their peers. By this time they were already acquired conceptual understanding in the sketching of parabolic func which helped them to use what they know to sketch hyper

3. Why did this happen?

When learnas have acquired conceptual understanding of certain concepts it becomes easy for them apply the knowledge in different context. Their procedural fluency in sketching of functions

4. What is the next step?

To use the key strategies of assessment for learning to foster strategic competence of sketching graphs among learners. Strategy four and five will be emphasised

27

Date: 29-10-2020

Grade/Class 10A

Subject MATHEMATICS

Topic of the day Functions

Questions

1. What did I expect would happen?

Learners will be able to sketch all the four function correctly and with ease. And get the time to do attempt the exponential graph which they n[illegible]

2. What actually happened?

Learners were able to sketch the graphs but not a[illegible] quicker as I expected. The new graph -expon[illegible] others managed to sketch but others failed to. But with the help of their peers they all managed to fix mistak[illegible]

3. Why did this happen?

Some of the learners haven't yet developed the strategic co[illegible] of sketching functions. The other problem is th[illegible] issue of time they couldn't sketch and correct all the functions in one period but peer-assessment work[illegible]

4. What is the next step?

To include the five key strategies in my lessons particularly strategy 4 and 5. Teach for conceptual understanding to help learners make the relations between conce[illegible]

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Appendix C: Approval from Limpopo Department of Education

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

CONFIDENTIAL

Ref: 2/2/2 Enq: Mabogo MG Tel No: 015 290 9365 E-mail:MabogoMG@edu.limpopo.gov.za

Sedibeng KM
P O Box 713
Dimpe Secondary School
0555

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.

2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"THE ENACTMENT OF ASSESSMENT FOR LEARNING TO ACCOUNT FOR LEARNERS MATHEMATICS UNDERSTANDING "**

3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

REQUEST FOR PERMISSION TO CONDUCT RESEARCH:SEDIBENG KM

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The heartland of southern Africa - development is about people!

Appendix D: Informed Consent for Principal

INFORMED CONSENT FORM for principal

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

Dear Principal

I request that you afford me a permission to collect data in Grade 10 mathematics classroom by video recording all the teaching and learning activities during my classroom interaction with the learners. I assure that the following conditions will be met:

1. Your real name will not be used at any point of information collection, or in the final writing up of the data.
2. Your participation in this research is voluntary. You have the right to withdraw at any point of the study, for any reason, and without any prejudice, and the information collected and records and reports written will be turned over to you.
3. You will receive a copy of the final report before it is handed in, so that you have the opportunity to suggest changes to the researcher, if necessary.
4. You were not forced into taking part, nor promised any form of incentive. This was a voluntary participation.
5. The collected information will be used for research purposes only.
6. I, the principal of Dimpe secondary school KGATLA M.V (print) grant permission to the researcher to conduct the study in the school. I understand and agree to the aforementioned terms and conditions.

Principal (signature) [signature] Date 21/09/2020

Researcher [signature] Date 21-09-2020

DIMPE SECONDARY SCHOOL
21 SEP 2020
STRIVE FOR EXCELLENCE

Appendix E: Informed Consent for parents of participants

PARENTAL ASSENT FORM

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

GO MOTSWADI WA NGWANA

KA BOIKOKOBETSO BJO BOGOLO, KE KGOPELA TUMELOLO YA GO GATISA NGWANA WA LENA DISWANTSHO LE DI VIDEO KA PHAPHOSING YAKA YA BORUTELO GO PHETHATAGATSA DITHUTO TSAKA LE UNIBESITHI YA LIMPOPO ,KE ITHUTELAGO DITHUTO TSA KA GODIMO (MASTERS IN MATHEMATICS EDUCATION). KE LE NETEFALETSA GORE MELAWANA YE E LATELAGO E TLA LATELWA:

1.LEBITSO LA NGWANA WA LENA LA NNETE, LE KA SE SOMISWE GE KE NGWALA RESETSHE YAKA KA DILO TSEO A DI BOLETSEGO GOBA A DI DIRILEGO KA NAKO YA GE KE GATISA DISWANTSHO LE DI VIDEO.

2. GO TSEYA KAROLO GA NGWANA WA LENA GA SE KGAPELETSO KE BOITHAOPI .BJALO KA MOTSWADI WA NGWANA, LENA LE TOKELO YA GO LESESA NGWANA WA LENA GO TSEYA KAROLO GE A SE SA NYAKA GOBA LENA LE SE SA NYAKA GORE KE MO TSEYE DISWANTSHO LE DI VIDEO. .

3. LE TLO FIWA COPY YA LETLAKALA LAKA LA RESETSHE PELE KE LE ROMELA UNIBESITHI YA LIMPOPO, KE TLA LE HLALOSETSA GORE KE NGWADILE ENG KA NGWANA WA LENA .LE DUMELELWA GO MPHA MAELE A GORE NKA FETOLA ENG , GE GO HLOKAGALA

4. GO KA SEBE LE MOPUTSO GE NGWANA WA LENA A TSEYA KAROLO MO DISWANTSHONG LE DI VIDEONG TSAKA

5. TSHOHLE TSE KE TLOGO DI GATISA KA NGWANA WA LENA,KE YA GO DI SOMISA MABAPI LE GO NGWALA RESETSHE YAKA YA MASTERS WA MATHEMATICS EDUCATION UNIBESITHING YA LIMPOPO FEELA.

E KABA LE FA NGWANA WA LENA TUMELELO YA GORE E BE KAROLO YA RESETSHE YAKA?:
E ✓ AOWA _____

NNA (LEINA LA MOTSWADI/MOHLOKOMEDI) Selly Makh... KE YA KWESISA EBILE KE DUMELELANA LE MELWANA YE E BWEILEGO. SIGNATURE ________
- LETSATSI 21-09-2020

RESEARCHER ________ DATE 21-09-2020

Appendix F: Informed Consent for Learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

Dear learner

I humbly request that you become part of my study when I collect data in Grade 10 mathematics classroom, by video recording all the teaching and learning activities during my classroom interaction with you. I assure that the following conditions will be met:

1. Your real name will not be used at any point of information collection, or in the final writing up of the data.
2. Your participation in this research is voluntary. You have the right to withdraw at any point of the study, for any reason, and without any prejudice, and the information collected and records and reports written will be turned over to you.
3. You will receive a copy of the final report before it is handed in, so that you have the opportunity to suggest changes to the researcher, if necessary.
4. You are not forced into taking part; no any form of incentive will be given to you for taking part. This is a voluntary participation.
5. The collected information will be used for research purposes only.

Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Madibana L.F Date 29/09/2020 I agree to the terms and conditions .

Researcher Sedib Date 29/09/2020

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

Dear learner

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5. The collected information will be used for research purposes only.

Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Mokoena H.R Date 29/09/2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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5. The collected information will be used for research purposes only.

Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Monyela J.B Date 29-09-2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

Dear learner

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5. The collected information will be used for research purposes only.

Do you grant permission to be quoted directly: Yes_X_ No ______ I agree to the terms.

Learner Seabi T.P Date 29.09.2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Lesudi M.M Date 29|09|20 I agree to the terms and conditions .

Researcher Sedib Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

Dear learner

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Monyeki JN Date 29-09-2020 I agree to the terms and conditions .

Researcher ____________ Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Motsheko T.C Date 29.09-2020 I agree to the terms and conditions .

Researcher ______________________ Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Maile M.C Date 29-09-2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes X No ______ I agree to the terms.

Learner Malete M.R. Date 29/09/2020 I agree to the terms and conditions .

Researcher ______________ Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Moruane M.E Date 29-09-2020 I agree to the terms and conditions .

Researcher Sedib Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Mokgohloa I.T Date 29.09.2020 I agree to the terms and conditions .

Researcher Sedn Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Macheko M.M Date 29September20 I agree to the terms and conditions .

Researcher Sedib Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Mochiko K. G Date 29/09/2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Mocheko S.C Date 29/09/2020 I agree to the terms and conditions .

Researcher Sedis Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Seanego P. M Date 29/09/2020 I agree to the terms and conditions .

Researcher Sedis Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner CHimo IE AE Date 29/09/2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes X No ______ I agree to the terms.

Learner Ramoroka R.B. Date 29-09-2020 I agree to the terms and conditions .

Researcher Sedibs Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes X No ______ I agree to the terms.

Learner Tapala J.M Date 29/09/2020 I agree to the terms and conditions .

Researcher Sedits Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Moyuguneng T.V Date 29/09/2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Learner Madibeng J.S Date 29/09/2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Motlohelo Mtett M.D. Date 29-09-2020 I agree to the terms and conditions .

Researcher Sedits Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Magwai Melidah Date 29.09.2020 I agree to the terms and conditions .

Researcher Sedi Date 29/09/2020

APPENDIX E: Informed Consent for learners

INFORMED CONSENT FORM for a learner

Researcher: Khutso Makhalangaka Sedibeng (cell number: 062 416 5443)

Dear learner

I humbly request that you become part of my study when I collect data in Grade 10 mathematics classroom, by video recording all the teaching and learning activities during my classroom interaction with you. I assure that the following conditions will be met:

1. Your real name will not be used at any point of information collection, or in the final writing up of the data.
2. Your participation in this research is voluntary. You have the right to withdraw at any point of the study, for any reason, and without any prejudice, and the information collected and records and reports written will be turned over to you.
3. You will receive a copy of the final report before it is handed in, so that you have the opportunity to suggest changes to the researcher, if necessary.
4. You are not forced into taking part; no any form of incentive will be given to you for taking part. This is a voluntary participation.
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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Isheri Pauline Lara Date 29/09/2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

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Learner Mokoena D. Date 29-09-2020 I agree to the terms and conditions .

Researcher [signature] Date 29/09/2020

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Do you grant permission to be quoted directly: Yes ✓ No ______ I agree to the terms.

Learner Mabasa K.G Date 29|09|2020 I agree to the terms and conditions .

Researcher ______________ Date 29/09/2020