SOCIOMATHEMATICAL NORMS CONSTITUTED FOR PROMOTING LEARNERS' PROFICIENCY IN MATHEMATICS

by

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RESEARCH THESIS

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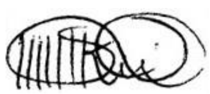
DECLARATION

I, L. L. Mokwana, declare that the research thesis hereby submitted to the University of Limpopo, for the degree of Doctor of Philosophy in Mathematics Education has not previously been submitted by me for any degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.



_______________________ 19 May 2021

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DEDICATION

This thesis is dedicated to my late mother, Betty Matsemela Mokwana.

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ABSTRACT

The qualitative study reported here sought to explore constitution of sociomathematical norms in a class where teaching was undertaken to promote learners' proficiency in mathematics. A case study research design in which I served as a teacher-researcher was adopted and focused on a Grade 11 mathematics class in which all learners were participants. The study was guided by three interrelated research questions and these questions were: (i) what are the sociomathematical norms constituted for promoting learners' proficiency in mathematics? (ii) how are the sociomathematical norms for promoting learners' proficiency in mathematics constituted and enacted in the classroom? (iii) how does the enactment of the constituted sociomathematical norms promote learners' proficiency in mathematics?

Three theories were employed during this study, namely: (i) sociocultural theory as a referent for classroom practice (Vygotsky, 1978). (ii) theoretical constructs of socio and sociomathematical norms found in the emergent approach by Cobb and Yackel (1996) as a lens through which data were analysed; and, (iii) the five interwoven strands of mathematical proficiency (Kilpatrick, Swafford & Findell, 2001) as standards for learners' mathematical development. The interplay of these three theories gave rise to the conceptualisation of a framework which best accounts for how the study unfolded.

Data were generated through video recording and participant observation. Data also emerged from classroom discussions when learners, in their pairs or small groups, worked through learning activities. These interactions, were audio and video recorded. Meanwhile, observation data were recorded in a researcher journal in which entries were made after each lesson. Data were analysed following Polkinghorne's (1995) narrative analysis of eventful data, followed by an analysis of narratives as the final step. During the analysis I listened to and watched the audio and video recordings a number of times, and then selected the excerpts which were

to be used for analysis. This was followed by the transcription of the selected audio and video data into text.

It was found that learners constituted and enacted the sociomathematical norms concerned with acceptable mathematical explanations, mathematical justifications and mathematically different solutions. The constitution and enactment of these sociomathematical norms provided learners with multiple opportunities to engage in classroom discourse which promoted their proficiency in mathematics. Furthermore, it became apparent that teachers' ability to foster productive sociomathematical norms is imperative for mathematics learning. As a result, I proposed a model for fostering the constitution of sociomathematical norms as teaching and learning unfolds in the natural setting of a classroom. The model consists of four major constructs, namely, identifying productive sociomathematical norms, knowing learners' existing sociomathematical norms, disrupting and deconstructing learners' sociomathematical norms and negotiating and authoring new sociomathematical norms.

Enactment of sociomathematical norms, on the other hand, emerged as learners engaged in mathematical classroom discourse and experienced the situation for challenges and justifications. The enactment of these sociomathematical norms reflected the unification of social norms and mathematical norms, as learners engaged in mathematical activity. In order to orchestrate the enactment of sociomathematical norms, teachers should first adopt social learning theories (sociocultural theory) as a referent for classroom practice. These social learning theories, if adopted, will ensure that learning is taking place within environments, which allows for social interactions to take place.

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CHAPTER 1: INTRODUCTION

1.1 INTRODUCTION

In this chapter I give an orientation to the study by first providing my background as the teacher-researcher, wherein I also give my encounters with mathematics and mathematics education at various stages of my schooling and career. In doing so, I keep these reflective pieces short but ensure that they succinctly lay unadorned my mathematics teaching and learning experiences which are relevant to the study I am reporting here. Second, I give an overview of the study including the context in which the study was undertaken, and finally, the structure of the thesis.

1.2 THE TEACHER – RESEARCHER

In this study I was the teacher who prepared and presented lessons in the Grade 11 mathematics classroom from which data were generated. Additionally, I was the researcher in pursuance of the study I am reporting here, hence, a teacher-researcher. In this section I present my experience in learning mathematics and later in teaching mathematics which are influential to the kind of mathematics education research that is close to my heart as well as this study in particular. As highlighted earlier, these experiences are organised according to the different stages of my schooling and career journey in which I encountered the learning and teaching of mathematics.

1.2.1 My First Encounter: My Primary and Secondary Schooling Years

My very first day at school is a memorable one, as I just joined my friends on their way to school. I was not prepared for going to school, as I was not registered, my actual school going year according to my age was the year which followed. My mother came looking for me and found me in the preschool classroom. Following a conversation with the teachers, it was agreed I should be allowed to attend school without paying fees as I was still to repeat preschool the following year and my mother could not afford to pay for the same school level twice.

After a few weeks in the school I become a top learner in the preschool class for both numeracy (mathematics) and literacy (speaking, reading and writing in mother tongue). I was then allowed to go to grade 1 the following year, meaning I was a year ahead of my schooling age. As I progressed with my primary education my love for mathematics developed and I become amongst the few learners who could do arithmetic mathematics using the Abacus, multiplication tables and later mentally.

In the early years of secondary school (grade 8 and 9), I continued to be a top learner in mathematics, until I wrote my grade 9 Common Task Assessments (CTA) in the year 2002. This was the first time I encountered mathematics which did not make sense to me, it was frustrating but I still passed at the end of the year. These frustrations resulted from various factors such as language and various real life contexts in which all the questions were imbedded in, just to mention a few. As such the paper required a lot of mathematical application but first, one had to come to grip with the context used. Even though Part A of the assessment was completed over a week with the allowance that we take it home, I still found it challenging. It was after grade 9 where I learned everything must make sense, and in the grades that followed I was very fussy about everything making sense and at time my classmates found it irritating as it did not matter for them. As long as we are able to follow teachers' worked examples and get the answers which were confirmed to be correct by the teachers it was enough for them.

Reflecting on some of these experiences from the time I was undergoing my training as secondary school mathematics teacher to date, I concluded that my learning of mathematics was more like a collection of rules and algorithms which had to be accepted by faith or at least because of a teacher's authority. A typical example is when I was in grade 10 and we were learning about powers and exponents, amongst others our teacher said '*any number exponent zero is one*'. I found this statement troubling as I had learned powers to mean repeated multiplication like $2^4 = 2 \times 2 \times 2 \times 2$. When I raised this concern with the teacher, I was told that it is a 'law of exponents' and all we needed to do was accept and apply it. Later in grade 11 when we dealt with powers involving rational exponents the 'law of exponents' kept increasing and it was overwhelming for me. Then I discovered that there is no solution for zero exponent zero (0^0) as I was using a calculator. Then I went back to challenge the law '*any number exponent zero is one*' again I lost the argument as the teacher asked me as to with which number do I start counting? It is important to note that, in grade 11 I had a different teacher from grade 10. My knowledge of number sets at the time failed me and I accepted we count from 1, I could not even think of questioning the variables which when the exponent was zero we still said the answer is 1. This is one of the many arguments which I had with my school teachers and only started making better sense when I was a preservice teacher.

The classroom talk inherent in the few examples above portrays the nature of classroom engagements which existed. All the time it was between me and a teacher, or one of my classmates and a teacher. Very seldom we engaged in a mathematical argument as learners in the presence of a teacher, even if we did, a teacher still served as the go-between person. Correspondingly, when inconclusive mathematical arguments that occurred between learners in the absence of a teacher or outside the mathematics classroom were taken to a teacher, a teacher served as a judge. Teachers mainly ruled by telling us who is correct without probing for or listening to reasons behind the arguments. These highlights the norms which governed the classroom talk, and justifies why other learners would get annoyed by persistent engagement as they have implicitly learned that a teacher's word is final.

1.2.2 My Second Encounter: A Preservice Mathematics Teacher

My journey of becoming a secondary school mathematics teacher, was one which I waited for anxiously. Everyone around me, my teachers, classmates, friends and family knew that I wanted to study towards being a teacher of mathematics. My excellence is school mathematics was one reason, the other was that I went through half of my grade 10 year without a mathematics and physical science teacher, but most importantly I learned that I enjoyed explaining mathematics to others and engaging in mathematical arguments.

When I got accepted to study for Bachelor of Education that was the beginning of my dream becoming true. It was at this stage when I discovered that for preservice teachers there are mathematics content modules and methodology modules. My content modules were taught by lecturers from the faculty of sciences, and the experience did not differ much from school experience. There were a lot of rules and theorems which we had to apply, if we were to prove them, the proof was expected in a particular way. Even though, I performed very well in mathematics content modules, in that I obtained distinctions in all modules from first to fourth year, I did not enjoy mathematics content as much as I enjoyed mathematics methodology.

The methodology modules on the other hand were intriguing, they asked many simple questions, using simple school mathematics as context, yet answering those questions was tough. These modules were very useful in getting me to think of the mathematics I learned in school and continue questioning the content I learned and how I learned it. In our first lecture, for the methodology module which focused on teaching for understanding, we were asked to explain the meaning of 3×2. With many of us starting by saying 6, but then grabbled until we got to seeing multiplication as repeated addition. This question grew to giving the meaning of $5(x-2)$ and so on until we handled the product of two binomials. In another instance where the lecturer referred to financial mathematics, we used the formula $A = P(1 + n.i)$ to get the answer. While explaining our algorithm we were asked why we gave the meaning of each variable but did not say anything about the ‘1’ in the formula. Well we said because it was not a variable, this was followed up by questions like can we put another number instead of ‘1’ why?

Needless to say we left many questions unanswered but too much thinking was initiated. It was during my teacher training years through the methodology courses that I learned about the instrumental and relational understanding (Skemp, 1976), mathematical process skills (National Council of Teacher for Mathematics (NCTM), 2000) and mathematical proficiency strands (Kilpatrick, Swafford & Findell, 2001) as some of the theoretical construct that could guide teaching mathematics for understanding. The latter readings shaped my understanding of how mathematics should be taught and I took an oath that in my teaching, they will forever be the dash board for my teaching. I told myself that upon graduating, I will always strive to teach my learners for relational understanding, so that they develop appropriate mathematical process skills and become mathematically proficient.

In the third and fourth year of my training I was engaged in school based teaching practice, these was the first time in which I had to put my teaching goal in motion. The first challenge was my mentor teachers discouraged how I facilitated learning, pointing out time to finish work schedule as a challenge. Since I was not willing to give my learners the same experience I had in school mathematics, I committed to do extra lessons. Fortunately, a majority of learners found how I question what they do interesting, yet wanted me to give them answers to the very questions I asked. It was not an easy journey as in some cases I lacked relational understanding of the content I was to teach and had to engage in searching for information as I prepared my lessons, or worse succumb to teaching instrumentally. All these trouble I went through was always applauded by my lecturers when they came to evaluate my teaching, hence I obtained distinctions for all lessons in which I was evaluated.

1.2.3 My Third Encounter: In-service Mathematics Teacher

I started teaching in multiracial former model c secondary school based in the Tshwane South Education District in Gauteng Province, the schools is located in Pretoria West. The first teaching load allocated to me was three grade 9 classes and two grade 10 classes. I was excited that I was not allocated grade 12, and will not have a pressure of rushing through lessons in order to complete the work schedule. The only struggle was getting learners to be more autonomous and not reliant on me as their teacher. The manner in which we engaged with the content was fascinating for learners, as the school was big and I did not teach all classes in the grade, my learners enjoyed asking their counterparts the conceptual questions which we engaged in during teaching and learning. Some learners named me 'Activity' while other named me 'Does that make sense', as in every lesson there was an activity and for every engagement I would ask if it made sense.

Two weeks in the school, my head of department (HoD) did a class visit, to observe me teaching. After the lesson, amongst others her comments were that; I do not use the chalkboard, I should let learners use the same method and save time. These comments disheartened me, but did not deter me from my teaching goal, I tried to justify my approach unfortunately she did not accept my justification. Matters became worse when my classes performed poorly during the March tests, the school principal was also a mathematics teacher and he then came to observe my teaching. After observations he hardly gave comments but would rather ask me questions regarding my facilitation decisions, I felt he supported me. Then for June examinations my classes' performance improved and then principal allocated me two grade 12 classes in the place of the grade 10 classes.

Exactly after three years of teaching I was appointed HoD in a rural school located in the Sekhukhune District of Limpopo Province. I immediately started advocating teaching for relational and conceptual understanding in my department. I taught mathematics to grade 11 and 12 learners, for those whom I taught in grade 11 their grade 12 year always became a very easy one as we had already developed rapport. Hence, grade 11 became the class in which learners had to readjust to a 'different' way of doing things. Interestingly, these made more and more learners to opt for mathematics and not mathematical literacy. The number of grade 12 learners in the school was consistent around 60 every year, when I arrived at the school in 2013 only 15 grade 12 learners enrolled for mathematics. Whereas, when I left the school four years later (2017), 47 grade 12 learners were doing mathematics. The mathematics pass rate ranged between 80% and 93% with the grade 12 overall pass rate ranging between 96% and 100%.

It was during tenure as HoD that I decided to further my studies, I opted to conduct research in my own classroom, in order to continuously evaluate my goal of teaching and document affordances and hindrances inherent to it. In particular, I would use my grade 11 class because these was where learners would encounter my teaching for the first time. As I completed my honours degree the mini-research focused on learners' contributions to (or retractions from) developing mathematical caring relations. Whereas, for my masters studies, I documented and described how learners become mathematically agentic. Both studies were conducted in my grade 11 classroom which I viewed as some sort of a 'laboratory' in which learners experience new teaching and learning experiences.

1.2.4 My Fourth Encounter: Mathematics Teacher Educator

My appointment as a lecturer started on part-time basis, and I was responsible for a combined method of mathematics, science and technology teaching module, which I offered to 1st and 2nd year Bachelor of Education students.

These modules gave students foundation of theories on teaching and learning of mathematics, sciences and technology. In my facilitation of these modules I was cautious not to approach them from a general education perspective and end up duplicating what students did in general education didactics modules. Hence, I ensured that a backdrop of school mathematics/science/technology content and storied lessons were used to enhance their understanding of the theories understudy. Most of these backdrops emanated from my own school classrooms (as I also taught physical sciences in school) and some constructed from how I learned or other colleagues commonly taught mathematics content.

The scenarios presented to students reflected what was usual to them and what was innovative but yet unusual to them. Then we would use the theories being learned to analyse the merits of each teaching approach and/or lesson. Even though I pushed students towards the teaching that I adopted, I indicated to them that I am not trying to clone them, rather want them to be even better than me. These students had more agency and autonomy than my high school learners and as such some would frankly not accept my approach to teaching. I always respected students' stance but asked them to support it with theoretical perspectives and other contemporary issues which forefront learners' development of mathematics with understanding.

Even after I was appointed a full time lecturer and allocated methodology of mathematics module for year 3 students, I continued engaging in teaching high school learners on weekends. Currently, I take about 6 of my 3rd year students to work with me on weekends. We prepare lessons together (though I take the lead) and at the beginning of the year I teach while they serve as assistants and gradually I allow them to lead other lessons. Finally, when they are in their 4th year they prepare as a pair or group and teach in pairs or individually. This project gives me an opportunity to continuously re-evaluate my demands about teaching and facilitating learning from my students. It also creates an opportunity of engaging in pragmatic research. Given the project described earlier one may wonder why I did not conduct my doctoral study in the same project. The study I am reporting here was accepted and approved while I was still a full time school teacher and part time lecturer, only later that year I was appointed full time and my work with students started the year which followed. I provide more details regarding the conceptualisation of the study in the section that follows.

1.3 OVERVIEW OF THE STUDY

1.3.1 Background of the Study

The study reported here was conceptualised while I was still working as an HoD and mathematics teacher in a secondary school. I had come across some reading on enculturation and classroom cultures during my master's studies. My interpretation of these readings, was that, when I meet a new cohort of learners in grade 11 as explained earlier in this chapter, I expect them to learn new/different mathematical classroom culture. These lead me to readings on social and sociomathematical norms, which I understood to best explain the mathematical classroom culture for different classrooms. For example, in coining the term sociomathematical norm, the proponents of these notion (Paul Cobb and Erna Yackel) studied two different mathematics classrooms and focused on *regularities in interaction patterns*. One classroom followed traditional teaching while the other followed reformed or inquiry based teaching.

Correspondingly, I regarded my classroom practice to be different, given my goal of teaching learners for relational understanding so that they develop appropriate mathematical process skills and become mathematically proficient. Hence, I pondered on what sociomathematical norms are actually constituted in such a classroom. Answering this question, meant I would be able to encourage teachers in my department to engender particular sociomathematical norms as I advocated teaching for relational and conceptual understanding as mentioned earlier in this chapter. Additionally, it would assist me to continuously improve my teaching towards achieving the goal identified. However, these personal question and reasons on their own would not be acceptable as basis for research, as a result I had to locate them within relevant literature as in the discussions in the sections that follow.

1.3.2 Positioning the Study in Literature

Poor performance in mathematics by South African learners has been of concern to everyone who has interest in education for a while now. This underperformance correlates with the notable upward trajectory of research on classroom social interactions aimed at explaining how mathematics is learned (Abdulhamid, 2016; Le Roux, Oliver & Murray, 2004). Cobb and Yackel (1996) argued that learning takes place as a result of social interactions and culture. Consequently, they define learning as “a constructive process that occurs while participating in, and contributing to, the practices of the local community” (p. 185).

The engagement of learners in social interactions makes it possible to account for learners’ mathematical development as it occurs in a social context of the classroom (Blankson & Blair, 2016; Bradley & Corwyn, 2016; Hodge & Cobb, 2019). Furthermore, learners develop social norms which are initiated by a teacher through explanation, justification and argumentation (Cobb & Yackel, 1996; Yackel, 2001). However, for these social norms to be regarded as sociomathematical, they should be specific to learners’ mathematical activity (Cobb & Yackel, 1996). Hence, sociomathematical norms describe the normative understandings related to mathematical reality (Yackel, Rasmussen & King, 2000).

Studies on sociomathematical norms have been conducted in both elementary and secondary school classrooms (Abdulhamid, 2016; Leveson, Torish

& Tsamir, 2009; Partanen & Kaasila, 2015; Planas & Gorgorió, 2004). However, most of these studies were not conducted in South African classrooms, with an exception of Abdulhamid's (2016) study, which was conducted in South African elementary classrooms. Correspondingly, Chuene (2011) noted a gap in creation of knowledge about normative aspects of mathematics classes at schools in South Africa. Hence, she argued that studies explaining the normative aspects of these classrooms are imperative in pursuance of bringing to the fore, knowledge of what is happening in these classes.

Additionally, Güven and Dede (2017) also argue for a need for studies that focus on sociomathematical norms in mathematics classrooms. They particularly argue for studies which seek to bring to the fore sociomathematical norms which are constituted in mathematics classrooms and how they are constituted. Hence, this study explored the constitution of sociomathematical norms, in a South African secondary school mathematics classroom, where teaching was undertaken in order to promote learners' proficiency in mathematics.

The constitution of sociomathematical norms takes place on the basis of learners engaging in mathematical activity for their mathematics learning. Therefore, stating the goal of teaching while researching on social and sociomathematical norms is imperative as it portrays the context in which such norms were studied. For example in studying social and sociomathematics norms Yackel and Cobb (1996) stated the goal of teaching for the classroom which they studied as to facilitate students' mathematical conceptual development. As a result, in the analysis they also clarified how students develop mathematical disposition and accounted for students' development of increasing intellectual autonomy in mathematics. Correspondingly, McClain (1995) and Simon (1995) also stated their goal of studying classroom social and sociomathematical norms. Simon (1995) stated the goal as to develop a model of mathematics teaching that is informed by a social constructivist view of learning. Whereas, McClain (1995) stated the goal as to identify aspects of effective reform teaching.

Mathematics learning in the context of the study reported here is defined as the achievement of the mathematical proficiency strands (Kilpatrick, Swafford & Findell, 2001). The interwoven strands of mathematical proficiency are; *conceptual understanding, procedural fluency, strategic competence, adaptive reasoning* and *productive disposition*. Kilpatrick et al. (2001) provided a description for each of the five intertwined strands of mathematics proficiency. These descriptions provided a framework for delineating what exactly learners are expected to do in demonstration of competence in, or achievement of, each strand. Furthermore, this framework afforded me an opportunity to develop a focused analysis and argument throughout the study, in terms of accounting for learners' mathematics learning. As a result, during analysis when I presented the sociomathematical norms which were constituted, I also described the way(s) in which such sociomathematical norms promoted learners' proficiency in mathematics.

My goal for teaching as highlighted earlier, is to teach my learners for relational understanding, so that they develop appropriate mathematical process skills and become mathematically proficient. This means I could have pursued the study and accounted for learners' development of relational understanding or mathematical process skills. However, I chose to focus on mathematical proficiency because in South Africa at the time I proposed this study a framework of Teaching Mathematics for Understanding (TMU) that is underpinned Kilpatrick et al.'s (2001) notion of mathematical proficiency was also proposed and later published (Department of Basic Education (DBE), 2018). While acknowledging that the TMU is not yet widely disseminated, making reference to it for this study is key in that it serves as standards for teaching mathematics in South Africa. Just like there are standards for teaching mathematics in the United States which are; *problem solving*, *communication*, *reasoning and proof*, *representation* and *connections*. Notably, this adds to the pragmatic significance of the study, because this study makes a contribution on the meaning of the TMU for my classroom practice from social and sociomathematical norms view point.

1.3.3 Research Problem Statement

The constitution of productive sociomathematical norms associated with reformed teaching practices contributes to conditions that make meaningful learning of mathematics possible (Yackel et. al., 2000). As a result, it is important for teachers to be aware of, and understand, the sociomathematical norms of their classrooms (Kang & Kim, 2016; Zembat & Yasa, 2015) and to be able to engender productive sociomathematical norms during teaching and learning (McClain & Cobb, 2001; Partanen & Kaasila, 2015; Yackel et al., 2000). This will enable teachers to effectively facilitate mathematics learning. Additionally, engendering productive sociomathematical norms associated with reformed teaching practices may reduce concerns about, and lead to an improvement of, the performance of learners in mathematics, since the constitution of sociomathematical norms directly influences our mathematical agenda (McClain & Cobb, 2001). Mathematical agenda refers to what and why we teach mathematics for, and in the context of this study teaching of mathematics was undertaken to promote learners' proficiency in mathematics.

However, there is a notable gap in the creation of knowledge about the normative aspects of mathematics classes at schools in South Africa (Chuene, 2011). In addition, there is paucity of studies which examine sociomathematical norms as one of the key aspects of mathematical classroom discourse that affords mathematics learning in classrooms (Widjaja, 2012). Correspondingly, Güven and Dede (2017) argued for studies that focus on determining which sociomathematical norms should be constituted and how they should be constituted.

As a result, this study explored the constitution of sociomathematical norms in a class where teaching was undertaken in order to promote learners' proficiency in mathematics. Through this exploration the sociomathematical norms which should be constituted, and how they were constituted, were documented and presented as a key aspect of mathematical classroom discourse that affords mathematics learning. Hence, knowledge of what is happening in South African mathematics classrooms was brought to the fore.

1.3.4 Purpose of the Study

The purpose of this qualitative case study was to explore the constitution of sociomathematical norms in a class where teaching was undertaken to promote learners' proficiency in mathematics.

1.3.5 Research Questions

The guiding research questions were:

- What are the sociomathematical norms constituted for promoting learners' proficiency in mathematics?

- How are the sociomathematical norms for promoting learners' proficiency in mathematics constituted and enacted in the classroom?

- How does the enactment of the constituted sociomathematical norms promote learners' proficiency in mathematics?

1.3.6 The Research Site

Initially (at proposal stage) this study was to be conducted in my own grade 11 mathematics classroom which was allocated to me for day to day teaching at the school which was my work station. However, by the time ethical clearance was issued I was no longer working as a school mathematics teacher, but a university mathematics education lecturer. As a result, I had to find a research site which I would be able access while executing my duties at the university. During in-school classroom support visits for my students, I visited a student who was placed in a school which I found learners being free to engage regardless of their answers being wrong or my presence as a visitor in their classroom. Then I decided that, such a school would be ideal for me conduct my research as it was no longer possible to conduct it in my own class as planned in the proposal stage. This decision is in harmony with purposive sampling. The only thing which changed was the research site, other plans such as collecting data from a grade 11 classroom and me being the teacher-researcher remained unchanged.

Data were generated from grade 11 mathematics classroom of a multiracial former model c secondary school, located in the Capricorn South District of Limpopo Province, Polokwane. Learners in this school did English as a home language, the school had eight (8) grade 11 mathematics classes allocated to three (3) teachers one of which was the HoD. The approval letter to my request to conduct research in the school had indicated that I would work with the HoD. The HoD was allocated three (3) grade 11 classes, since I collected data during normal school hours, the HoD allocated one (1) class for me to teach as I collect data. According to the HoD, this was the class which would not be intimidated by the presence of video cameras. However, I was involved in teaching of the other two (2) classes although I did not video record lessons from them, I taught them as the HoD wanted all his classes to experience the same teaching. During lessons the HoD was assisting in video recording for the class in which I collected data from, and we often engaged in team teaching in the other classes.

1.4 STRUCTURE OF THE THESIS

This thesis consists of eight chapters. In chapter one, I explain what prompted me to undertake this study and locate this decision within literature. In chapter two I present the theoretical framework which I adopted as lens through which data were analysed which also serves as the overarching framework underpinning all sections of the thesis. In chapter three I discuss my philosophical stance in relation to mathematics learning, in doing so I first touch on classroom based research and then discuss theories of learning that are pivotal to this study. In chapter four, I present an analysis of the literature on classroom social and sociomathematical norms. In chapter five, I present the research methodology and methods which I adopted in pursuance of this study. In chapter six and seven, I present the findings of the study, along with my analysis and interpretation of the findings. Finally, in chapter eight I present the conclusion by presenting a discussion which responds to the research questions, after which I provide recommendations, as well as suggest areas for future research.

1.5 SUMMARY

In this chapter I presented a background and introduction to the study by first describing myself as the teacher-researcher and the encounters I had with mathematics and mathematics education which influence the type of research I conduct. I then presented the overview of the study by presenting the background of the study, positioning the study in literature and the research problem statement. I then highlighted the purpose of the study, as well as the research questions which the study sought to address. Finally, I concluded the chapter by presenting an outline of the thesis and explaining what each chapter of this thesis will deal with.

CHAPTER 2: THEORETICAL FRAMEWORK

2.1 INTRODUCTION

In this chapter, mainly I present the theoretical constructs of social and sociomathematical norms found in an emergent approach by Cobb and Yackel (1996) which I adopted as a lens through which I analysed data. However, I set the scene by first discussing two theories from which Cobb and Yackel's (1996) perspective emerged. Even though the purpose of this first discussion is to offer a theoretical backdrop of the analytical framework, I also explicate my understanding of these theories and their implication for my study. Thereafter, I present the interpretive framework, clearly outlining its constructs and stating how they guided data analysis. The discussions in this chapter also frames and underpins the engagement with literature in chapters 3 and 4 that follows.

2.2 THEORETICAL ORIGINS OF THE INTERPRETIVE FRAMEWORK

Cobb and Yackel's interpretation of classroom practice was initially underpinned by von Glasersfeld's (1984) perspective of constructivism. This was the case as their intention for studying classroom practice was to explain students' mathematical activity and learning in individualistic psychological terms (Cobb & Yackel, 1996). However, as they reflected on their classroom experiences, a need to broaden their interpretive stance by developing a sociological perspective on mathematical activity arouse (Yackel & Cobb, 1996). Consequently, their perspective on interpreting classrooms evolved as they developed a sociological perspective which was guided by Blumer's (1969) notion of symbolic interactionism. Hence, the interpretive framework adopted here, which Cobb and Yackel (1996) refer to as an emergent approach, emerged.

2.2.1 CONSTRUCTIVISM

It might seem contradictory that the interpretive framework adopted in this study was underpinned by cognitive constructivism, yet the theory employed as a referent for classroom practice was sociocultural theory. It is for this reason that in discussing constructivism, I also argue for how this theory resonates with sociocultural theory which I adopted as a referent for classroom practice as outlined in Chapter 3. Notably, Cobb (1996) in his article on "*Where is the mind? A coordination of sociocultural and cognitive constructivist perspectives"* also supported this assertion. Even though, Cobb's intention in this article was to argue for the sociocultural perspective, as a way of interpreting the sociological aspects of classroom practice, here it is used for a different reason. The sociological perspective of classroom practice is interpreted guided by symbolic interactionism (Cobb & Yackel, 1996; Yackel & Cobb, 1996), which indicates that the attempted to use sociocultural perspective was unable to account for all pragmatic classroom interactions. Hence, in this study it only guides classroom practice and not used to interpret the sociological perspective to mathematics learning.

In describing constructivism, von Glasersfeld (1992), put forward debates about; the nature of knowledge, the role of social interactions during knowledge construction and the implications of radical constructivism on teaching. To put forth these arguments he wrote:

> Radical constructivism is an attempt to develop a theory of knowing that is not made illusionary from the outset by the traditional assumption that the cognizing activity should lead to a 'true' representation of a world that exist in itself and by itself independently of the cognizing agent. Instead, radical constructivism assumes that the cognizing activity is instrumental and neither does it concern anything but the experiential world of the knower. This experiential world is constituted and structured by the knower's own way and means of perceiving and conceiving, and in this elementary sense it is always irrevocably subjective (von Glasersfeld, 1992, p. 1).

Inherent to this argument is that learning occurs within an individual learner's cognitive realm, wherein learners continuously reorganise their cognitive schemas. The social interactions are treated as a periphery which triggers the cognitive process which learners undergoes during learning. This view is in harmony with the Piagetian constructivist view that learning mathematics is a process in which learners reorganise their activity to resolve situations that they find problematic. In the current study this constructivist perspective is acknowledged as it holds the view that learners' inappropriate constructions should not be taken as errors or misconceptions to be eliminated and replaced by mathematically acceptable ones. Instead, such constructions should be expected because they are part of a learners' mathematical development and learning process (Ferrini-Mundy & Graham, 1994). However, this view fails to account for social interactions which takes place in the classroom, as part of the whole learning process but not merely a periphery.

Therefore, multiple perspectives that would reflect both the cognitive and sociocultural perspective on learning are relevant. With these perspectives to learning, both enculturation and self-reorganisation processes occur among learners while participating in cultural practices in frequent interaction with others. A comment made by Sfard (2002) regarding the participation and acquisition approaches to cognition is also applicable in this case.

> No theory is built on a single metaphor. However, of those metaphors that can be identified, one is usually the most prominent and influential. Also, not all of the differences between the different approaches are necessitated by the respective metaphors. Some of the entailments are optional and sustained by mere habit. (Sfard, 2002, p. 23)

The need for a cognitive and social perspective to learning, is justified by the question which could be raised when learning is only viewed from a sociocultural theoretical point. The question is; how does the social become part of learners' mind? Whereas, in the same way, following cognitive view to learning, one may ask;

how are actions reified to become mental mathematical objects that can themselves be acted upon? This is a challenge as following either cognitive constructivist or sociocultural perspective leaves a question to ponder.

Cobb (1996) addresses the latter challenge, firstly by recognising that both theories highlight the importance of the role that is played by activity in mathematical learning and development. Secondly, he contrasts the two theories against each other through coordinating the process of internalisation that was introduced by Vygotsky, and the process of empirical and reflective abstraction encouraged by von Glasersfeld. In illustrating Vygotsky's notion of internalisation, Rogoff (in Cobb, 1996) argued that as children observe and participate with others they are already engaged in a social activity. By taking the view that children are in social activity, and the view that the interpersonal aspects of their operations are integral to their intrapersonal aspect, then a separate process of internalisation cannot exist. According to Cobb (1996), this illustrates that as learners are involved in classroom activity they are active participants – they contribute to the development of mathematical classroom practices from the onset. A conclusion would then be to view "mathematics learning as a process of active construction that occurs when children engage in classroom mathematical practices, frequently while interacting with others" (Cobb, 1996, p. 41).

Reflective abstraction is a means by which learners reorganise their initially informal mathematics ideas (Cobb, 1996). These reorganisations of ideas take place as learners participate in cultural practices from which they abstract meaning. According to Cobb (1996), this therefore suggests that when reflective abstraction is defined, emphasis should be placed on the fact that "it involves the reification of sensory-motor and conceptual activity and that it occurs while engaging in cultural practices, frequently while interacting with others" (Cobb, 1996, p. 43). Using Rogoff and von Glaserfeld's work, Cobb (1996) then concluded that the

> … view of learning as acculturation via guided participation implicitly assumes an actively constructing child. Conversely, …[the] view of learning as cognitive self-

> organization implicitly assumes that the child is participating in cultural practices. In effect, active individual construction constitutes the background against which guided participation in cultural practices come to the fore for [sociocultural theorists], and this participation is the background against which self-organization comes to the fore for [cognitive theorists] (Cobb, 1996, p. 43).

Cobb (1996) also contrasts sociocultural perspectives and cognitive perspectives by focusing on potential links between sociocultural analysis provided by Saxe (2015) and cognitive analysis by Steffe and von Glasersfeld (1988). He compares how the two theorists would analyse a mathematical activity. A socioculturist would view a learner involved in such an activity as appropriating or internalising a cultural form and be faced with having to explain how the cultural form becomes cognitive. The cognitive theorist will circumvent this by not stressing internalisation of the cultural form but rather stressing that a learner reorganises his or her own activity. As a result, there would be no need to conceive a process of internalisation from a cultural plane to a cognitive plane. Thus the sociocultural perspective approach to learning complements the cognitive perspective and vice versa.

2.2.2 SYMBOLIC INTERACTIONISM

The notion of symbolic interactionism by Blumer (1969) is one of the theories underpinning the emergent approach, in particular the sociological perspective of the interpretive framework (Cobb & Yackel, 1996; Yackel & Cobb, 1996). In explaining symbolic interactionism, Blumer (1969) acknowledges it's assumed similarity to the unification of psychological and sociological interpretations of human group life and conduct. However, proclaims that this notion distinctively differs from psychological and sociological interpretations in studying human group life and conduct and hence continuously contrasts these interpretation as he lays down what constitute symbolic interactionism. Blumer (1969) indicates that, just like psychological and sociological analysis, symbolic interactionism rests in three premises. First, human beings act towards objects (inclusive of objects in their physical world and other human beings), on the basis of the meanings these objects have for them. Second, the meaning of the objects is derived from the social interactions that human beings have with one another. Third, these meanings are handled in and modified through, an interpretation process used by the human being in dealing with the objects they encounter.

In unpacking the first premise, that humans act on objects on the basis of the meaning they have to them, Blumer (1969) starts by lamenting how this premise is taken for granted by psychologists and sociologists. In putting his lamentation to the fore he writes:

> …this simple view is ignored or played down in practically all of the thought and work in contemporary social science and psychological science. Meaning is either taken for granted and thus pushed aside as unimportant or it is regarded as a mere neutral link between the factors responsible for human behaviour and this behaviour as the product of such factors. (Blumer, 1969, p. 2)

He further cautions that, ignoring the meanings of the objects which people act towards, falsifies the behaviour under study, as human behaviour should not be treated as a product of factors that play upon human beings. Thus, concludes that the position of symbolic interactionism is that, “meanings that things [objects] have for human beings are central in their own right.” (p. 3). This view is understated by both psychologists and sociologists as illustrated, hence the notion of symbolic interactionism. To understand succinctly, how this view distinctively differs from that of psychologists and sociologists, Blumer (1969) asserts that all three premises should be taken into consideration as they have an interplay. Hence, I will first discuss the other two premises before presenting my understanding of the implications symbolic interactionism hold for analysing classroom practice.

The second premise is concerned sources of meaning and the process of meaning formation. Blumer (1969) states that meanings of objects is traditionally accounted for as being intrinsic to the object that has it or being a natural part of the objective makeup of the object. The examples he cites are similar to stating that a book being a book in itself and a calculator being a calculator, a pen a pen, and so forth. He then argues that, these sources of meaning, which are the objects themselves only involves a mere observation, and as such are not reflective of the process involved in the formation of meaning. He posits that this view is entrenched in traditional social and psychological sciences. Therefore, he indicates that symbolic interactionism views meaning as having a different source and also acknowledges meaning as being a process. He concludes on this thought by saying:

> …it [symbolic interactionism] sees meaning as arising in the process of interaction between people. The meaning of a thing [object] for a person grows out of the ways in which other persons act toward the person with regard to the thing [object]. Their actions operate to define the thing for the person. Thus, symbolic interactionism sees meaning as social products, as creations that are formed in and through the defining activities of people as they interact. (p. 4 – 5)

The third premise is concerned with how meanings are handled in and modified through, an interpretation process which has two steps. Firstly, actors pointing out objects which have meaning to themselves, which indicates them the objects that they are acting towards. “The making of such indications is an internalised social process in that the actor is interacting with the self” (Blumer, 1969, p. 5). Blumer (1969) clarifies this interactions with the self as different from those in psychology which refers to the interplay between psychological elements, and points out that instead he refers to a person engaging in a process of communicating with himself.

Secondly, “the actor selects, checks, suspends, regroups and transforms the meaning in the light of the situation in which he is placed and the direction of his actions" (Blumer, 1969, p. 5). This results in a formative process in which meanings are used and revised for guidance and formation of action. Blumer (1969) cautions that even though meaning of things is formed and derived by a person from social interactions, "it is a mistake to think that the use of meaning by a person is but an application of the meaning so derived" (p. 5). This caution echoes his assertion that the use of meanings in a person's actions is a result of an interpretation process explained earlier.

The general perspective of symbolic interactionism is that in studying human life, one must see the activities of the collectively as being formed through a process of designation and interpretation. Therefore, for this study, the classroom participants (myself and the learners) formed a human society of the classroom community. Then, from a symbolic interactionism perspective the human society is viewed as people engaged in living (in our case learning). The learning, is a process of ongoing activity in which lines of action in the countless situations encountered by the participants are developed. The learners and I were caught up in a vast process of interaction in which we had to fit our developing actions to one another. This process of interaction consists in making indications to others of what to do and interpreting the indications as made by others (Blumer, 1969).

Our classroom social interactions too place in world of (mathematical) objects (and ourselves as objects) which guided our orientation and action by the meaning of these of objects. These objects, were formed, sustained, weakened, and transformed as we interacted with one another. Blumer (1969) asserts that "this process should be seen, of course, in the differentiated character which it necessarily has by virtue of the fact that people cluster in different groups, belong to different associations, and occupy different positions" (p. 21). This means our interactions differed in approach as a result of a learner interacting with another at their desk, during whole class discussion or interacting with me.

2.3 INTERPRETIVE FRAMEWORK

An emergent approach by Cobb and Yackel (1996), which is the interpretive framework that guided data analysis is summarised in Figure 2 – 1 below and consists of two interrelated perspectives; the social perspective and the psychological perspective. The psychological perspective is underpinned by constructivism whereas the social perspective is underpinned by symbolic interactionism. This approach allowed for a coordinated analysis of collective classroom processes (social perspective) and the individual learners' activity (psychological perspective) as they participated in, and contributed to, the development of these collective processes. Each of the two perspective has three constructs which have an assumed inter-relationship with one another.

Cobb and Yackel (1996) recommend this interpretive framework which views learning as a "process that occurs while participating in, and contributing to, the practices of the local community" (p. 185). This makes an indirect link between the individual and the social because participation can enable and constrain learning but does not determine learning. But with this approach the unit of analysis is a local community such as a classroom.

Furthermore the emergent approach views communication as a

> process of mutual adaptation that gives rise to shifts and slides of meaning as the teacher and students coordinate their individual activities in the process of constituting the practices of the classroom activity (Cobb & Yackel, 1996, p. 186).

A teacher's role is therefore not only to proactively support learners' individual constructions but also to proactively support the evolution of mathematical practices. This, according to Cobb and Yackel (1996), will enable learners to gradually participate effectively in the mathematical practices of the wider society.

Figure 2 – 1 below represents this framework that explicitly coordinates two distinct viewpoints on classroom activity. There is an assumed relationship between each construct in the social perspective and the corresponding construct in the psychological perspective. For example, looking at classroom social norms raw means that "a teacher who initiates and guides the renegotiation of classroom social norms is simultaneously supporting the individual students' reorganisation of the corresponding beliefs" (Cobb & Yackel, 1996, p. 177).

SOCIAL PERSPECTIVE	PSYCHOLOGICAL PERSPECTIVE
Classroom social norms	Beliefs about own role, others' role, and general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Figure 2 – 1: An interpretive framework for analysing individual and collective activity at classroom level (Cobb & Yackel, 1996, p. 177).

Cobb and Yackel (1996) emphasised that "neither the social norms nor individual students' beliefs are given primacy over the other" (p. 178). They argued that social norms and beliefs should be seen as reflexive and, as a result, cannot exist independently of each other. The same applies for all other constructs, which implies that mathematical beliefs and values develop concurrently with sociomathematical norms. Therefore, it is neither a case of a change in sociomathematical norms causing a change in individual learners' mathematical beliefs and values nor a case of learners first reorganising their mathematical beliefs and values and then contributing to the evolution of sociomathematical norms.

During the analysis I constructed arguments about the constitution of social and sociomathematical norms considering the social perspective of the emergent approach as this approach deals with patterns of participation. The psychological perspective, on the other hand, was considered when I accounted for how the constitution of sociomathematical norms promoted the learners' proficiency in mathematics. This, however, should not be interpreted to mean that the social perspective influences the psychological perspective but, just as my approach to analysis, because the two perspectives coexist.

2.3.1 CLASSROOM SOCIAL NORMS

Classroom social norms are concerned with explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreements and questioning alternatives in situations which a conflict regarding interpretations or solutions exists (Cobb & Yackel, 1996). According to Bowers, Cobb, and McClain (1999), social norms concern individuals' obligations and expectations regarding participation in the classroom. Examples of social norms include explaining solutions, making sense of explanations that are given by others, indicating understanding and non-understanding, and questioning alternatives in cases where there is conflict in interpretations.

> Social norms are not psychological processes or entities that can be attributed to any particular individual. Instead, they characterize regularities in communal or collective classroom activity and are considered to be jointly established by the teacher and students as members of the classroom community. (Cobb and Yackel, 1996, p. 178).

This would then mean that both teachers and learners contribute to the way these norms evolve as they reorganise their beliefs (Bowers, et al., 1999). This evolution takes place as learners reorganise their beliefs and conversely the reorganisation of these beliefs is enabled and constrained by evolving classroom social norms (Cobb & Yackel, 1996).

In the social analysis I explicated how these and other classroom social norms, which emerged in the classroom, were constituted and enacted. In the psychological analysis I focused on individual learners' activity as they participated in these normative aspects of their classroom and highlighted the reorganisation of their beliefs in process. Doing this, I was aligning to Cobb and Yackel's (1996) assertion that "classroom social norms are seen to evolve as students reorganise their beliefs, and, conversely, the reorganisation of these beliefs is seen to be enabled and constrained by evolving social norms" (p. 178).

2.3.2 CLASSROOM SOCIOMATHEMATICAL NORMS

Unlike the social norms that are applicable to any classroom in which learning is taking place, sociomathematical norms are specific to mathematics. Sociomathematical norms, according to Cobb and Yackel (1996), include what counts as different, sophisticated, efficient mathematical solutions, and an acceptable mathematical explanation. They are useful in revealing the type of intellectual autonomy that is fostered by teachers in mathematics classrooms. As teachers guide their learners' reorganisation of their sociomathematical norms, they are at the same time preparing a framework that should support their learners' development of specifically mathematical norms (Bowers, et al., 1999).

These sociomathematical norms which emerged in the classroom co-developed with learners' mathematical beliefs and values. It is the analysis of learners' reorganisations of their mathematical beliefs and values that allowed for an account of their mathematical proficiency to be made. The analysis of the renegotiation of the sociomathematical norms allowed for documentation of how they were continuously constituted.

2.3.3 CLASSROOM MATHEMATICAL PRACTICES

Classroom mathematical practices constitute the immediate, local situations of learner's development (Cobb & Yackel, 1996) and correlate with students' ways of interpreting and solving specific instructional civilities. The analysis of these practices documents how social situations in classrooms, where learners develop, learn and evolve. This evolution takes place as learners and their teacher discuss problems and solutions, and also evolve by means of symbolising, arguing and validating specific task situations (Bowers, et al.,1999). Cobb and Yackel (1996) observed that with the emergent approach, learners contribute to the evolution of classroom practices as they reorganise their individual mathematical activity. Conversely, students' reorganisations are also enabled and constrained by their participation in the mathematical practices.

Therefore, the psychological analysis brings to the fore two aspects; heterogeneity in the activities of members of a classroom community and qualitative differences in individual learners' mathematical interpretations, even as they participate in the same mathematical practice (Cobb & Yackel, 1996). These qualitative differences in learners' mathematical interpretations account for learners' mathematical development which, in this study, referred to the achievement of mathematical proficiency strands. The social analysis, on the other hand, brings to the fore "taken-as-shared" meanings that are jointly established as teachers and learners coordinate their individual activities (Cobb & Yackel, 1996).

2.4 SUMMARY

In this chapter I first discussed two theories which underpinned the interpretive framework by Cobb and Yackel (1996) which I adopted as a lens through which I analysed data. Thereafter, I presented the interpretive framework together with its two perspectives which have an assumed relation. Finally, explained the individual constructs and explained how they each guided the analysis of data.

CHAPTER 3: STANCE ON MATHEMATICS LEARNING

3.1 INTRODUCTION

In this chapter, I present my stand point with regard to the learning (and teaching) of mathematics and illustrate how it governs my classroom practice. This presentation affords the reader the opportunity to create an image of the classroom environment in which data for this study were constructed. I begin this presentation by discussing an argument on mathematics classroom practices being guided by theory. Then present the sociocultural theory (Vygotsky, 1978) which I use as referent in my own classroom. Thereafter I present the forms of mathematics classroom interactions which typify my classroom as a result of the guiding theory and finally conclude the chapter by explicating how learners' mathematical development is accounted for in my classroom and in this study.

3.2 CLASSROOM-BASED RESEARCH

In conducting classroom-based research (whether own classroom or that of others) teachers' position about what reality is and how knowledge is constructed needs to be taken into account. This is to make sure that a researcher does not fault teachers for lack of particular aspects in the classroom which do not apply to teachers' world view. In a way, this provides a context through which the classroom environment must be understood and critiqued.

The sentiments alluded above are supported by various classroom-based qualitative studies particularly in mathematics education which explicitly acknowledge the theory used as a referent in the classrooms which are studied. Though all classroom-based, these studies differ in their research foci, some investigated teachers' actions (Chen, 2002; Haney & MacArthur, 2002; Kinnucan-

Welsch & Jenlink, 1998), whereas others assessed students' constructions (Pirie & Martin, 2000) and interactions (Yackel & Bowers, 1997) and others investigated student-teacher interactions (McClain & Cobb, 2001; Simon, 1995). Even though all the studies cited here focused on particular theories, the researchers indicated that the classrooms in which they were conducted followed constructivism as referent. In providing more details on this point, I consider classroom-based studies from the last two categories, as the study reported here focused on social and sociomathematical norms which are constituted during classroom interactions and such interactions involve both learners' interactions and learner-teacher interactions.

Pierie and Martin's (2000) study focused on the notion of *folding back* and *collecting* as described by Pirie and Kieren (1994) in their theory on the growth of understanding. Yackel and Bowers' (1997) study focused on students' development of conceptual foundation for place value numeration, their study involved two third grade classrooms. Simon (1995) studied his own classroom and analysed his teaching of mathematics to preservice teachers in order to develop a model of mathematics teaching informed by a social constructivist view. McClain and Cobb (2001), on the other hand identified aspects of effective reform teaching by analysing the instructional practices of a first grade teacher. In the study reported here, I studied the sociomathematical norms constituted for promoting learners' proficiency in mathematics in my 'own' classroom which followed sociocultural theory as a referent.

3.3 SOCIOCULTURAL THEORY

In this thesis I report on the sociomathematical norms constituted for promoting learners' proficiency in mathematics as explicated in Chapter 1. The context in which sociomathematical norms are to be understood for this study is foreshadowed in Chapter 2 and discussed at length in Chapter 4. As highlighted earlier in this chapter, with classroom based research, teachers' paradigm must be acknowledged. It is for this reason that a shared understanding of a social learning theory, which guided the classroom in which data were generated, be developed.

The sociocultural theory by Vygotsky (1978) was adopted as a referent for classroom practice. Hence, in this section I explicate what it means to learn mathematics within a sociocultural perspective. This is the case because the constitution of sociomathematical norms in the reported study was documented during learners' mathematics learning experience. Therefore, an understanding of what it means to learn mathematics needed to be gained.

In presenting the sociocultural theory which elucidates the relationship between learning and development, Vygotsky (1978) started by acknowledging existing conceptions of this relationship. He argued that the relationship can be summarised through three major theoretical positions. Firstly, development and learning are independent of each other. Secondly, learning is development. The third position is a combination of the first and second positions. Vygotsky used this third position, which he explicitly rejected, to build his theory on, gave rise to his argument about the zone of proximal development (ZPD). He regarded ZPD as the distance between what learners can do on their own and with the assistance of their peers or teachers. I return to this in more detail later in this section and also explain its implications to teaching and learning.

Vygotsky (1978), regarded learning as the acquisition of abilities to think about a variety of things. Furthermore, he argued that the environment should be the starting point of learning, and that learner-centred learning should be designed within learners' zone of proximal development. On the other hand, he rejected assumptions and views that learning is purely external and that having learned one thing, leads to being able to do other things which are entirely unrelated. As a result, he asserted that "human learning presupposes a specific social nature".

Consequently, this social nature which learning depends on, leads to the sociocultural learning theory.

From a sociocultural perspective, individual learning takes place as a result of social interactions and culture (Cobb & Yackel, 1996). Culture refers to both the classroom culture and the culture of learners within and outside of their classroom. Cobb and Yackel (1996) state that culture contributes to learning, while learners also contribute to the culture in the classroom. This means that learners and the culture of the classroom have an influence over each other. Social interactions, on the other hand, refer to the conversations (in this case mathematical conversations) that learners engage in with each other or with a teacher. It is this view of learning that was adopted in the reported study.

Furthermore, Cobb and Yackel (1996) define learning as "a constructive process that occurs while participating in and contributing to the practices of the local community" (p. 185). They, however, make it clear in their work that the construction of an idea will be different for each learner even within the same environment. For me this means that the mental processes (cognition) that learners engage in will be different. Cobb and Yackel's (1996) view of learning corresponds with that of Vygotsky. Then again, Vygotsky posited that the mind does not, and cannot, exist outside the social practices (Packer & Goicoechea, 2000). As such, learners' mental processes are not the only aspects of learning which are significant, but also the social interaction which triggers such processes.

Vygotsky's sociocultural theory has three major attributes concerning learning. Firstly, cognition exists between and among people in social learning settings, and that, from these social settings, learners move ideas into their psychological realm (Forman, 2003). Secondly, the way in which information is learned depends on whether it was within a learner's zone of proximal development (ZPD) or not, which represents the difference between learner's assisted and unassisted performance on a task (Vygotsky, 1978). Thirdly, the way in which information moves from the social plane to the individual plan, which is referred to as semiotic mediation. Semiotic mediation, according to Forman (2003), refers to interaction through language, diagrams, pictures and actions. In view of this, I considered Cobb and Bowers' (1999) view of learning as "a process in which learners actively reorganise their ways of participating in classroom practices" (p. 9) to be more meaningful. Therefore, in summary, learning involves learners, social interactions in the classroom and the culture within and beyond the classroom. This is the view of learning which the present study adopted. Therefore, the three overarching constructs which underpin Vygotsky's sociocultural theory are; *zone of proximal development* (ZPD), *scaffolding* and *semiotic mediation*, which I explicate in sections that follow.

3.3.1 Zone of Proximal Development

The ZPD is defined as "the distance between the actual development level as determined by independent problem solving and level of potential development as determined through problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Though there are various models in literature which attempt to depict the ZPD, I found that the model by McLeod (2018) captured it best. McLeod's (2018) model of ZPD (Figure 3 – 1) also incorporates an illustration of how learning progresses within the ZPD, an element that is not included in other models found in literature.

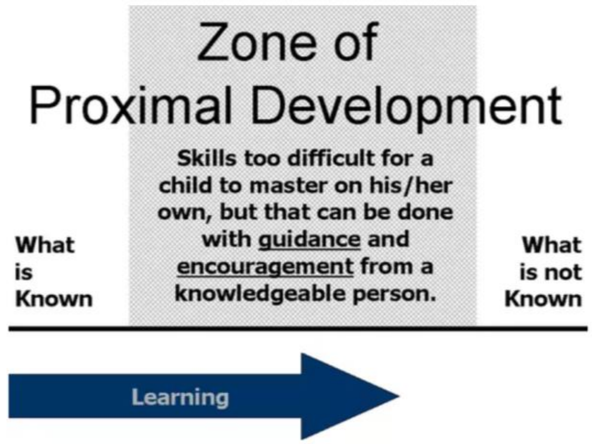


Figure 3 – 1: A Model of the Zone of Proximal Development (McLeod, 2018, p. 1)

Learning starts from what learners already know. For this study, which took place in a Grade 11 mathematics classroom, this notion refers to learners' mathematical knowledge developed from previous grades, their pre-conceptions as well as their misconceptions. Learning takes place only when there is a shift in learners' ability to perform tasks with assistance and, later, without assistance, as well as their ability to move towards learning new things. Vinney (2019) argues that, if ZPD is applied as a reciprocal teaching strategy, then it leads to learners executing four skills, namely, summarising, questioning, clarifying and predicting. Initially these skills are displayed or modelled by teachers and, gradually, learners take over the responsibility for utilising these skills themselves (Vinney, 2019). Hence, success in learning within the ZPD requires the presence of someone with the requisite knowledge and skills to guide learners and to create the opportunities for social interactions that allow learners to observe and practice their skills (Cherry, 2019).

In this study, social and sociomathematical norms were constituted through my guidance and eventually learners enacted these norms without my guidance. For example, during the initial stages of data collection I was the one who called on learners to share their solutions, which led to me and other learners challenging the presented solutions and learners who presented clarifying their solutions. As time passed this situation of challenges and justification became the norm in the classroom, in that it was enacted by learners and dominated their discourse. When teaching is based on the ZPD then teachers plays the role of providing scaffolds and mediating learning for learners (Cherry, 2019; Christmas, Kudzai, & Josiah, 2013). The notions of scaffolding and mediation are discussed in detail in the following sections.

3.3.2 Scaffolding

Scaffolding refers to the activities provided by a teacher, or a more competent peer, to support a learner (McLeod, 2018) who is attempting to learn something new in the zone of proximal development (Vinney, 2019). Even though, scaffolding is almost synonymous to ZPD (McLeod, 2018; Vinney, 2019) it should be noted that scaffolding is not mentioned in Vygotsky's writing. This term was coined by Wood, Bruner and Ross in 1976 in an attempt to operationalise the concept of teaching within the ZPD. They asserted that:

> Scaffolding is a process that enables a child or novice to solve a task or achieve a goal that would be beyond his unassisted efforts. The scaffolds require the adults controlling those elements of the task that are initially beyond the learner's capability, thus permitting them to concentrate upon and complete only those elements that are within his range of competence (Wood et al., 1976, p. 90).

Providing learners with support for learning within their ZPD but gradually removing the support (scaffold) and allowing learners to take responsibility of their own learning is at the core of scaffolding. This support is what van de Pol, Volman and Beishuizen (2010) refer to as *contingency*, which also refers to "responsive, tailored, differentiated, titrated, or calibrated support" (p. 4). Van de Pol et al. (2010) further argued that scaffolding is a participatory interactive stage and involves interactions between a teacher and a learner, where both must be actively engaged. Figure 3 – 2 presents a conceptual model for scaffolding by van de Pol and colleagues (2010). Teaching and learning starts in Time 1 where a teacher offers great support and a learner takes minimal responsibility for learning. As time progresses to Time 2, the support provided by a teacher fades away and learners' responsibilities grow. This transfer of responsibility is facilitated through *contingency*.

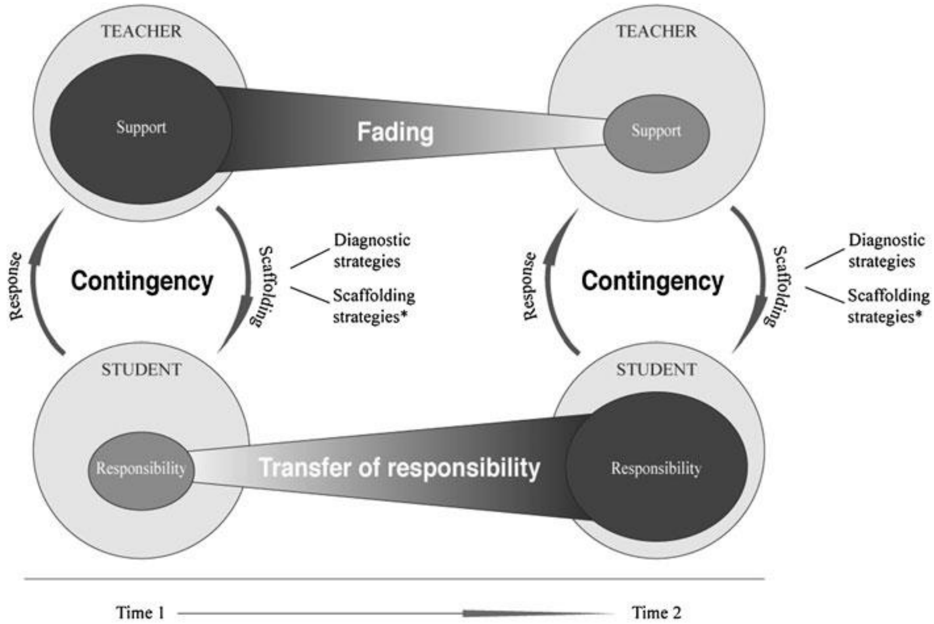


Figure 3 – 2: Conceptual model of scaffolding (van de Pol et al., 2010, p. 274)

For this study the goals of scaffolding were twofold. Firstly, scaffolding was used for learners to establish sociomathematical norms in order to make meaningful learning of mathematics possible. The establishment of such norms, in turn, resulted in the creation of a mathematical classroom discourse that afforded mathematics-learning opportunities to learners. Secondly, scaffolding was used to provide a premise from which to analyse how the establishment and enactment of sociomathematical norms promoted learners' proficiency in mathematics. Correspondingly, scaffolding explains the social and participatory nature of teaching and learning as it occurs within the ZPD (Christmas et al., 2013). Furthermore, scaffolding is an instructional structure in which teachers model the desired goal, and gradually shifts the responsibility to learners (Christmas et al., 2013).

During teaching and learning, the responsibility of initiating challenges and justification was fully mine, and learners' responsibility was limited to clarifying their solutions. This is an indication of Time 1, in accordance with van de Pol et al.'s (2010) model. As time went by, gradually some learners started taking the responsibility of sharing their solutions, and then demanded that other learners justify and clarify their solutions. This is indicative of the fading nature of support by a teacher and growth in responsibility by learners. Finally, the responsibility for initiating the challenges and justification was taken by the majority of learners in the classroom. This corresponds to Time 2 on the model, where a teacher's support in minimal. During this participatory interactive stage, I was able to support learners towards the development of normative aspects about what counts as acceptable mathematical explanations, reasons and justifications. When the scaffold was removed learners continued to enact the constituted norms.

Teaching within the ZPD is undertaken in order to foster the development of knowledge and/or skills (Christmas et al., 2013). In this study teaching within the ZPD was undertaken to foster the development mathematical knowledge and interactions skills, which lead to meaningful learning of mathematics. At the stage where learners take on more responsibility towards the development of the envisaged knowledge and skills, teachers can remove the scaffold (Christmas et al., 2013). This is an indication that a learner has benefited from the period of assisted learning and has internalised the desired goals. As result learners are able to perform the desired tasks without assistance. Since learning is continuous, it means after such achievement learners' ZPD will move forward on a learning continuum and a teacher will continue to build scaffolds towards achieving the 'new' goals. In this study the scaffolds were removed and learners continued to renegotiate and re-establish ‘new’ sociomathematical norms, which were later widely endorsed by the whole class and were enacted in learners’ mathematical classroom discourse.

3.3.3 Semiotic Mediation

Dixon-Krauss (1996) emphasised the notion that, “from a Vygotskian perspective, the teacher’s role is mediating the child’s learning activity as they share knowledge through social interaction” (p. 18). Vygotsky refered to mediation as the role played by more knowledgeable others (teachers or more competent peers) in selecting and shaping learning experiences presented to learners (Christmas et al., 2013). This process also underpins the movement of information from the social plane to individual learners’ cognitive realm and involves the use of tools or artefacts (Forman, 2003). These artefacts, which Vygotsky also refers to as mediators (Christmas et al., 2013), are enablers of learning, including, among others, language, diagrams, pictures and actions (Forman, 2003). The artefacts are of great significance to sociocultural theory as they are the constitutive part of thinking and sensing (Leung & Bolite-Frant, 2015) and assist learners to move into and through their ZPD (Chirstmas et al., 2013).

Bussi, Corni, Mariani and Falcade (2012) (see Figure 3 – 3), modelled semiotic mediation to illustrate the relationship between the use of artefacts (tools) and learning (Leung & Bolite-Frant, 2015). The lower part of the model deals with mathematical knowledge to be mediated by teachers. In this study the envisaged mathematical knowledge was summarised into three broad areas, namely, mathematical explanations, mathematical justifications and mathematical reasons. Considering the knowledge to be taught (bottom left vertex of the model), I designed learning activities (Tasks – top left vertex of the model) which were presented in the form of mathematical texts (bottom right vertex of the model) through the use of artefacts. Subsequently, learners engaged in carrying out the learning activities, through participation, interpreting the mathematical text. Hence, this led to learners using the artefacts in order to produce their own texts and arguments (top right vertex of the model), situated within the task and mathematical texts.

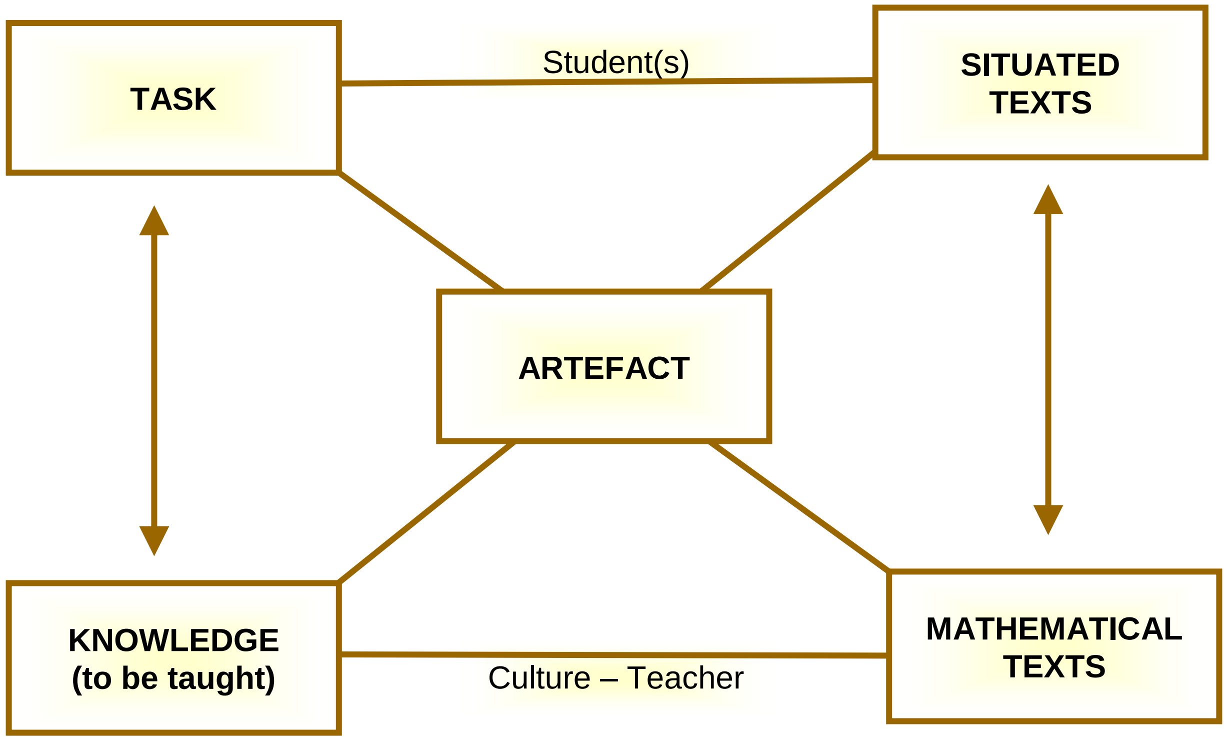


Figure 3 – 3: Elements and links between the elements of the theoretical constructs of semiotic mediation (Bussi et al., 2012, p. 6)

Christmas and colleagues (2013) raised two broad questions. It is imperative that both questions are answered in attempting to make sense of semiotic mediation. Firstly, “what kind of involvement on the part of the adult (knowledgeable other) is effective in enhancing the child’s performance?” Secondly, “what changes in the child’s performance can be brought about by the introduction of the child to symbolic tools and mediators such as language and learning media?” (p. 373). These questions are, to some extent, aligned to the purpose of this study in that the study was concerned with how learners constitute and enact the sociomathematical norms engendered by a teacher. In addition, I, as the researcher, was concerned about how the development and enactment of such sociomathematical norms would promote learners' proficiency in mathematics.

Semiotic mediation was, therefore, pivotal to lesson preparation during the study. I became conscious of the artefacts which learners would use in order to learn the desired mathematical knowledge and skills. Even though the choice of mathematical content was dictated by the work schedule provided in the mathematics policy document, the design of learning activities was guided by the notion of semiotic mediation. Mathematical activities were designed and presented as mathematical texts and symbols, which served as tools through which learners' learning was mediated. Therefore, as learners engaged with the tasks, they produced mathematical texts which were situated within the tasks, which afforded me an opportunity to come to come to grips with, and challenge, what initially counted as mathematically acceptable explanations, reasons and justifications. Through continued interaction with the learners a 'taken-as-shared' meaning of acceptable mathematical explanations, reasons and justification was constituted.

3.4 CLASSROOM INTERACTIONS

The earlier sections of this chapter portrayed classroom social interactions as the overarching issue. In discussing on classroom-based research, the studies cited involved classroom social interactions and Vygotsky also believed that interactions are an essential part of the learning process (Cherry, 2019). As a result, the scaffolding and mediation processes take place through various interactions that occur within learners' zone of proximal development. In this section I start by summarising the social interactions in classroom based studies which used constructivism as referent, then present the social interactions for the classroom in which data were generated for the study reported here.

In Simon's (1995) study, social interactions revealed themselves in how he was engaging his students in ways that led to mutual adaptation of both his knowing and that of students. In this regard, the role of social interactions became that of offering individual participants means of reflecting on and adjusting their own individual conceptual systems in view of what was transpiring during the interactions. Social interactions in this context not only become means of resolving common problems but also become learning experiences in their own right.

Meanwhile, Pirie and Kieren's (1994) study offered a detailed description of how, through their notion of the complementarities of acting and expressing, social interactions between a teacher and a student can contribute towards the student's growth in understanding. Their study, therefore, is unique in that it helps clarify some ways in which teachers can attempt to make sense of the covert forms of interactions that students engage themselves in during their active knowing activities. Pirie and Martin's (2000) study, on the hand, offered a practical perspective of how those ideas of acting and expressing can be used in a classroom learning environment. By listening carefully to students' voices whilst also taking note of tone, gestures, and all other forms of body language, the two researchers could make more sense of the status of the students' active knowing activities. At the end of the day all overt forms of interactions are useful only if they facilitated those covert forms of interactions that take place in our cognitions.

These studies in different ways assisted me in developing an awareness of various forms of social interactions that could take place in a classroom, and the opportunities for learning that they offer. In describing the classroom social interactions which took place in my classroom, Masha's (2004) model of classroom interactions best captured them. Even though, he developed the model while his interest was on learners' formation of self-identities in relation to mathematics as they engaged in the different forms of interactions, for this study the model is adopted for a different purpose. It serves to explicate the different classroom social interactions which learners in my classroom engage in during the learning of mathematics. My study is concerned with constitution of sociomathematical norms, therefore at the centre learners would be reformulating their identities with regard to participation in classroom mathematical activities, this argument fits in well with the theoretical discussions detailed in Chapter 2 earlier.

During teaching and learning, four types of interactions occurred in the mathematics classroom in which data for the study was generated. These interactions were; *student-material*, *student-student*, *student-teacher* and *whole class* interactions, as modelled by Masha (2004) in Figure 3 – 4 below. Through the model, it can be deduced that teaching and learning were mainly driven by learners' interpretations of the content. Furthermore, it can be deduced that lessons were highly interactive, allowing learners to continuously reflect on their learning and the adequacy of the learning material. This was the case because learning was conceptualised as a socially constructed activity that required an agent, a committed human being who makes the decision of engaging themselves in the activity of learning (Stentoft & Valero, 2010). To make these deductions clear, I discuss each of the four types of interactions that occurred in more details, care should be taken that the order in which the interactions are discussed is not an implication of how they unfold in the classroom. These interactions do not occur in a linear order and in some instances a certain form of interactions becomes a subset of the other form.

3.4.1 Student-Material Interaction

The material given to learners was designed in such a way that concept formation is driven by means of questions. A series of questions, ranging from those easily accessible to the more advanced questions, were raised. In the process learners were asked a number of reflective questions that required them to generate theories on what they were learning. To this end, semiotic mediation and scaffolding took place. In addition, it should be notable that the various interactions did not unfold independently of each but, instead, were interconnected. Subsequently, a number of questions were raised that required learners to put into practice their emerging theories.

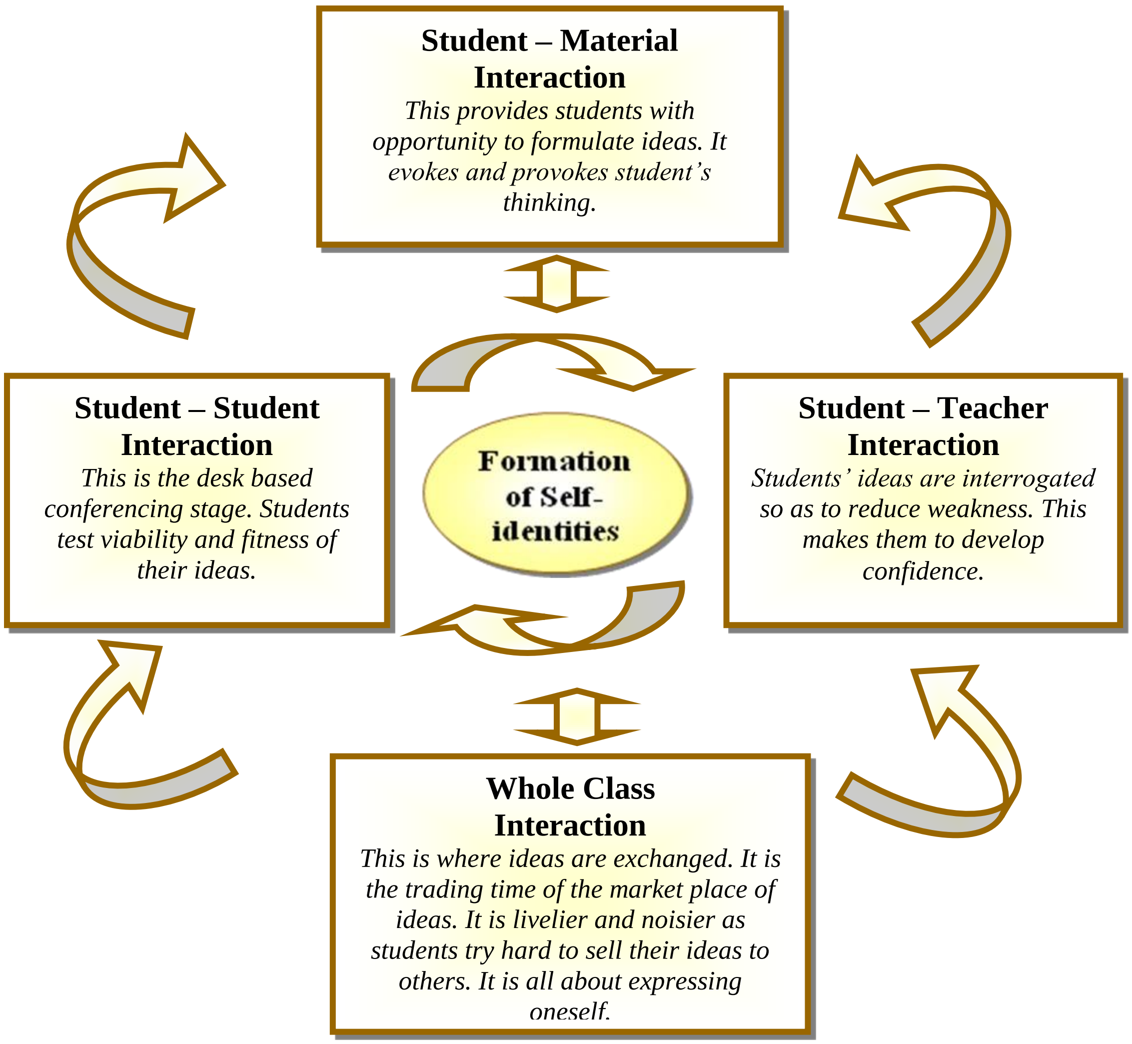


Figure 3 – 4: Classroom interactions (Masha, 2004)

These engagements with learning material (or mathematical texts) created an opportunity for mathematical norms to be learned implicitly, and for learners to depict their own mathematical norms. Since these norms are discipline based, some were acceptable, some partially acceptable and required reorganisation, whereas others were not acceptable at all, as they did not conform to the mathematics disciplinary knowledge.

Interacting with material, made each learner become aware of their own thoughts, ideas, position, and challenges. This served as the point of departure in engaging with the student-student interaction stage or with the student-teacher interaction stage. This entry into engagement became apparent when learners shared their solutions to the various problems given in the learning material. As they shared their attempts to find solutions, an opportunity for other learners and me to challenge these solutions was created and learners were expected to justify their solutions. This situation of challenges and justifications which unfolded, facilitated the constitution of social and sociomathematical norms.

3.4.2 Student-Student Interaction

Engaging in student-student interaction learners had opportunities to test their emerging ideas. While always ready to engage in a robust debate, the learners were also ready to reformulate or adapt to new ideas that come as a result of this interaction. It was at this stage that their mathematical communication skills were put to a robust test. Hence, sociomathematical norms that concerned what counts as a mathematically acceptable answer, a different answer, an explanation, reason and justification, were constituted and enacted.

3.4.3 Student-Teacher Interaction

Through the learners' responses to the questions raised in learning material, I was able to gauge the current knowledge of individual learners on the issues raised. Learners' responses acted as a foundation for engaging them in further questions that were aimed at further exploring their responses. The foundation from which I could mediate and scaffold learners' learning was also established during this stage. In some cases, opportunities for contradicting situations were provided in order to underpin the sociomathematical norms of what counts as mathematically correct and different answers. The core of this form of interactions was an opportunity to make learners more aware of their thoughts and how robust and viable these thoughts were. The idea was to inculcate reflective process into thought formation, which allowed for re-negotiation of various norms and shaped how the learners participated in the classroom discourse.

3.5 MATHEMATICAL PROFICIENCY

Kilpatrick and colleagues (2001) pioneered the notion of mathematical proficiency. In their work, they used five intertwined strands; *conceptual understanding, procedural fluency, strategic competence, adaptive reasoning* and *productive disposition* to explicate what it means to know and do mathematics. Their work also underpinned the South African Framework of Teaching Mathematics for Understanding (TMU) (DBE, 2018). This framework adopted all the strands of mathematical proficiency as outlined by Kilpatrick and colleagues (2001), with the exception of productive disposition. The mathematical proficiency strands provide the basis on which delineating what exactly are learners expected to do in demonstration of competence in knowing and doing mathematics is done.

Earlier in this thesis I argued that the constitution of sociomathematical norms is based on learners' engagement in mathematical activity undertaken towards their mathematics learning. Mathematics learning, in the context of this study, was defined as achievement of the mathematical proficiency strands (Kilpatrick et al., 2001), as outlined in the TMU framework (DBE, 2018). Kilpatrick and colleagues provided a description for each of the five intertwined strands of mathematics proficiency. Furthermore, the TMU framework represented the intertwined nature of the strands, as depicted in Figure 3 – 5 below. In the following sub sections I provide explanations of each of the strands and captures a range of learners' abilities, which I looked for during analysis, in order to account for mathematics learning, alongside with constitution and enactment of sociomathematical norms.

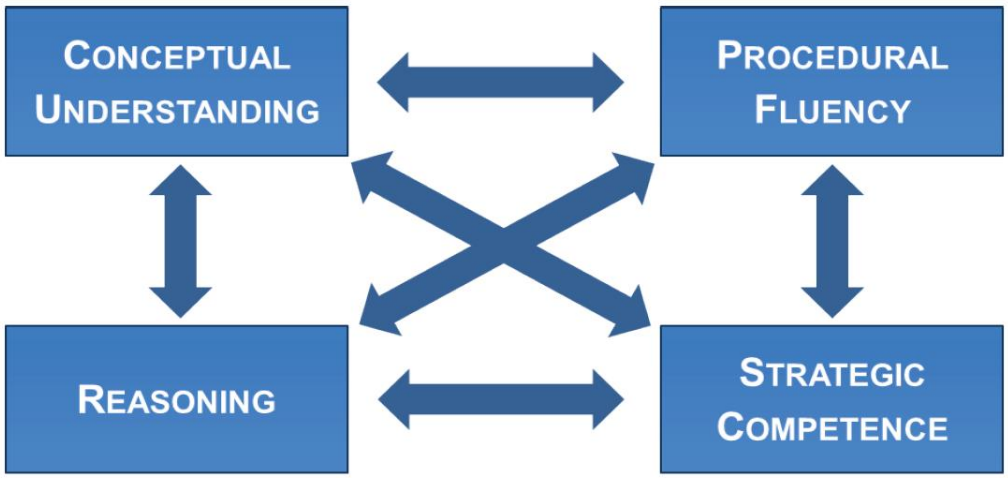


Figure 3 – 5: Model of South African Framework of Teaching Mathematics for Understanding (DBE, 2018, p. 9)

3.5.1 Conceptual understanding

Conceptual understanding refers to the "comprehension of mathematical concepts, operations and relations" (Kilpatrick et al., 2001, p. 116). The understanding referred here is synonymous with Skemp's (1976) *relational understanding*. Conceptual understanding is achieved when learners are able to; (i) see mathematics as a connected web of concepts, (ii) explain the relationship between different concepts and make links between concepts and related procedures and (iii) apply ideas and justify their thinking (DBE, 2018).

3.5.2 Procedural fluency

Procedural fluency is defined as "a skill in carrying out procedures flexibly, accurately, efficiently and appropriately" (Kilpatrick et al., 2001, p. 116). Procedural fluency it is knowledge and the use of rules and procedures in carrying out mathematical processes (Van De Walle, Karp, & Williams, 2010). As a result, procedural fluency is achieved when learners are able to (i) carryout mathematical procedures accurately and efficiently, and (ii) know when to use a particular procedure (DBE, 2018). In accounting for learners' fluency in mathematical procedures, Graven and Stott's (2012) argument for a spectrum extended from restricted procedural fluency towards elaborated and fully developed flexible fluency was taken into account.

3.5.3 Strategic competence

Kilpatrick and colleagues (2001) regarded strategic competence as the "ability to formulate, represent, and solve mathematical problems" (p. 116). Competence in this strand is demonstrated when learners are able to (i) identify and use appropriate strategies and (ii) devise their own strategies to solve mathematical problems (DBE, 2018). The ability to devise their own strategies requires of learners to have understanding of concepts involved in a particular problem, and the ability to use identified and devised strategies requires learners to be fluent in carrying out procedures (Van De Walle et al., 2010). This is indicative of the intertwined nature of the mathematical proficiency strands and supports the assertion that the strands do not exist in isolation of each other.

3.5.4 Adaptive reasoning

Adaptive reasoning is regarded as the "capacity for logical thought, reflection, explanation and justification" (Kilpatrick et al., 2001, p. 116). This strand, which involves both inductive and deductive reasoning, is achieved when learners are able to (i) explain and justify their mathematical ideas, and (ii) communicate their mathematical ideas through appropriate mathematical language and symbols (DBE, 2018). This strand involves the ability to evaluate own work and adapt it, as and when needed (Van De Walle et al., 2010).

3.5.5 Productive disposition

Even though this strand is not included in the TMU framework, it has been retained in this study for two reasons. First, it addressed one's belief in relation to mathematical activity and that resonates with the psychological perspective of Cobb and Yackel's (1996) emergent approach which was adopted as a lens through which data were analysed. Secondly, Van De Walle and colleagues (2010) argues that it is the glue that holds all the other strands together. Since, one has to develop a positive self-concept in relation to the subject for them to learn all skills and knowledge embodied by the subject.

Productive disposition refers to "a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick et al., 2001, p. 116). This strand has to do with ones' gratification as they engage in mathematical activity, and can be observed when one takes the responsibility of initiating discussions by raising and offering answers, and by accepting arguments that are efficient to them (Chuene, 2011). In so doing, one will be engaging in the process of co-constituting sociomathematical norms with the rest of the classroom community.

3.6 SUMMARY

In this chapter I argued for the need to acknowledge teachers' paradigm when conducting classroom-based research. Then presented the sociocultural theory as a referent for classroom practice (Vygotsky, 1978), and the forms of classrooms social interactions which typified the classroom from which data were generated. Finally, I discussed the five interwoven strands of mathematical proficiency (Kilpatrick et al., 2001) as standards for learners' mathematical development, supporting this discussion by referring to TMU framework (DBE, 2018) which provides a pragmatic view of notion of mathematical proficiency.

CHAPTER 4: CLASSROOM NORMS

4.1 INTRODUCTION

In this chapter I present a critical and analytical literature review on norms, in general, in order to delineate what sociomathematical norms are in the context of this study. I start this chapter by first discussing general classroom social norms and how they emerge in the classroom. I then explicate what mathematical norms, and how sociomathematical norms are constituted from the unification of both social and mathematical norms. Thereafter, I present an analysis of studies focusing on sociomathematical norms, raising arguments about their limitations and, in turn, highlighting the significance of this study. Finally, I present an argument for the need to link the development of sociomathematical norms with mathematical proficiency. The latter allows for construction of an explanation about the extent to which learners' mathematical development is advanced through development of sociomathematical norms.

4.2 CLASSROOM SOCIAL NORMS

The dictionary defines a norm as something that is usual, typical, or standard. In a classroom context, norms refer to taken-as-shared suppositions, assumptions and interpretations (Cobb, Wood, Yackel, & McNeal, 1992; Cole & Wertsch, 1996). Correspondingly, Turpen and Finkelstein (2010) argue that classroom norms are "a shared meaning system that gives sense or coherence to the community's collective activity" (p. 1). Furthermore, they assert that classroom norms are constructed through a collection of fine-grained classroom practices over time. Much and Shweder (1978) identified *regulations*, *conventions*, *morals*, *truths*, and *instructions* as five types of classroom norms. In order to make a distinction between these norms, they referred to their historicity, their source, and the consequences of transgressing them.

Regulations and conventions are historical norms; however, they differ with respect to their source and the consequences of transgressing them. Regulations are established by a specifiable authority who can alter them, while transgressing them will result in a penalty. For example, a teacher may tell learners to always work in groups of six members. This directive is established by a teacher, who has authority in the classroom and can change the directive, hence this directive is a regulation. The source for conventions, on the other hand, is not specifiable and the consequences of transgressing them is social disapproval. Unlike regulations and conventions, norms regarded as either morals, truths, or instructions are treated as being ahistorical by members of a community (Much & Shweder, 1978). The consequence of transgressing a *moral* norm is moral culpability. Cobb et al. (1992) argue that a sense of culpability is the one which differentiates morals from truths and instructions. The consequence of breaching a *truth* is error per se, whereas that of transgressing an *instruction* is ineffectiveness.

Classroom norms enacted in traditional teacher-centred classrooms differ from those enacted in reformed or inquiry based, learner-centred kinds of classrooms. In traditional classrooms learners are expected to come to grips with knowledge that a teacher already has. Correspondingly, a teacher's role is to explain and clarify, while learners' role is to try to figure out what a teacher has in mind. In reformed classrooms, on the other hand, the classroom norms enacted include; explaining and justifying one's solutions, trying to understand other learners' reasoning, asking questions if one does not understand, and challenging arguments which one does not agree with. For this study I focused on classroom norms enacted in a learning centred classroom, which was also the focus of this literature review.

In terms of identification of classroom norms during classroom discourse, Much and Shweder (1978) speak of two basic acts, which are: accusations and accounts. They argue that, during analysis of classroom discourse, attention needs to be paid to the acts and the speech of the classroom community members. In the case where one member acts or speaks in such way that would suggest that another member has transgressed a rule, this is an accusation. Subsequently, the other member will have to account for the act or speech which they are being accused of. Much and Shweder coined this phrase to describe these situations as *situations of accountability*, and noted that these situations are representative of the normative aspects of the classroom. In trying to explicate Much and Shweder's notion of situations of accountability, Cobb et. al. (1992), speak of *situations for justification* where the two basic acts involved are challenges and justifications. This is the case where one member challenges other members' solutions to questions/activities, subsequently, the other member will have to justify their attempt. It is through this process, as modelled in Figure 4 – 1 hereunder, that classroom norms enacted in a learning centred classroom, as highlighted earlier, will emerge.

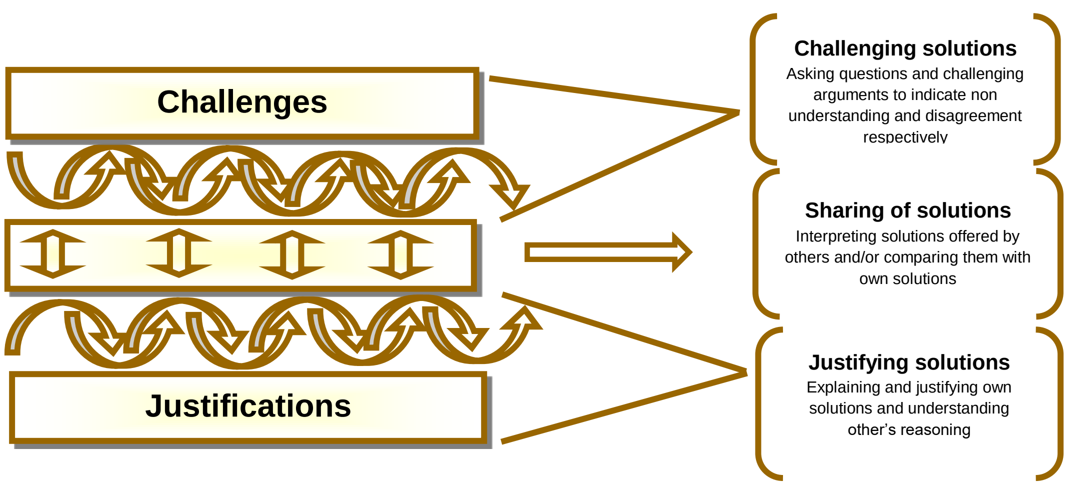


Figure 4 – 1: The emergence of classroom norms

The emergence of classroom norms as modelled in Figure 4 – 1 involves three intertwined stages which do not occur in a linear manner. During these stages learners continuously engage in a process of reflection. The first stage is that of sharing solutions. This is the stage where learners explain their solutions in order to clarify, and not to justify, them, while other learners interpret these solutions and compare them to their own. During this stage, normative aspects of what counts as correct, acceptable and different answers emerge. The situation for justifications arises through learners' reflective sharing and comparison of solutions. Here learners challenge presented solutions by asking questions when they do not understand and challenging arguments to indicate disagreement. During this stage, the normative aspects of interpreting, questioning and challenging other's responses emerge. Finally, learners, whose solutions are challenged, attempt to understand the reasoning of others and provide justifications for their own solutions. It should be noted that the moment of sharing is pivotal and exists throughout learners' interactions, regardless of whether they are challenging or justifying solutions. This process continues until consensus is reached and a "taken-as-shared" meaning is developed and agreed upon.

Classroom norms are understood from a sociological perspective, since they are socially negotiated and collectively agreed upon (Turpen & Finkelstein, 2010). Hence, they are often referred to as social norms which are concerned with an individual's obligations and expectations while participating in classroom social interactions (Bowers, Cobb, & McClain, 1999; McClain & Cobb, 2001; Yackel et al., 2000). These obligations and expectations during classroom participation constitute the classroom culture. Classroom culture is learned implicitly through observation and participation, and not by deliberate study. Therefore, understanding norms from a sociocultural perspective is more pertinent than perceiving them as being only sociological.

Classroom social norms are not specific to mathematics and, as such they can be enacted in a classroom for any other subject (Cobb & Yackel, 1996). Examples of social norms include explaining and justifying solutions, attempting to making sense of explanations that are given by others, indicating understanding and non-understanding, and questioning alternatives in cases where there is conflict in interpretations or solutions (Cobb & Yackel, 1996; McClain & Cobb, 2001). Smith, Hillen and Catania (2007) added accountability, clarity and respect as norms which might be established and enacted as learners generate and share multiple solutions to provided tasks. All the examples cited here outline patterns of interactions which can exist as learners engage with each other or with a teacher, regardless of the subject under discussion. Hence, these examples of what McClain and Cobb (2001) refer to as classroom participation structure.

The development of the notion of participation structures of classrooms in recent literature resulted in an emergence of new social norms, other than those cited earlier. These norms explicitly place specific demands on students or teachers, and on both, in some cases. Students are expected to make an effort to provide answers by explaining their thinking, while not embarrassing those learners who make invalid contributions (Güven & Dede, 2017; McClain & Cobb, 2001). Furthermore, students are expected to engage in sense-making (Pang, 2005) and take the responsibility of initiating discussions by raising and offering answers, and by accepting arguments that are efficient to them (Chuene, 2011). Teachers, on the other hand, are expected to comment on, or to re-describe students' contributions, and to build their arguments on students' ideas (McClain & Cobb, 2001; Wadjaja, 2012). Finally, both students and teachers are expected to carefully use the language of teaching and learning, and that of the discipline; and to strive towards achieving getting good grades (Güven & Dede, 2017).

The existence of social norms in every classroom arises from the patterns of interactions or structures of participation in that particular classroom. It is, therefore, accepted that social norms are not unique to mathematics classrooms. Even though social norms exist in every classroom, they are not similar, due to the different participation structures or patterns of interactions. Correspondingly; Yackel and colleagues (2000) argued that all classrooms have social norms which differ in nature and, hence, the nature of these social norms is what makes the classrooms different. Although this is the case, not every pattern of interaction or participation structure constitutes a norm, unless the pattern of action or participation structure is observed at least three times (Park, 2015). This view is supported by Turpen and Finkelstein (2010) who argued that norms are established through repeated engagement in social practices.

Similarly, Sánchez and García (2014) assumed the existence of rules in every classroom, in contrast to participation structures or patterns of interactions. They refer to these rules as structures which regulate discourse in the classroom, and concluded that these rules are regarded as norms if they are covert. Their view of norms, just like that of Cobb and Yackel (1996), is located within the interactionist perspective. In spite of this, unlike in most other studies (e.g. Chuene, 2011; Güven & Dede, 2017; Wadjaja, 2012), they adopted Sfard's perspective of a norm and not that of Cobb and Yackel. Yackel and Cobb define social norms from a sociocultural point of view.

> Social norms are not psychological process or entities that can be attributed to any particular individual. Instead, they characterise in communal or collective classroom activity and are considered to be jointly established by the teacher and students as members of the classroom community (Cobb & Yackel, 1996, p. 178).

Cobb and Yackel's (1996) view of norms is adopted in this study, since the view is underpinned from a sociocultural perspective. Correspondingly, the classroom in which data were generated for this study adopted a sociocultural perspective to teaching and learning. This theoretical perspective to teaching and learning is presented and discussed in detail in Chapter 3. Here I highlight why Cobb and Yackel's view of norms was adopted in relation to the sociocultural perspectivie to teaching and learning.

A sociocultural perspective implies comprehending the social systems of (mathematics) classrooms that are organised as shared practices, which shape, in different ways, how learners are expected, and allowed, to act and participate. This implication is in agreement with Cobb and Yackel's definition of learning as "a constructive process that occurs while participating in and contributing to the practices of the local community" (p. 185). Furthermore, Cobb and Bowers (1999) argue that learning is "a process in which learners actively reorganise their ways of participating in classroom practices" (p. 9). In short, from a sociocultural perspective, learning is dependent on other learners, teachers, the social interactions in the classroom, and the culture within the classroom.

4.3 MATHEMATICAL NORMS

There is a distinction between the mathematical norms discussed here and sociomathematical norms discussed in the section that follows. This distinction is suggested by the use of these two terms, mathematical and sociomathematical norms, by different authors in literature. Some authors use mathematical or sociomathematical norms, in particular, without mentioning the other. While other authors use both terms with an 'or' in between, implying that they mean the same thing. However yet other authors use the terms with an 'and' in between, implying that they mean different things. It is the covert, multiple meanings attached to these two terms which made it important for me to attempt to delineate them for the purpose of this study.

Sekiguchi (2005) states that, in their work, they used the term mathematical norms instead of the term sociomathematical norms, arguing that mathematics is intrinsically sociocultural and, as such, the prefix "socio-" is redundant. The latter view suggests that the two terms refer to the same thing, and is in contrast with the view of McClain and Cobb (2001), who used the terms mathematical norms and sociomathematical norms to describe two different dimensions of classroom action. I am of the view that mathematical norms and sociomathematical norms are different. However, in this section I am not attempting to delineate the difference between the two categories of norms through comparison but, instead, I explicate what mathematical norms are referred to in the context of this study.

Mathematical norms refer to meta-knowledge of mathematics (Sekiguchi, 2005). As such, mathematical norms embrace the three dimensions of knowledge (Anderson & Krathwohl, 2001) about mathematics. These three dimensions of knowledge are factual, conceptual and procedural (Anderson & Krathwohl, 2001) and are also standard to teaching and learning (doing mathematics). Accepting that a norm is understood as something that is usual, typical, or standard, as mentioned at the beginning of this chapter, then, using the three dimensions of knowledge, I provide an explanation of what mathematical norms are within the context of this study. In the explanation, I first provide a definition of each of the three knowledge dimensions, I then provide an example from mathematics to explicate my interpretation of the dimension, and, finally, I provide examples of mathematical norms which can be constructed from each domain.

Factual knowledge refers to knowledge about basic and discipline specific elements, such as terminology, symbols, and notations (Anderson & Krathwohl, 2001). These are the elements which a learner must know and be familiar with in order to understand a discipline and to solve problems from that discipline (Pratama & Retnawati, 2018). For example, when representing figures, points and graphs in a Cartesian plane, the horizontal and vertical axis are labelled as the x-axis and y-axis, respectively. This is a fact which is standard. Similarly, knowledge of mathematics concepts, such as knowing that where two functions intersect on a Cartesian plane are equal, constitute mathematical norms.

Procedural knowledge, on the other hand, refers to knowledge about algorithms, techniques to solve problems and being able to choose the right procedure on the basis of a particular set of criteria (Anderson & Krathwohl, 2001; Pratama & Retnawati, 2018). For example, knowing that, when carrying out procedures such as factorisation, it is standard that first a highest common factor is identified, before other procedures of factorisation, such as grouping pairs, can be used. In the same way, knowing specific facts about methods of factorising the difference between two squares, sum or difference of two cubes and perfect square trinomials are also regarded as procedural knowledge. Therefore, knowledge of the facts, concepts and procedures in mathematics constitute what is normative about doing mathematics and, hence, are referred to as mathematical norms.

Mathematical norms, just like social norms, are not explicitly taught by teachers to learners. Instead they are learned implicitly as learners engage with learning material, either designed by a teacher or presented in text books. Additionally, mathematical norms are learned as teachers speak and write mathematics during teaching and learning. It should be noted that the manner in which some of these norms are perceived by learners may be different to how they are supposed to be perceived in the discipline. As a result, such learners' perceptions about knowledge of mathematics would not be regarded as normative but rather as misconceptions, as they do not provide an understanding and knowledge of what is standard in the discipline.

Examples of misconceptions which may result from learners' perceptions of the mathematics they read, or are exposed to by a teacher, may include the use of the equals sign and the interpretation of equivalence. Numerous researchers argue that there are multiple interpretations of the equals sign by learners to mean things such as 'do something', 'the answer is' or 'the next step is' instead of viewing it as a relational symbol to compare numbers or expressions (Essien, 2009; Machaba, 2017). Another example of a misconception is when equivalence is interpreted to mean the same, instead of equal to. For example, , and are equivalent but mean different things. If used conventionally they could be interpreted to mean the same thing. The first two are fractions made of a numbers from as set of integers while the third fraction is a rational number or negative fraction. The mathematical norms discussed here and the social norms discussed in the previous section, together form sociomathematical norms, which are discussed in the section that follows.

4.4 SOCIOMATHEMATICAL NORMS

The term sociomathematical norms was first coined by Voigt (1995) and emerged as an extension of studies from general classroom social norms to the normative aspects established on the basis of mathematical argumentation (Cobb & Yackel, 1996). Unlike social norms, sociomathematical norms, on the other hand, are not universal, but are specific to mathematical activity (Kazemi & Stipek, 2001). They are established on the basis of mathematical arguments, and state normative aspects related to mathematical activity (Yackel et al., 2000). However, they are different from mathematical content and mathematical norms. Sociomathematical norms are concerned with how the mathematics community makes decisions, talks about, and analyses the mathematical aspects of the activities at hand (Güven & Dede, 2017). Additionally, sociomathematical norms provide criteria for the evaluation of mathematical activities and discourse unrelated to any particular mathematical idea (Cobb, Stephan, McClain & Gravemeijer, 2010) but instead related to mathematical norms.

Sociomathematical norms in this study are viewed as the unification of general classroom social norms and mathematical norms, as illustrated in Figure 4 – 2 hereunder.

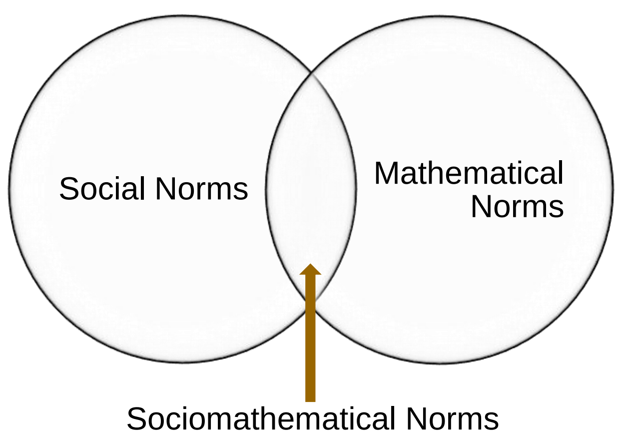


Figure 4 – 2: Representation sociomathematical norms in relation to social and mathematical norms

When learners engage in situations for challenges and justification in order to develop the "taken-as-shared" meanings, normative aspects of their classroom social interactions emerge, as explained earlier in this chapter. However, if these acts for challenges and justifications are based on learners' meta-knowledge about doing mathematics, explicated as mathematical norms in the latter section, then sociomathematical norms are constituted. This is because learners are engaged in social interactions which are specific to mathematical activities and, as such, the *words* (objects), *visual mediators* and *routines* (Sfard, 2008) for their discourse are mathematical in nature, hence sociomathematical norms emerge. In the same vein as social norms, sociomathematical norms are not static and, as such, are continuously renegotiated and re-endorsed by the classroom community, while mathematical norms, on the other hand, are static in that they conform to what is standard in the discipline.

Examples of sociomathematical norms include what counts as a mathematically different, sophisticated, efficient, and acceptable solution (Cobb & Yackel, 1996). Mathematically different solutions are observable in interactions where learners present different solutions to the same task (Chuene, 2011). However, for solutions to be accepted as mathematically different; their difference should be mathematically sound (Widjaja, 2012). As learners compare and argue their solutions in order to come up with a mathematically efficient one, they reach consensus when their discourse lends to a mathematically acceptable solution (Chuene, 2011). The establishment of the sociomathematical norms explained here fosters meaningful learning of mathematics (Yackel et al., 2000).

Yackel and Cobb (1996) further clarifies that:

> Issues concerning what counts as different, sophisticated, efficient, and elegant solutions involve a taken-as-shared sense of *when* it is appropriate to contribute to a discussion. In contrast, the sociomathematical norm of what counts as an acceptable explanation and justification deals with the actual *process* by which students contribute (p. 461).

Embodied in this clarification is that, the students' taken-as-shared sense of when to make a contribution depends on their comparison of their own contribution with those that have been offered already. The comparison draws focuses on the difference, sophistication, efficiency, and elegance of the solutions. For example, efficiency of solutions has to do with their viability for application. If a student has offered a long winded solution which maybe laborious, it would be expected that students make additional contributions provided their contribution are a shorter version of what has been offered already. Therefore, students are not expected to make additional contributions when it is not necessary.

The importance of sociomathematical norms in mathematics learning has been recognised, as evidenced by the decade-long notable upward trajectory of research focusing on these norms. These studies were conducted in both elementary (Abdulhamid, 2016; Kang & Kim, 2016) and secondary (Partanen & Kaasila, 2015) classrooms while other studies were conducted at institutions of higher learning (Güven & Dede, 2017; Sánchez & García, 2014). All the studies cited here adopted Yackel and Cobb's (1996) view of sociomathematical norms, as is the case in the reported study. However, the prominent difference between these studies was in the purpose and the research questions the studies sought to address.

The studies cited earlier sought to identity, explore, describe, examine and/or investigate the sociomathematical norms in various classrooms. Güven and Dede (2017) identified social and sociomathematical norms in classrooms at university level. Similarly, Chuene (2011) explored the enactment of social and sociomathematical norms at university level. The difference between the studies is that, in the former study, participants were students being trained to become teachers, while in the latter study the participants were students enrolled in a Bachelor of Science degree. Even though the reported study also sought to explore the enactment of sociomathematical norms, it also accounts for how these norms were established. Furthermore, the study was conducted in a secondary classroom in which teaching was undertaken in order to promote proficiency in mathematics.

Through her engagement with the data, Chuene (2011) concluded that social and sociomathematical norms were enacted as students (a) contributed to discussions through initiating discussions, (b) took responsibility for raising and offering answers, and (c) accepted arguments that are efficient to them. However, it remains unclear whether these norms are social or sociomathematical, even though Chuene mentioned that she finds the distinction between the two fuzzy. This is understandable as sociomathematical norms are often regarded as a subset of social norms and, as result, it can be considered acceptable that sociomathematical norms are social. However, identifying them as either is necessary, due to their widely acknowledged differences. Caution should be taken with regard to social norms, as they apply to every classroom and not necessarily only to a mathematics classroom. Hence, without the mention of mathematics along with these norms, it can be argued that these norms can exist in any classroom and, hence, be interpreted as social. Additionally, Chuene (2011) adopted the emergent approach as an analytical lens. This further signifies the need to differentiate social norms from sociomathematical norms. The emergent approach presents social norms and sociomathematical norms as separate, yet intertwined constructs of a mathematics classroom.

In the same way, the reported study also adopted the emergent approach as a lens through which data were analysed. However, in presenting findings, the establishment and enactment of social and sociomathematical norms was delineated, taking into account what mathematical norms are. Correspondingly, this approach to reporting, which reflects awareness to the distinction between social and sociomathematical norms, is notable in literature, even though no mention of mathematical norms is made. This is evident in the works of Güven and Dede (2017), McClain and Cobb (2001), Widjaja (2012) and Yackel et al. (2000), amongst others. Notwithstanding the fact that these studies adopted a similar reporting approach, the difference was their explanations of how the sociomathematical norms supported learners' mathematics learning, or the absence thereof.

Güven and Dede (2017) offered no explanation about how the sociomathematical norms which were at play in the classroom they researched advanced students' mathematics learning. This was the case as their study did not focus on how sociomathematical norms can advance students' mathematics learning; nonetheless they argued that student teachers' awareness of norms could benefit the classrooms in which they are going to teach. Inherent Güven and Dede (2017) argument is a claim that development and enactment of sociomathematical norms has the potential to support students' mathematical learning. The notion that sociomathematical norms support students' mathematics learning is also supported by the results of studies conducted by Van Zoest, Stockero and Taylor (2012) and Wadjaja (2012).

Van Zoest and colleagues (2012) investigated the durability of professional and sociomathematical norms; while Wadjaja (2012) examined the enactment of social and sociomathematical norms. Wadjaja also indicated that, in his study, interest was also placed on the impact of social and sociomathematical norms on students' learning. Even though Van Zoest and colleagues (2012) did not explicitly point out their interest about this impact, they still argued for its existence. Hence, they concluded that the establishment of norms that make effective teaching and learning possible should be the focus of teacher education. Correspondingly, the reported study used empirical data to explicate how the development and enactment of sociomathematical norms supports effective mathematics learning in the classroom.

In the reported study, sociomathematical norms were explored, unrelated to specific mathematics content, but in relation to the emergent "taken-as-shared" meanings about meta-knowledge of doing mathematics. This was the case as Güven and Dede (2017) asserted that sociomathematical norms are different from mathematics content. Therefore, the development and enactment of sociomathematical norms can support learning in different content areas. On the other hand, Wadjaja's (2012) study focused on the enactment of sociomathematical norms specific to students' learning of data representation. The research questions posed show that the inquiry was specific to data representation and not mathematical classroom discourse, in general. Therefore, the reported study made a significance contribution to close this gap of focusing on a single concept of mathematics, by bringing to the fore sociomathematical norms which afford learners' effective mathematics learning unrelated to specific mathematics content.

Mathematics learning in the context of the reported study is defined as the achievement of the mathematical proficiency strands (Kilpatrick et. al., 2001). Kilpatrick and colleagues provided a description for each of the five intertwined strands of mathematics proficiency. This provides a framework for delineating what exactly are learners expected to do in demonstration of competence in, or achievement of, each strand. Furthermore, this framework afforded me an opportunity to develop a focused analysis and argument throughout the research. Mathematical proficiency strands identified by Kilpatrick et al. (2001) except for productive disposition, as the basis for mathematics learning are also reflected in the South African Framework of Teaching Mathematics for Understanding (TMU) (DBE, 2018). The notion of mathematical proficiency was discussed in detail in Chapter 3, as a theoretical construct for mathematics learning.

Studies which found that sociomathematical norms support mathematics learning, do not offer an explanation of what they refer to as mathematics learning. This, in turn, opens their results and arguments up to different interpretations and understandings of what students are expected to do in order to indicate learning of mathematics learning. It is important for teachers to be aware of, and understand, sociomathematical norms (Kang & Kim, 2016; Zembat & Yasa, 2015) and to be able to engender them during teaching and learning (McClain & Cobb, 2001; Partanen & Kaasila, 2015; Yackel et al., 2000). This awareness and understanding, along with ability to engender sociomathematical norms, will enable teachers to effectively facilitate mathematics learning, because the establishment of sociomathematical norms directly influences our mathematical agenda (McClain & Cobb, 2001). Furthermore, the establishment and enactment of sociomathematical norms contributes the conditions that make meaningful learning of mathematics possible (Yackel et. al., 2000).

Correspondingly, Zembat and Yasa (2015) explored teachers' understanding of sociomathematical norms without observing their classrooms. They found that teachers' understanding of sociomathematical norms is not analogous to scenarios illustrating the particular sociomathematical norms reflected. This contradiction between the expressed understanding of teachers and the application of sociomathematical norms is an indication of a need to observe classroom practice while studying norms. Similarly, in the reported study, persistent classroom observations were used, amongst other data generation methods.

Kang and Kim (2016) investigated the relationship between teachers' construction of sociomathematical norms and their mathematical beliefs. They concluded that mathematical beliefs were reflected in making instructional decisions and contributed to the establishment of sociomathematical norms. This relationship is an indication of the fact that there are particular sociomathematical norms which teachers engender in their classrooms, and such norms are underpinned by their mathematical beliefs. The relationship highlighted by Kang and Kim (2016) corresponds to the assumed relationship between social and psychological perspectives of classroom practice, as captured in Cobb and Yackel's (1996) emergent approach. In the same way, teachers' mathematical teaching philosophy and conceptions about mathematics learning build on their mathematical beliefs which, in term, will determine the types of sociomathematical norms they engender in their classrooms. For the study reported here, philosophical understandings of mathematics learning are underpinned by the sociocultural perspective as discussed in Chapter 3. As such, the sociocultural perspective reflects my mathematical beliefs, which have an influence on the sociomathematical norms that I negotiate in the classroom.

Even though this study sought to explore sociomathematical norms which were constituted (or emerged) during mathematics learning, it can still be argued that I negotiated particular norms. These negotiated norms were the result of his mathematical beliefs as reflected in his conception of mathematics learning in a social setting. For this study, I was aware of the norms he proactively negotiated during time spent in the classroom. However, in other classrooms this act can be unconscious due to teachers' unawareness of norms and the mathematical beliefs guiding their instructional decisions. Negotiating sociomathematical norms creates opportunities for teaching for understanding (Schoenfeld & Kilpatrick, 2008); and simultaneously supports students' development of both a mathematical disposition and intellectual autonomy (McClain & Cobb, 2001).

Notwithstanding the fact that teachers can negotiate particular sociomathematical norms; they should also guide the emergence of other norms proactively during mathematical classroom discourse (McClain & Cobb, 2001). Guiding teaching and learning to allow for emergence of sociomathematical norms which are not directly influenced by a teacher is essential (Chuene, 2011). Occasionally, the sociomathematical norms negotiated by a teacher differs from those preferred by students and this stifles learning (Levenson, Tirosh & Tsamir, 2009). In the same vein, at times, sociomathematical norms negotiated by a teacher are perceived differently learners.

The emergence of sociomathematical norms in the case of this study resulted from an analysis of learners' patterns of participation when teaching was undertaken to promote proficiency in mathematics. I structured teaching and learning activities such that they afforded learners multiple opportunities to develop towards achieving the mathematical proficiency strands (Christiansen & Ally, 2013; Kilpatrick et al., 2001; Schoenfeld & Kilpatrick, 2008). Correspondingly, Partanen and Kaasila (2015) described the production of sociomathematical norms that were relevant to the investigation of a small group approach. Similarly, in this study the I report on sociomathematical norms that were relevant to developing learners' mathematical proficiency.

4.5 SUMMARY

In this chapter I discussed general classroom social norms and how they emerge in the classroom. I then explicated what mathematical norms are, and how sociomathematical norms are constituted from the unification of social and mathematical norms. Thereafter, I presented an analysis of studies focusing on sociomathematical norms, raised arguments about their limitations and highlighted their significance in terms of this study. Finally, I argued for a need to link the development of sociomathematical norms with mathematical proficiency.

CHAPTER 5: METHODOLOGY

5.1 INTRODUCTION

In this chapter, I outline the research approach, which I adopted in order to undertake this study. My choice of approach is defended by the sociocultural perspective, which I employed as a referent for classroom practice as outlined in Chapter 3 and the emergent approach employed as a lens through which I analysed data discussed in Chapter 2. I then present the design of the study, after which I describe the participants in the study, before discussing the data gathering techniques, which I employed in the study. Furthermore, discuss the data analysis method applied and deal with the actual data analysis process as it unfolded. In this regard, I found audio and video data transcription to be a very slow and tedious exercise. Nevertheless, I transcribed the data and undertook the analysis. I describe and discuss how I applied various quality criteria throughout the study and, finally, I deal with the ethical issues which I considered throughout the study.

5.2 RESEARCH APPROACH

It was made clear that the study used sociocultural perspective as a referent for classroom practice (see chapter 3) and the emergent approach as the lens through which I analysed data (see chapter 2). Owing to the philosophical assumptions made by sociocultural perspective and emergent approach in terms of epistemology and ontology, a qualitative research approach was deemed more suitable. Creswell (2012) defines a qualitative research approach as an inquiry approach that is useful for exploring and understanding a central phenomenon. In the context of this study, the central phenomenon explored was learners' constitution of sociomathematical norms.

My choice of a qualitative research approach was guided by the purpose of the study, which was explore the constitution of sociomathematical norms in a class where teaching was undertaken to promote learners' proficiency in mathematics. As a result, I considered two detailed definitions for qualitative research, one from Creswell (2007) and the other from Denzin and Lincoln (2005), which are based on philosophical assumptions and on strategy of inquiry, respectively. Creswell (2007) defines qualitative research, outlining its philosophical assumptions, as follows:

> Qualitative research begins with assumptions, a worldview, the possible theoretical lens, and the study of research problems inquiring into the meaning individuals or groups ascribe to a social or human problem. To study this problem, qualitative researchers use an emerging qualitative approach to inquiry, the collection of data in a natural setting sensitive the people and places understudy, and data analysis that is inductive and establishes patterns or themes. (p. 37)

This definition describes the process of qualitative research, from philosophical assumptions to the procedures, involved throughout the study. In line with the definition by Creswell (2007), this study also began with assumptions related to why mathematics is taught and what is it taught for? This led to my considering worldviews about teaching and learning and, ultimately, to finding harmony in the sociocultural perspective by Vygotsky (1972). The problem being studied was to consider the meanings which learners ascribe to their mathematical activity. Consequently, data collection took place in the natural setting of the classroom. Finally, the data analysis was inductive and grounded in the data, patterns and themes as learners' constitution of sociomathematical norms emerged. On the other hand, Denzin and Lincoln (2005) outline qualitative research as a strategy of inquiry, which they define as follows:

> Qualitative research is a situated activity that locates the observer in the world. It consists of set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative

> research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of meanings people bring to them (p. 3)

This definition is relevant because it captures a number of methodological issues which are evident in this report. These methodological issues include the strategy of inquiry which was employed in the study, methods which were used for data collection and the setting in which data were collected. These aspects will be discussed in more detail later in this chapter. Due to the philosophical assumptions made in this study, and the exploratory and descriptive nature of this study, a qualitative approach was adopted. I intended to offer thick description of how learners constitute sociomathematical norms in a class where teaching was undertaken to promote learners' proficiency in mathematics.

5.3 STUDY DESIGN

Learners' constitution of sociomathematical norms in a sociocultural mathematics classroom is not a predetermined and a predictable phenomenon. Rather, it is a contemporary phenomenon, deeply rooted in actions taken by learners during their mathematics learning experience. Therefore, studying this phenomenon should be done in a real-life context and, in this case, the context was a mathematics classroom. Studies conducted in real-life contexts are best done using the case study approach (Merriam, 1998; Stake, 1995; Yin, 2003). Case study as a research design has been thoroughly explored by three seminal authors, namely; Merriam, Stake and Yin (Brown, 2008; Ebneyamini & Moghadam, 2018; Harrison, Birks, Franklin & Mills, 2017; and Yazan, 2015), in particular. Hence, I will discuss what case studies are as conceptualised by these distinguished authors.

Yin (2003) defined a case study as an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident. On the other hand, Stake (1995) defines a case study as a form of research defined by interest in an individual case, and not by the method of inquiry used. Unlike Yin and Stake, Meriam (1998) does not offer a precise definition of a case study. Instead she clearly defines a “case” and creatively discusses characteristics of a case study (Yazan, 2015). Using Meriam’s definition of a case and her discussion of the characteristics of case study, then a case study can be defined as an intensive, holistic, descriptive and analysis of a bound phenomenon or social unit.

It would appear that all these definitions of a case study fit well with my study, as they all place emphasis on the phenomenon being studied within its real-life context. However, that is not the case. The views of Merriam, Stake and Yin of case study differ because of their ontological and epistemological commitments. Correspondingly, Harrison and colleagues (2017) stated that, philosophically, case study research can be re-orientated from a realist or positivist perspective to a relativist or interpretivist perspective. This suggests that case study research is not aligned to a specific ontological, epistemological or methodological position, since researchers who subscribe to realist or positivist perspective hold a view that there is a single reality, which is independent of the individual and can be studied and measured. On the other hand, researchers who subscribe to a relativist or interpretivist perspective hold a view that there are multiple realities and meanings, which depend on individuals and are co-created by the researcher (Lincoln, Lynham & Guba, 2011; Yin, 2014). Therefore, careful consideration of the different case study approaches is required when choosing a methodological position in order to determine the design that best aligns with the researcher’s worldview (Harrison et al., 2017).

In terms of taking a stance on reality or meaning, Harrison et al. (2017) and Yazan (2015) argue that Yin’s approach to case study leans towards a positivist philosophical stance. In particular, Harrison et al. (2017) stated that the traits of post-positivism are evident in Yin’s approach to case study, as Yin defined case study as a form of empirical inquiry. In addition, Yin (2014) himself describes his approach to case study as using a "realist perspective" (p. 17). This worldview is not aligned to a worldview underpinned by the sociocultural perspective which was adopted in this study. Furthermore, Yin's worldview cannot be relevant since norms in the context of this study are social phenomena. Sociomathematical norms, on the other hand, are social phenomena that emerge during the (re-) construction of mathematical knowledge. It is, therefore, needless for me to say that reality or meaning is socially constructed. Merriam and Stake both subscribe to the constructivist paradigm (Njie & Asimiran, 2014; Yazan, 2015). In particular, Merriam is a pragmatic constructivist and Stake is a relativist-constructivist or interpretivist (Harrison et al., 2017). It is these worldviews that are aligned to the one adopted in this study and, as such, I find myself in harmony with the view of a case study as seen through Merriam and Stake's lens.

In terms of views of the bounded nature of the case study, I will discuss the views of Stake and Merriam only, since their views of case study are epistemologically relevant to my study. Stake views a case study as an integrated system (Yazan, 2015), which means multiple cases and various contexts. In my study the case is one of enactment of sociomathematical norms which is bounded in a sociocultural mathematics classroom. It is, therefore, clear that the case study described in this study is the one which is viewed as a bounded system (Merriam, 1998). For this reason, the case study approach adopted in this study was that described by Merriam (1998). However, some of Stake's (1995) positions with regard to data collection and analysis were adopted in order to strengthen the arguments constructed in this thesis. Brown (2008) suggested that the approaches of Yin, Merriam and Stake to case study rest along the quantitative – qualitative continuum, where postpositivist methodology of Yin (2014) sits at one end, Stake's (1995) interpretivist design sits at the other end, and Merriam's (1998) pragmatic constructivist methodology, which draws elements from both, rests towards the centre.

Merriam (1998) further explains a case study in terms of three distinctive attributes, namely, particularistic, descriptive and heuristic. She explains particularistic as a focus on a particular situation, event, programme or phenomenon. This, according to Brown (2008), suggests what should be done in a similar situation. Descriptive case study is explained as a study which yields rich and thick description of the phenomenon studied. Lastly, heuristic case study is a study which clarifies the phenomenon under study. Brown (2008) and Yazan (2015) contend that case study allows for a broader understanding on the phenomenon.

These traits are interpreted as types of case studies (Brown, 2008) and, often, as characteristics of a case study (Yazan, 2015). For the reported study, I did not choose either one of the attributes, nor did I subscribe to all of them because of the purpose of the study, which was to explore learners' enactment of sociomathematical norms, in a class where teaching was undertaken in order to promote proficiency in mathematics; making the study to have descriptive elements. The use of case study as a research design in studying sociomathematical norms is not foreign to research in mathematics education. For example, studies focusing on sociomathematical norms employed single case study design (Widjaja, 2012), multiple case study design (Güven & Dede, 2017) and interpretive case study design (Chuene, 2011).

5.4 CHOOSING PARTICIPANTS

Sampling is defined by Merriam (1998) as the selection of a research site, time when the research is undertaken, people to participate in the research and the selectino of events in a field research. Purposive sampling, which is a non-random method of sampling where the researcher selects information rich case for conducting an in-depth study, was employed (Cohen, Manion, & Morrison, 2000). This was the case because Merriam indicated that a non-probability sample is effective when, the researcher is exploring and describing what is occurring. She suggested that such a purposive sample has a logic and a power, and provides rich information.

Merriam (1998) further explained that purposive sampling places emphasis on a criterion-based selection of information-rich cases from which a researcher can discover, understand and gain more insight into issues crucial to the study. This is an indication that purposive sampling is consistent with case study design. The participants in the study comprised 23 Grade 11 mathematics learners from an urban school, located in the Capricorn District of the Limpopo Province, South Africa. This class was allocated to me in order for me to teach the learners at this school for the 12-week period of data collection.

Although the sample consisted of 23 learners, the data and the findings were based on the sample's grain-sizes. The grain-sizes were made up of either groups or pairs of learners working together (with me) on the teaching and learning activities in the natural setting of a mathematics classroom. Instances analysed did not represent the experiences of each group, nor of all individual learners. Since my interest in undertaking this study was to describe how sociomathematical norms are constituted and enacted in the classroom, the strength of instances analysed was in showcasing when and in what ways were the sociomathematical norms constituted and enacted. Therefore, external validity was traded off for transferability (Guba & Lincoln, 1989).

5.5 DATA GATHERING TECHNIQUES

There are a variety of data collection methods used in qualitative research which have been documented by several authors. For example, Creswell (2012), Maree (2017), McMillan and Schumacher (2010), and Struwig and Stead (2013). The common methods, which made it onto the list of all the authors cited here, are interviews and observations. These methods are the prominent data collection methods in case studies, as noted by Merriam (1998) and Stake (1995).

In this study data collection took place during teaching and learning. The lessons were video recorded using two video recorders. One video recorder was used by me as I moved from one pair or group of learners to another, engaging with the learners on the basis of their mathematics learning activities. The other video camera was used by the head of the mathematics department at the school, who was always with me in the class. He recorded pair and groups interactions which occurred when I was not part of the interactions and also the whole class discussions which I led.

Data also emerged from classroom discussions, when students, in their groups or pairs, worked through activities given to them as part of their day-to-day mathematics learning. The topic under discussion was guided by the activities I supplied to learners. These activities were aligned to the subject content to be taught in Grade 11, as prescribed by the Curriculum Assessment and Policy Statement (CAPS) (DBE, 2011) and the Annual Teaching Plan (ATP) provided to the school by the school district. Needless to say, the research agenda did not shape the data which emerged.

Mathematics education researchers have been using technology to capture and study audio and audio coupled with visual images of teachers and learners engaged in mathematical activity. Correspondingly, Erickson (1992) posits that in educational research, the use of audio and video technology for studying interactions has intellectual antecedents in several analytic approaches. These approaches dates back to the 1950s in studies which involved detailed transcripts of cinema film of naturally occurring interactions, this was regarded as the context analysis approach. Erickson (1992) further states that the approach of *ethnography of* *communication* used audio and video recordings to study the moment-by-moment organization of the conduct of interaction.

Currently in the mathematics education research community, the capability of videotaping to record the moment-by-moment unfolding of sounds and sights of a phenomenon has made it a powerful and widespread tool (Powell, Francisco, & Maher, 2003). Employing video as data, researchers have been able to contribute thick descriptions of classroom interactions on mathematical tasks in a classroom settings. Some descriptions have emerged from large-scale, video-based international surveys of classroom instruction, such as ones from the Videotape Classroom Study, a component of the Trends in International Mathematics and Science Study (TIMSS) (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999).

There is a notable upward trajectory of studies which use videotape data to examine classroom mathematical activity. However, Powell and colleagues (2003) notes that in reporting such studies "some authors discuss video-related methodological issues implicitly when reporting results of their research, while others do so explicitly, raising important methodological and theoretical issues concerning the use of video recording in data collection, analysis and interpretation, as well as presentation and ethical issues" (Powell et al., 2003 p. 407). Mathematics education researchers who have started conversations to articulate explicit methodological and theoretical issues and questions pertaining to videotape in research amongst others includes Cobb and Whitenack, (1996); Davis, Maher, and Martino, (1992); Hall, (2000); and Pirie, (1996).

Despite the popularity of video data, Hall (2000) claims that little is known and written about the use of videotape for "collecting, watching, and interpreting video as a stable source of data for research and presentation purposes" (p. 647). Methodologically, video technology lends itself to a strict application or a mixture of qualitative and quantitative approaches in both data collection and analyses (Powell, 2003). A salient reason for this is, as Pirie (1996) observed in discussing video recording in mathematics education, that videotaping a classroom phenomenon is likely to be “the least intrusive, yet most inclusive, way of studying the phenomenon” (p. 554).

The use of video recordings allowed for repeated observation to be made, hence it fits to say participant observation was also used as data collection technique. Observation is defined as the process of gathering information by observing people or places (Creswell, 2012) in their real-life context (Struwig & Stead, 2013). The use of observation in case studies is supported by Cohen and colleagues (2000) when they assert that “whatever the problem or the approach, at the heart of every case study lies a method of observation” (p. 185). In the same vein, Merriam (1998) also mentions the notion that observations are important in case studies. However, she cautions that they are subjective data sources, hence she recommends that their use be carefully considered.

For Merriam (1998), this subjectivity may affect the credibility of the data collected, which poses a threat to the validity of the findings. She argues that the researcher mostly enters the situation without an observation schedule and engages in constructing an understanding of the research environment through self-interpretation of what happens. For this reason, qualitative research often produces results which are “an interpretation by the researcher of others’ views filtered through his or her own” (p. 23). The qualitative research paradigm, however, acknowledges the researcher’s subjectivity (Guba & Lincoln, 1989) and subjectivity is, therefore, not a threat to the validity of the findings as such. Later in this chapter I will discuss how credibility of the collected data was ensured in this study.

Observations can be conducted with or without the researcher being a participant (Creswell, 2012; Gatley, 2001). Participant observation, which Creswell (2012) describes as the role adopted by the researcher when they take part in the situation they are observing, was employed in this study. Therefore, as a researcher, and through a close interaction with the learners, I became a “passionate participant” (Guba & Lincoln, 1994, p.115).

During observations made as learners engaged in activities there were instances when I was a silent participant observer. This means that I was a member of the group who did not participate in conversation. Although my presence in the group did not affect the data produced, it kept students on task. Entries were made in my journal during group conversations. This was, however, mostly done later, after the lesson, while reflecting on the lesson. The reason was that I was, for the most part, immersed (by listening) in the learners’ discussions, that I forgot to immediately make entries in my research journal.

5.6 TEACHING PLAN

From the time I arrived at the school for data collection, the teaching plan was as captured in the table below. I assumed the duty of teaching the learners during their normal mathematics periods as assigned on the schools’ teaching time table. The HoD taught the learners during morning and afternoon sessions, hence I did not take the learners through all the content as in the table below. In particular I handled; analytical geometry in term 2 and trigonometry in term 3, other sections were handled by the HoD, who also served as the subject teacher for the classes which I taught the one from which I collected data.

Date	Main Topic	Content
	TERM 2	
3 Weeks 07 – 25 May 2018	Analytical Geometry	Derive and apply: 1. the equation of a line through two given points; 2. the equation of a line through one point and parallel or perpendicular to a given line; and 3. the inclination (θ) of a line, where is the gradient of the line
2 Weeks	Number patterns	Patterns: Investigate number patterns leading to those where there is a constant second

28 May – 08 June 2018		difference between consecutive terms, and the general term is therefore quadratic.
TERM 3		
2 Weeks 16 – 27 July 2018	Functions	1. Investigate the effect of the parameter *k* on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = tan(kx)$ 2. Investigate the effect of the parameter *p* on the graphs of the functions defined by $y = \sin(x + p)$, $y = \cos(x + p)$ and $y = tan(x + p)$ 3. Draw sketch graphs defined by: $y = a \sin k(x + p)$, $y = a \cos k(x + p)$ and $y = a\ tan\ k(x + p)$ at most two parameters at a time.
3 Weeks 30 July – 17 August 2018	Trigonometry	1. Prove and apply the sine, cosine and area rules. 2. Solve problems in two dimensions using the sine, cosine and area rules.
1 Week 20 – 24 August 2018	Measurement	1. Revise the volume and surface areas of right-prisms and cylinders. 2. Study the effect on volume and surface areas when multiplying any dimension by a constant factor k. 3. Calculate volume and surface areas of spheres, right prisms, right cones and combination of those objects (figures).

It should be noted that even though I spent 12 weeks at the school with the learners approximately only 8 weeks was used for teaching the content on analytical geometry and trigonometry reflected in the teaching plan. The rest of the time in between was used to do revision with learners in preparation for midyear examination and also to do remedial work after the examination itself. Correspondingly, the Learning Activities included in Annexure F are based on analytical geometry as well as trigonometry and covers all the concepts in those sections as prescribed by CAPS but also building on concepts learned in previous grades. These learning activities therefore do not represent a single period or lesson but instead a series of lessons for each topic for over four weeks.

5.7 DATA ANALYSIS

Data analysis, according to Merriam (1998), is “the process of making sense out of the data. Making sense out of the data involves consolidating, reducing, and interpreting what people have said and what the researcher has seen and read – it is the process of making meaning.” (p. 178). This view of data analysis is consistent with the systematic analysis of data in qualitative research (Creswell, 2012; Kim, 2016; Struwig & Stead, 2013). Systematic qualitative data analysis involves identifying concepts from the raw data through coding which involves linking similar or related codes to form a category, identifying repeated patterns from categories and creating themes that represent similar patterns (Kim, 2016). However, in this study, narrative analysis was adopted instead of the “traditional” qualitative data analysis.

Narrative analysis is “an act of finding narrative meaning” (Kim, 2016, p. 190).Narrative meaning was defined by Polkinghorne (1995) as a cognitive process that organises human experiences into temporally meaningful episodes. Narrative analysis concerns different aspects of experience that involve human actions or events (Kim, 2016). Additionally, Kim posits that narrative analysis is a meaning-finding act used in an attempt to elicit implications for better understanding of human actions. Therefore, the reported study adopted narrative data as conceptualised in the work of Polkinghorne.

Polkinghorne (1995) divides narrative data analysis into two categories, namely, analysis of narratives and narrative analysis. Analysis of narratives follows on after the systematic qualitative analysis of data has been completed, as the final step, as discussed earlier. Furthermore, Polkinghorne posits that, through this analysis, common themes are examined in storied data, and organised into several categories, using stories as data. Since the data collected during this study were in the form of actions, events and text, Polkinghorne’s narrative analysis of eventful data was adopted in order to make meaning of the data. This narrative analysis was followed by an analysis of the narratives, where common themes were examined in the stories that emerged and then organised into several excerpts.

In narrative analysis the search is for data that will reveal the uniqueness of the bounded system (or the individual), and provide an understanding of the bounded system's idiosyncrasy and its particular complexity (Polkinghorne, 1995). In this approach to analysis, instead of separating data, data are synthesised into a coherent developmental accounts by linking the themes which emerge during data sorting, when interpreting the data. Hence, the purpose of this analysis was, according to Kim (2016), to help understand why and how things happened and why the participants acted in the way they did.

5.7.1 Selection of Excerpts

In preparation for analysis I watched the videos a number of times until I reached a stage where I accepted the picture that they portrayed (Chuene, 2011). I had an opportunity to go through videos which were recorded by the HoD and myself for same lesson. I then selected excerpts of the video recordings which I was going to use for analysis. With respect to data analysed in chapters 6 and 7 I decided to use only recording which I did from groups of learners, and not those done by the HoD for learners who worked in groups in my absence as they could not sustain their discussions and hence their footage did not provide rich data. Whereas those recorded by me had rich data in that I probed the learners in the quest of determining what was normative about how did they mathematics. Subsequently, challenged their ways of doing mathematics for cases which I regarded as not productive in accordance with my goal for teaching as articulated in Chapter 1.

Excerpts of video data which were recorded by the HoD and were considered for analysis, are those which captured the whole class discussion which

I facilitated during the lessons. The whole class discussions are moments of classroom interactions during the lessons which occurred at stages where I saw it fit to engage learners in a whole class discussion on the basis of what transpired in their groups.

5.7.2 Transcription of Video Records into Textual Data

Excerpt selection was followed by transcription of all the selected video data into text (Creswell, 2012), which was followed by reading through the textual data several times. The choice of the excerpts was guided by their strength in fulfilling the purpose of the study and in response to the research question. In the same way, data collected through observation were also read through several times. Finally, the findings were presented through related excerpts of discussions that were found to be substantive to the narrative.

In constructing the episodes captured under the various excerpts, the utterances were cleaned, in that repeated words and short pauses were removed. Additionally, in cases where learners spoke almost at the same time, the utterances of the one who started first were captured first to completion before capturing the those of the other one, unless there is a clear pause in between. This is harmonious with the narrative analysis as described by Polkinghorne (1995), as he posits that the researcher re-stories the episodes to produce a coherent account of what transpired.

5.8 QUALITY CRITERIA

The quality of a qualitative research is measured by its trustworthiness. A study is said to be trustworthy if, and only if, the reader of the research report judges it to be so (Rolfe, 2006). Trustworthiness of qualitative research is generally questioned by positivists, as their concepts of validity and reliability cannot be addressed in the same way in naturalistic studies (Shenton, 2004). Qualitative investigators, like Guba and Lincoln (1985, 1989), address these concepts. However, they prefer to use different terminology to distance themselves from the positivist paradigm. They divide trustworthiness into credibility, dependability, transferability and confirmability. In the discussion which follows I offer a brief explanation of each and explain how each was addressed in this study.

5.8.1 Credibility

Credibility is defined as the extent to which data, data analysis and conclusions are believable and trustworthy (McMillan, 2012). Positivist researchers refer to internal validity, which ensures that their studies measure or test what they actually intended to test (Shenton, 2004). Shenton further suggests that Merriam (1998) refers to credibility by stating that the qualitative researcher's equivalent concept deals with the question, "How congruent are the findings with reality?" Even though both internal validity and credibility deals with issues of reality in research, the reality these concepts refer to differs due to epistemological assumptions. The assumption in qualitative research is that reality is multidimensional and ever-changing and not a single, fixed and measured phenomenon, as it is in quantitative research (Merriam & Tisdell, 2015).

Guba and Lincoln (1989) elaborated on seven techniques which qualitative researchers can employ in order to address credibility. The techniques included: prolonged engagement, persistent observation, peer debriefing, negative case analysis, progressive subjectivity, member check and triangulation. For this study, three techniques to enhance credibility were employed, namely, prolonged engagement, persistent observation and triangulation.

5.8.1.1 Prolonged engagement

Credibility of data collected was ensured through prolonged engagement with the learners I taught, and through which a rapport developed (Guba & Lincoln, 1989).

This was possible because the data providers were the learners I taught mathematics to on a daily basis for a period of 12 weeks. I, therefore, did not expect learners to deliberately mislead, distort or misconstrue their discussions or actions (Chuene, 2011).

5.8.1.2 Persistent observation

Persistent observation is a technique which ensures depth of experience and understanding during prolonged engagement (Williams, 2011). Its purpose is "to identify those characteristics and elements in the situation that are the most relevant to the problem or issue being pursued and focusing on them in detail" (Guba & Lincoln, 1985, p. 304). Persistent observation provides an opportunity for the understanding of the participants' views and minimises the effects of the researcher's presence (Anney, 2014). The prolonged engagement allowed persistent observation to take place which result in an in-depth study (Bitsch, 2005).

5.8.1.3 Data triangulation

Data triangulation refers to the use of a variety of methods in collecting data (Bitsch, 2005; Merriam, 1998). Data triangulation helps the researcher to reduce bias and affords the researcher to cross examine the integrity of the participants' responses (Anney, 2014). This is where most qualitative researchers employ several data collection techniques in a study but, usually, select one as the central method of analysis (McMillan & Schumacher, 2010). Even though both observations and interviews were used for data collection, observation was the central method. The methods employed permitted triangulation of the data collected, resulting in increased credibility of findings. This process conformed to the case study approach as a research design (Cohen et al*.,* 2000). Since data were video recorded, this afforded me an opportunity to view the videos as many times as possible and make other observations while I was in class.

5.8.2 Dependability

Dependability is defined as “the stability of findings over time” (Bitsch, 2005, p. 86). According to Merriam (1998), dependability refers to the extent to which research findings with similar subjects and in a similar context are consistent. Positivist researchers refer to dependability as reliability, which they regard as “a technique to show that, if the work were repeated, in the same context, with the same methods and with the same participants, similar results would be obtained” (Shenton, 2004, p. 71). Shenton acknowledges that such a provision is problematic to qualitative researchers due to the changing nature of the phenomena under study. This is so because it is unlikely that participants in a later similar study will provide identical responses, as their understandings may have changed or developed due to their reflection on the initial research (Carcary, 2009). Guba and Lincoln (1985) put forward the notion of an inquiry audit as a technique to establish dependability.

Inquiry audit is a process where a researcher who was not involved in the research process examines both the process and product of the research study for consistency (Guba & Lincoln, 1985). Guba and Lincoln, further assert that “since there can be no validity without reliability (and thus no credibility without dependability), a demonstration of the former is sufficient to establish the latter” (p. 316). This means that, if a researcher has demonstrated the credibility of the data collected, then this is enough to regard the data as dependable.

5.8.3 Transferability

Transferability is defined as the extent to which the findings of one qualitative study can be applied to other situations (Guba & Lincoln, 1985; Merriam, 1998). In the naturalistic paradigm, transferability of findings cannot be specified by the researcher, only the reader can determine whether the findings are applicable to a new situation (Guba & Lincoln, 1985). This can only be achieved when the researcher provides a “thick description” of the study (Guba & Lincoln, 1985; Merriam, 1998; Stake, 1995).

Thick description refers to a detailed account of field experiences in which the researcher makes explicit the patterns of cultural and social relationships, and puts them in context (Holloway, 1997). Thick description enables judgement of how well a research context fits with other contexts (Anney, 2014). Without thick description it becomes difficult for the reader to determine the extent to which the findings are true (Shenton, 2004).

5.8.4 Confirmability

Confirmability refers to the extent to which the researcher can demonstrate the neutrality of their research interpretations (Guba & Lincoln, 1985). This means, according to Anney (2014), the extent to which findings of a study can be corroborated by other researchers. Guba and Lincoln (1985) recommend a confirmability audit and triangulation as techniques for establishing confirmability. The interpretation of triangulation is still that of using multiple data sources during data collection, as discussed earlier in this section. In the same vein, a confirmability audit is the process where a reviewer examines the process and product of the research, as discussed earlier in this section.

5.9 ETHICAL CONSIDERATIONS

A letter requesting permission to collect data from a Grade 11 classroom, during normal teaching hours was submitted to the principal and to the Department of Education. In the letter I informed them about the study, its purpose and the envisaged participants. After permission had been granted, with the assistance of the school, consent to participate in the study was requested from the parents and the learners who belonged in the class I was allocated during the data collection period. I informed the learners that I would be recording the lessons, including their interactions, during mathematics periods. The following ethical principles which were shared with both the school management and with the participants: informed consent, confidentiality and dual role of the teacher-researcher (Orb, Eisenhauer, & Wynaden, 2001; Leedy & Ormrod, 2005).

5.9.1 Informed Consent

The participants exercised their rights as autonomous persons to voluntarily accept or refuse to participate in the study. They were informed of the nature of the study and what would be expected of them, in order to allow them to make an informed consent regarding their participation. They were informed of their right to freely participate or withdraw their participation from the study at any time, without penalty. Given that data was collected during normal teaching hours, the participants were informed that, if they did not want their group interactions recorded, they should say so without fear. In the case of whole class discussions they were informed that their identity would be held confidential when reporting on the study. For this study all the 23 learners in the Grade 11 mathematics class accepted my invitation to participate.

5.9.2 Confidentiality

Information collected from the participants was treated as confidential by using pseudonyms when reporting. I have to acknowledge that confidentiality in a case study is often a challenge due to the thick descriptions which are given. At times these descriptions may reveal the identity of the participants, or even the research site. Consequently, in cases where photographs are included, the faces of the learners are hidden and the pictures are converted to black and white, to prevent other people from identifying the school by the colours of the school uniform.

5.9.3 Dual Role of the Teacher Researcher

Entering the classroom as both the teacher and researcher presented some challenges. However, I took an oath that I would always take the lead and, as such, I did not allow the research agenda to interfere with the normal teaching of the learners. I followed the Annual Teaching Plan (ATP) in preparation of my lessons, and the head of the mathematics department was always with me in the classroom during teaching. While he also assisted through taking video records, he ensured that the day to day learning activities of the learners took place.

5.10 SUMMARY

In this chapter I presented the research approach which guided the study and the study design which was adopted. I then described the participants in the study and discussed the data gathering techniques which I employed. Furthermore, I discussed the qualitative data analysis method which I employed, outlined the quality criteria and, lastly, dealt with the ethical issues which were considered throughout the study.

CHAPTER 6: CONSTITUTED SOCIOMATHEMATICAL NORMS

6.1 INTRODUCTION

In this chapter, I present my analysis of data following the interpretive framework developed by Cobb and Yackel (1996) which I discussed at length earlier in Chapter 2. During the analysis of data, I focused on two constructs of the social perspective of Cobb and Yackel's (1996) emergent approach which I adopted as a theoretical lens through which I viewed the data. These constructs are; social norms and sociomathematical norms. However, their corresponding constructs from the psychological perspective were also considered during the analysis. As I indicated in Chapter 2, that the two perspectives coexist.

The analysis here focuses on the first two research questions, which are; (1) what are the sociomathematical norms constituted for promoting learners' proficiency in mathematics? and (2) how are the sociomathematical norms for promoting learners' proficiency in mathematics constituted and enacted in the classroom? Data associated with the third research question is analysed in Chapter 7.

6.2 FOCAL POINT OF THE ANALYSIS

During the analysis I focused on two issues. Firstly, I focused on the classroom social and sociomathematical norms and considered learners' mathematical beliefs and values, as well as their beliefs about their roles, roles of others, and the general nature of mathematical activity. Secondly, I focused on how the constitution and enactment of sociomathematical norms reorganised the mathematical classroom practice and considered learners' mathematical conception and activity. This led to an analysis of learners' achievement of mathematical proficiency stands which is indicative of their mathematical development.

The data is presented according to learning activities, that is, only data generated from a single learning activity was considered at a time. I chose to present the analysis this way in order to maintain the coherence of the classroom social interactions which took place in the classroom as learners engaged in the activity. Not all interactions are captured here, only excerpts of interactions which have the strength to showcase aspects of the classroom social interaction considered important for answering the research questions were selected. Correspondingly, Yackel and Cobb (1996) posits that "episodes [in my case excerpts] have been selected for the clarifying and explanatory power" (p. 459).

The analysis in this chapter is presented in two parts, each part represents data generated from a single learning activity, the justification of the chosen activity and excerpts of data is provided in the section that follows. Each part of the analysis here is presented in under three sub-headings. Sub-heading 1 is concerned with coming to grips with the learners' existing sociomathematical norms and initiating the renegotiation of such norms, as well as engendering new norms.

Sub-heading 2 documents and describes how I challenged the learners' existing sociomathematical norms and how that, in turn, created opportunities for negotiating new norms, as well as engaging in classroom discourse which could lead to promoting proficiency in mathematics. Sub-heading 3 captures the shedding off authority as learners authored new sociomathematical norms. Here learners were pressed or challenged to change from what was normative to them and jointly constitute new norms, which could be considered as taken-as-shared for the classroom. Under all the headings, an analysis of multiple opportunities created, used and/or missed for promoting proficiency in mathematics is also included. In cases where such opportunities were used, a discussion on learners' mathematical development is also included. While, in cases where such opportunities were not used, a discussion of possible mathematics learning which could have taken place is included.

6.3 PROLOGUE OF ACTIVITIES AND EXCERPTS

Data presented and analysed here emanates from two learning activities which were both based on analytical geometry. It should be noted that the activities do not represent a single lesson as, but instead were completed over 2 weeks in at least 10 periods of 45 minutes each. Learning Activity 1 (Figure 6 – 1) is chosen for analysis as it was the very first activity which the class encountered as part of my teaching. As such, it is best suited to tease out the learners' existing sociomathematical norms.

Correspondingly, the excepts chosen for analysis in part 1, emanates from item 1 of the learning activity and represents classroom engagements which took place over two days. Since data was generated in a natural setting of a classroom, it was impossible for me to detach from the learning experiences as I was the teacher responsible for facilitating learning. Hence, my interactions with the learners challenged what seemed to be their existing sociomathematical norms at the time and initiated the co-constitution of new sociomathematical norms. Even though, the analysis focuses only on item 1 of the learning activity, the excerpts are representative of what transpired almost throughout the activity.

Part 2 of the analysis is based on Learning Activity 2 (Figure 6 – 2) which was handled in the second week of data collection. Though the focus of the analysis is the same, but now the engagements show much different patterns of interactions as compared to learning activity 1. Although it offered insight of what learners were used to, but it was skewed more to how the learners defend their positions and also respond in shifting towards constituting new sociomathematical norms.

6.4 ANALYSIS PART 1: LEARNING ACTIVITY 1

Learning Activity 1

1. L$(-1\,;3)$, M$(7\,;1)$ and N$(x\,;2)$ are points in a Cartesian plane.
   Calculate x if:
   1.1 the gradient of MN is $\frac{1}{2}$.
   1.2 N is the midpoint of LM.
   1.3 the length of MN is $\sqrt{2}$ units.
   1.4 line MN is perpendicular to the x-axis.
   1.5 L, M and N lies on the same straight line.
2. Determine the values of x and y if M$(2\,;\,-3)$ is the midpoint of P$(3\,;8)$ and Q$(x\,;y)$.
3. E$(-4\,;\,-1)$, F$(2\,;3)$ and G$(6\,;\,-3)$ are vertices of a triangle.
   Prove that:
   3.1 ∆EFG is right-angled.
   3.2 $F\hat{E}G = 45°$
4. W$(0\,;\,4)$, X$(5\,;3)$ and V$(2\,;\,1)$ are vertices of a triangle.
   4.1 Prove that $\hat{V} = 90°$
   4.2 Determine the coordinates of Y and Z, if WXYZ is a rhombus with diagonals WY and XZ intersecting at V.
   4.3 Prove that WXYZ is a square.
5. A$(0\,;7)$, B$(9\,;9)$, C$(9\,;5)$ and D$(0\,;3)$ are vertices of a quadrilateral. Prove that ABCD is a parallelogram.

Figure 6 – 1: Learning activity 1 – assessment of learning focusing on grade 10 analytical geometry

I was expected to take the class through the learning of Grade 11 analytical geometry. Since it was expected that learners had learned this section in Grade 10, I decided to start with an activity that assessed what was learned in Grade 10. According to CAPS, it is expected that learners in Grade 10 achieve the following:

> Represent geometric figures in a Cartesian co-ordinate system, and derive and apply, for any two point $(x_1; y_1)$ and $(x_2; y_2)$, a formula for calculating:
>
> - the distance between the two points;
> - the gradient of the line segment joining the two points;
> - conditions for parallel and perpendicular lines; and
> - the co-ordinates of the mid-point of the line segment joining the two points (DBE, 2011, p. 15).

I requested the learners to form groups and to work through the learning activity provided. There were four groups with the number of members ranging from four to five. All groups consisted of members of mixed gender and of mixed abilities. The learning activity which learners were requested to work on is as captured in Figure 6 – 1 above.

6.4.1 Learners' existing sociomathematical norms

After the learners had formed the groups and had settled, there was total silence. Learners started attempting the activity individually. There were no forms of classroom interactions between the learners themselves or between me and the learners. As I observed learners' attempts to solve the problem, I noted, with regard to item 1.1. of the learning activity, that there were two different entry points into the solution by the learners. Some learners started off by writing the general equation of a straight line '$y = mx + c$', while others used the formula for calculating a gradient '$m = \frac{y_2 - y_1}{x_2 - x_1}$' as a starting point. The second entry point was the one I expected from the learners. As a result, I become more interested in the first entry point. Consequently, I started engaging with one group of learners, which consisted of four members. Initially two learners took part in the discussion while the other two were merely listening, but joined in at a later stage.

Excerpt 1

1.1 T What is this equation that you have written, for?

1.2 L1 It is for the equation of a straight line.

1.3 T Why do you want to use it?

1.4 L1 Because we are given the gradient and points on the line MN

1.5 T What do you intent to do with this information?

1.6 L1 I want to find the equation of the straight line.

1.7 L2 Why? What is the question?

Pause

1.8 L1 The question is we must calculate the value of x.

1.9 T So how do you intend to use the equation of the line that you are calculating in order to calculate the value x?

1.10 L1 Eish sir, I see I made a mistake, I must use the gradient formula because I am given gradient.

1.11 L2 Sir look at how I have done it (the attempt was as below), but I am stuck. My problem is that I do not know how to do this calculation going further but I know that the value of x is 9

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2-1}{x-7}$$
$$= \frac{1}{x-7}$$

1.12 T How do you know that the answer is 9?

1.13 L2 I think I guessed, so that is why I am asking you to help?

1.14 T I hear that, just tell me how you guessed the 9?

1.15 L2 Okay sir, the gradient is $\frac{1}{2}$ so already in the numerator I have 1 (referring to $\frac{1}{x-7}$) and I need 2 in the denominator. So I know that 9 – 7 = 2. But sir I know I must show the steps, so I don't know how to do it.

1.16 T L1, what do you think about her calculation?

1.17 L1 Sir, I was not paying attention because I wanted to do mine first so that I will be able to compare with her.

1.18 T Take a look at her calculation and listen to her concerns then share your thoughts with us.

Even though L1 was not done attempting to respond to the question he had had already written his new incomplete working out as below:

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{1}{2} = \frac{2-1}{x-7}$$

1.19 L1 (Looking at Leaner 2's attempt) You did not substitute the gradient of MN by ½

1.20 L2 Why should I do that, because I am calculating it?

1.21 L1 You used the information given about M and N but you did not use the ½ in your calculation, and the ½ which is the gradient is already given.

1.22 L2 Is he correct sir? (Looking at me)

The manner in which the discussion in excerpt 1 started (1.1 – 1.6) is an indication of my awareness of, and my attempt to, work within L1's ZPD. I started this discussion with an endeavour to discover L1's current knowledge and understanding, in order to provide scaffolds, if the need arose. However, L2's involvement in the discussion (1.7) disrupted what I had developed in mind about L1's approach to the question. L2 asked L1 two questions, the first question (why?) called for L1 to justify his approach. Even though justification of answers constitutes a social norm (Cobb & Yackel, 1996), at this point this was not the case as both Park (2015) and Sfard (2008) asserted that a particular practice must be observed at least three times before it can be regarded as normative. Additionally, L2's second question muted the creation of an opportunity for challenges and justification, which is a process through which norms are interactively constituted. This was case here as the question called for L1 to revisit the activity, suggesting that according to L2, he had missed what was asked. Hence, L1 did not respond to the why question. Instead, after re-examining the question, he abandoned his initial approach and opted to use the gradient formula (1.10). Even though my follow up question, posed after he revisited the question (1.9), explicitly asked how he would use the equation of the straight line to fulfil what was asked. L1 did not think about it but interpreted my question posed as an additional reason for him to justify that his initial approach was not acceptable.

In reflecting on the discussion thus far, it is clear that L2 played a role of the more knowledgeable other. However, her assistance provided to L1 was framed by her own approach to the question. I argue that L2 indirectly told L1 to use gradient formula or to change his approach because she did not give him time to justify his approach. This claim is supported by how quickly L2 moved to her own attempt (1.11), without asking further questions, after L1 had indicated that he was going to use the gradient formula (1.9) It could be L2 did this indirect telling as she had observed that I was attempting to assist L1 through questioning. Furthermore, an opportunity for engendering a sociomathematical norm of what counts as a mathematically different solution was missed. Here, I failed to mediate learning for L1, allowing him to carry on with his initial approach, as I wanted to accommodate L2's contribution as she was assuming the responsibility of assisting others, a responsibility which, in traditional classrooms, lies with a teacher.

In the same group, L2 presented her attempt (1.11) and claimed that she was stuck and needed help. Since she had already alluded the fact that the answer was 9, my interest turned to how she knew it was 9 (1.12), while she was more concerned with the algorithm. The explanation she offered (1.15) made mathematical sense and for me it was acceptable. Unfortunately for her, she regarded the process she had followed as guessing. It should be noted that, in her explanation, *conceptual understanding* (Kilpatrick et al., 2001) is reflected. Conceptual understanding has to do with the learners' comprehension of mathematical concepts, operations and relations (Kilpatrick et al., 2001) and the ability to apply ideas and justify their thinking (DBE, 2018). Correspondingly, L2 demonstrated an understanding of the equals sign as a mathematical operation and the concept of equivalence, in order to get to a correct answer. The conceptual explanation was, however, not sufficient for her. I had thought the omission of substitution of $\frac{1}{2}$ could be the reason. However, this was not the case per se as, even after L1 had indicated that she had omitted that (1.16 – 1.22), she still wanted to have an algorithm representative of her explanation systematically (in a step by step fashion).

The classroom social interactions which took place in the classroom as, captured in Excerpt 1, create opportunities for interactively constituting sociomathematical norms which could lead to promoting proficiency in mathematics. In this except, L1 had approached the question differently (1.6), that is, by finding the equation of a straight line first. Although this approach was abandoned as a result of his conversation with L2 (1.7), I tried to steer him back to this approach (1.9), but did not succeed. Since, at the time, the idea was to become aware of the classroom sociomathematical norms, I did not insist that he continued using the approach. However, I recognise that this was an opportunity to engender a sociomathematical norm of what counts as mathematically different solutions.

During the whole class discussion, I could have flagged the idea of first finding the equation of the straight line with the rest of the learners. However, the discourse was entrenched in issues about acceptable mathematical explanations and justifications. This opportunity, if explored, could have led to the promotion of proficiency in mathematics. As McClain and Cobb (2001) asserted, the establishment of sociomathematical norms directly influences our mathematical agenda. A link between two broad fields of mathematics (analytical geometry and functions) would have been made explicit. This link would have led to a discourse that involved the learners' ability to connect mathematical concepts, hence it could have promoted *conceptual understanding* (Kilpatrick et al., 2001). Furthermore, the link would have demanded that learners bring to the fore their knowledge about straight lines and finding points on straight lines. To achieve this, engagement in reflections involving mathematical representations were going to be imperative, hence promoting *adaptive reasoning* (Kilpatrick et al., 2001). The ability to think about this approach, which is not necessarily the expected one, is a reflection of *strategic competence* (Kilpatrick et al., 2001).

6.4.2 Disrupting learners' existing sociomathematical norms

At this stage it became apparent that, for L2, a systematic algorithm seems to be what counted as an acceptable mathematical explanation. As a result of observing other groups, I learned that the all the groups attempted the question and obtained the correct answer. Generally, the learners substituted m_{MN} by $\frac{1}{2}$ and the coordinates of M and N, and then did a cross multiplication, which produced a linear equation, which they solved and got 9 as the answer. At this point, I invited learners to participate in a whole class discussion. Since the idea in this lesson was to get to know what is normative for the class already, I decided to invite the whole class into a discussion (2.1) based on L2 conceptual explanation, in order to establish whether it was acceptable to them, as it was to me, or whether it was not acceptable to them, as it was not acceptable to L2. The whole class discussion, which was a bit rowdy, unfolded as captured in excerpt 2 below:

Excerpt 2

2.1 T Ok class, how did you respond to question 1.1. of the learning activity?

(There was noise for some time as learners concluded their discussion and I was also repeating the question)

After some time…

2.2 L3 We used the gradient formula, substituted the given coordinates of M and N, and the gradient of MN by $\frac{1}{2}$.

2.3 T (Wrote what the learner said on the board and as well as the final answer as below)

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{1}{2} = \frac{2 - 1}{x - 7}$$
$$x = 9$$

2.4 Ls Where did you get the 9? … you must show all steps. Do a reverse calculation from $x = 9$ and take us back where we started.

2.5 T We expect the same fractions on both sides of the equal sign, right?

2.6 Ls Yes

2.7 T Why is that the case?

2.8 L5 Because in step 2 we have two fractions with an equal sign meaning that they are equal.

2.9 T Okay, good! Can we all see that the numerator 1 on both sides?

2.10 Ls Yes sir

2.11 T In order to get equal fractions, what should the denominator be?

2.12 Ls Two (2).

2.13 T Good! I have a certain number (pointing at x) which I don't know, but know that when I subtract 7 from it (pointing at $x - 7$) the answer must be 2. What is that number?

2.14 Ls (Reluctantly) 9 (with others arguing that it is still guessing)

2.15 L4 But sir will they give us all the marks even if we did not show all the steps?

2.16 T Let us go back a little bit (I wrote on the board □ - 7 = 2)

Some learners started laughing and mumbling

2.17 T What would you write in that box while in primary school, say grade 1?

2.18 Ls 9 (with others laughing)

2.19 T Is that guessing?

Some learners were saying yes with some saying no, while others saying but we are in secondary school now.

2.20 T Then how □ - 7 = 2 different from $x - 7 = 2$?

2.21 L2 No sir, actually it is the same thing, just expressed differently.

2.22 L4 But sir still we must do all the steps.

The class shouted yes

2.23 T Why? Since it is clear where our answer comes from?

2.24 Ls For marks

2.25 T Okay, let us prolong the calculation just like most of you did then;

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{2} = \frac{2-1}{x-7}$$

$$\frac{1}{2} = \frac{1}{x-7}$$

$$x - 7 = 2$$

$$x = 2 + 7$$

$$x = 9$$

How many marks do you think this calculation is worth?

2.26 Ls 3 marks … 2 marks.

2.27 T Let us look at how marks will be allocated if it worth 3 marks

$m_{MN} = \frac{y_2-y_1}{x_2-x_1}$ ✓ for the formula

$\frac{1}{2} = \frac{2-1}{x-7}$ ✓ for correct substitution

$\frac{1}{2} = \frac{1}{x-7}$

$x - 7 = 2$

$x = 2 + 7$

$x = 9$ ✓ for the answer

2.28 T Can anyone explain how $x - 7 = 2$ was obtained?

2.29 L4 We did a cross multiplication.

2.30	T	What if I said we equated the denominators?
Silence		
2.31	L2	That will be correct still, I guess.
2.32	L4	But sir can we at any time equate the denominators or numerators?
2.33	T	I don't know. What do you think?
2.34	L4	Aah! But sir you are the teacher, so you must know.
2.35	T	Tell me what you think first. What do others think?
Silence		
2.36	T	Can we at any given time equate the numerators or denominators?
Silence		
2.37	L2	I think in this case we equated them because we are sure that the numerators are equal.
2.38	L3	I agree, if the numerators are equal then we can equate the denominators, if the denominators are equal then we can equate the
2.39	L4	But why?
Silence		
2.40	L2	Because we are sure that the two fractions are equal, isn't we wrote an equal sign between them?
2.41	L4	Sir is that correct?
2.42	T	Do you agree with them or not?
2.43	L4	But sir….(mumbling)
2.45	T	Okay guys, let's continue working on the rest of the questions.

When writing L3's response (2.2) on the board, I deliberately wrote down the final answer (2.3) in order to disrupt what, at the time, seemed to be the learners' sociomathematical norm of what counts as an acceptable mathematical explanation. The learners' reaction to this disruption (2.4) confirmed that, indeed, the learners felt that it is normative to present solutions in a way that steps are linked algorithmically, not conceptually, per se. This was evident as the learners put forward their demands for the attempt to be acceptable. They wanted all steps to be made explicit, while other learners wanted a reverse calculation to be done. These two demands are reflective of the learners' norms in relation to acceptable mathematical justifications. This is the case as, not only did the learners ask where I got the 9 (2.4), but also provided ways in which they expected the justification.

The first expectation, showing all steps, could be seen as aligned to the mathematical proficiency strand of procedural fluency. Unfortunately for this class, this does not seem to be case, as they were separating procedures from understanding. Kilpatrick and colleagues (2001) emphasised the intertwined nature of the strand, an aspect that is lacking this case. As a result, these learners have *restricted procedural fluency* (Graven & Stott, 2012). In unpacking procedural fluency, Kilpatrick and colleagues (2001) also made mention of procedures carried out ‘efficiently’. Efficiency, according to the dictionary, refers to ‘achieving maximum production with minimum wasted effort’. The learners’ preferred way of working out (2.25) is clearly not an efficient one. Therefore, this norm does not promote learners’ proficiency in mathematics.

The second expectation, that of doing a reverse calculation, is concerning and, yet again, shows the learners’ lack of understanding required when doing mathematics. This approach is generally used during the teaching of factorisation, where finding the product is the reverse operation. Although prone to creating misconceptions, this thinking is prevalent in teaching of mathematics. Other examples include multiplication and division, addition and subtraction, differentiating and integrating, exponents and surds, just to mention a few. However, if over generalised and applied without understanding they mislead learners, and it seems even in this case, learners over generalised this idea. This could be regarded as *instrumental understanding* (Skemp, 1976). Justification of answers has to do adaptive reasoning and, in this case, the learners preferred approach to justification lacks logical thought (Kilpatrick et al., 2001).

Through a question and answer type of discourse (2.5 – 2.22), I attempted to provide a conceptual justification for the answer. This was an attempt to start negotiating with learners in order for them to reconsider their normative stance on what counts as acceptable mathematical explanations. However, this was blandly rejected by the learners (2.22) and the majority of the class supported this rejection.

When I probed to determine why the learners insisted on taking long steps (2.23), the learners indicated it was for marks (2.24). This response was indicative of the fact that we had reached a consensus that there was no guessing involved in getting to the answer. Furthermore, the solution, together with its explanation and justification, was acceptable. After disrupting learners' concerns about marks, by showing them that the other steps which they insist should be there, were not worth any marks (2.26), the class gave up on the argument. This does not mean they had accepted that long steps were not necessarily important.

As noted in Except 1 earlier, and even in the discussion in Excerpt 2, there were a number of opportunities for promoting proficiency in mathematics. As in Excerpt 1, these opportunities were not explored fully as the idea was to get to know the learners' existing sociomathematical norms. There were two classroom sociomathematical norms which were at play during the discussion, namely, what counts as an acceptable mathematical explanation and justification. Unlike in Excerpt 1, though, I was trying to understand these norms from the learners' perspective while, at the same time, an inconclusive renegotiation of these norms took place. The negotiation of these norms was discussed in earlier in this section. Here the focus is on opportunities for promoting proficiency in mathematics. One of the traits of mathematical proficiency is the ability to translate between various forms of mathematical representation (NCTM, 2000). In particular, the discussion yielded the mathematical idea presented as words (2.13), symbols (2.16) and algebraically (2.20). This multidimensional way of looking at the same mathematical idea involves the capacity for logical thought and reflection, which are steeped in *adaptive reasoning*.

The learners engaged in reflecting on what was under discussion. Throughout their discussion (2.32 – 2.38) the learners were attempting to make sense, while, at the same time, they were attempting to come up with a way to generalise their argument. L4's question (2.32), was actually inquiring whether this argument could be applied to any equations involving fractions. As mentioned earlier, learners tend to rush to generalisation without a proper understanding of the concepts and, hence, end up applying their conjectures, even in cases where they cannot be applied. This question (2.32), if explored, could have opened new arguments, which would have assisted learners to develop a better understanding of fractions, equivalence and number sense, in particular. For example, in scaffolding for L4, within his ZPD, I could have given the equation; $\frac{1}{2} = \frac{x}{x-7}$ and asked him his own question "can we at anytime equate the denominators or numerator?" If he was to follow this argument, then two different values of x were going to be obtained from the numerator and the denominator. Through mediation, the argument could have been steered to $x = -7$, then the right side of the equation would be $\frac{-7}{-14}$. This would now bring a new dimension into the discussion, as learners would have had yet another opportunity to think about the equals sign to mean 'equal' and not necessarily 'same'. It is these forms of engagement which could lead to a relational understanding (Skemp, 1976). Yet again, using the learners' ideas to mediate learning is desirable (Schoenfeld & Kilpatrick, 2008), as such ideas are within their ZDP and, through assistance, they are more likely to pursue them on their own.

6.4.3 Constitution of 'new' productive sociomathematical norms

In this section of the analysis the focus was on instances where learners' are pressed or challenged to deviate from their usual sociomathematical norms due to the nature of the mathematical problem at hand. As much as learners have their own acceptable ways of presenting, explaining and justifying their solutions, these problems create a dilemma for them, as they do not conform to these norms. Therefore, they are faced with dilemmas in that the explanations and justifications they have are not their preferred ones and, yet, they could not come up with their preferred explanations and justifications in this case. In this case, though I offered the learners scaffolds to help them to access the mathematics, I did not put forward the sociomathematical norms as rules to be followed but instead allowed for them to be interactively constituted as we engaged in mathematical talk. Within a sociocultural perspective, such instances of renegotiations of ways of doing things is expected, as culture is not static but, instead, it is volatile and it evolves.

Excerpt 3 below captures the discussions based on the item 1.4. of learning activity 1, which first began in group 3 and was later taken to the whole class as the three other groups were also struggling with the same issue. Learners were requested to calculate x if line MN is perpendicular to the x-axis, while given their coordinates as M$(7\,;1)$ and N$(x\,;2)$. In all the four groups there was at least one learner who wrote '$m_1 \times m_2 = -1$', however, the learner could not proceed with the calculation. This way of approaching questions was also seen in Excerpt 1. For these learners, it seems that their choice of procedures to follow is not informed by the conception of what is given to, or required of, them. Instead, they memorise facts about mathematical relations and associate such facts with terms and formulae.

Excerpt 3

3.1	L9	Sir the information given is incomplete for us to answer this question.
3.2	T	Okay, tell us why you are saying so.
3.3	L9	You see sir if we use the formula $m_1 \times m_2 = -1$, we will use coordinates of M and N for gradient 1, so for gradient 2 which is for the x-axis we must be given either coordinates of some points there or the actual gradient.
3.4	T	What could be the other way to answer without using that formula?
3.5	L12	No sir, there is no other way, if the say lines are perpendicular, we must use that formula. If they are parallel we equate the gradients.
3.6	L9	That is why I am saying the information given is not enough.
3.7	T	So you want coordinates of points on the x-axis or the gradient of the x-axis?
3.8	L9	Yes sir
3.9	T	Draw a Cartesian plane and indicate x-axis as well y-axis, check if you cannot get the coordinates that you want.

Later

3.10	L9	We cannot just pick points on the x-axis because we don't know if MN is

perpendicular to x-axis on the negative or positive side (pointing on her Cartesian plane)

3.11 L11 But we can use any points to calculate the gradient as it the same throughout the x-axis.

3.12 L9 Okay lets take (2 ; 0) and (3 ; 0), ahh the gradient will be 0. If we substitute in the formula then everything is going to be zero.

3.13 L12 Then we can say no solution because we will be getting 0 = - 1 which is Impossible.

3.14 T No, there is a way to answer that question, forget the formula a little bit.

3.15 L9 Tell us what to do, at least give us a hint then.

3.16 T No, talk about it and find a way as a group.

Later

3.17 L9 Sir, we got it (excited)

3.18 T That is good let me see your calculation

3.19 L9 There is no calculation but we understand it and can explain it, is there any group which got it?

3.20 T Not as yet

3.21 L9 Can I explain it in front to everyone?

3.22 T Yes, go ahead

3.23 L9 Guys, guys listen I want to explain 1.4. to you

Few seconds later…

3.24 L9 The question says, we must calculate x if line MN is perpendicular to the x-axis. So we cannot use the formula for gradients of perpendicular lines because we are given information about only 1 line. Then, in our group we draw the Cartesian plane (drawing it on the board), look at the x-axis and realised it has a gradient of zero (0), see why we cannot use the formula?

3.25 Ls No (shouting)

3.26 L9 Check if you substitute by zero on $m_1 \times m_2 = -1$, then you will be left with 0 = – 1. So what we did was to try plot the points M and N on the Cartesian plane. It was easy to plot M(7 ; 1) because we know the exact point. So if a we draw a line passing through M such that it is perpendicular to the x-axis, it is a like a vertical line (drawing on the board). On this line the value of x at any point is 7, so even where y is 2, the value x is 7.

3.27 L4 So if it was asked in the test what would you write, if they said show all calculations?

3.28 L9 I will just write $x = 7$, because there is no calculation that I can do here.

3.29 L4 What if you just guessed or copied the answer from another person?

3.30 L12 Then you will write an explanation that a line perpendicular to the x-axis is a vertical line and has a constant x value throughout.

When the learners claimed that the information provided was insufficient for question to be responded to (3.1), I asked them why they thought this (3.2). My question called on them to justify their claim. Interestingly, the learners provided a procedural justification (3.3). That is, their reason focused on how the formula (or procedure) they chose could not be used on the basis of the information at their disposal. If learners possess relational understanding (Skemp, 1976), their choice of procedure must be informed by their understanding of the question and the mathematical concepts involved. Furthermore, learners are expected to be able to identify appropriate strategies (DBE, 2018). In this case the learners failed to identify an appropriate strategy, which was an indication that they lacked the strategic competence to do so (Kilpatrick et al., 2001).

If an appropriate strategy is not readily available, in order to demonstrate strategic competence learners are expected to devise their own strategies (DBE, 2018). Furthermore, these abilities require of them to have both the understanding of the concepts and the ability to carry out the procedures (Van De Walle et al., 2010). Devising their own strategies appeared to be something foreign to this class, as everyone (line L9) expected me to provide them with a strategy (3.15). However, I offered the learners scaffolds and mediated their learning through leading questions (3.4, 3.7, and 3.9). Learners engaged in a discourse that allowed for various mathematical concepts and arguments to be reflected upon (3.10 – 3.15). Through this back and forth discussion, learners grappled with devising their own strategy but mostly they fell back to their procedural reasoning (3.12 – 3.13). Finally, they broke through and arrived at the answer. To corroborate their existing norms, I requested to see their workings out (3.18). They indicated that they did not have workings out but, instead, had an understandable explanation (3.19). This was the first sociomathematical norm constituted by this group and they were ready to share their attempt with the whole class (3.21). Since the norm of sharing and comparing solutions was already enacted within the groups, L9 wanted to expand this norm across the groups (3.21).

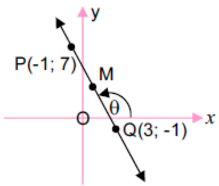
In presenting the group's attempt, L9 started by justifying why they transgressed an endorsed norm (3.24), a justification which the class rejected (3.25). He then demonstrated how their usual approach could not work, and presented how his group approached the question (3.26). Even though most learners seemed to be receptive to the explanation, there was an undertone of dissatisfaction, which L4's question (3.27) confirmed. However, L12 justified the explanation and addressed how the attempt could be represented in text. This represents, yet again, another shift from previously endorsed norms to authoring new norms. Previously, learners wanted solutions to be presented to them in a step by step manner for the purposes of marking. Now, inherent in L4's questions (3.27 and 3.29), it was clear that focus was no longer on marks but on presenting solutions textually and in a way that made sense to anyone who read them. Here again, there is another sociomathematical norm that the class constituted, namely, that acceptable mathematical explanations must be understandable when presented textually.

In terms of promoting proficiency in mathematics, which is the core reason for engaging in mathematical classroom discourse in this study, two opportunities were created. Firstly, learners had to strive towards strategic competence, in this case, by not choosing an appropriate procedure but, instead, by devising a new one. Secondly, the learners had to engage in adaptive reasoning. This was done through navigating multiple representations, and by proving and disproving certain procedures. Implicitly, learners learned the limitations of first choosing a formula based on certain phrases or bits of information given, instead of examining all the given information as a whole.

6.5 ANALYSIS PART 2: LEARNING ACTIVITY 2

Learning Activity 2

1. P(- 1 ; 7) and Q(3 ; -1) are two points on a straight line in a Cartesian plane as shown in the diagram below.



Determine:

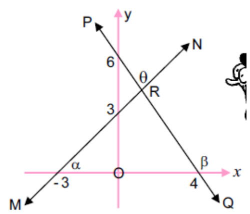
1.1 the equation of line PQ.

1.2 the size of θ.

1.3 the equation of the line which is parallel to PQ and passes through the point (-5 ; 1).

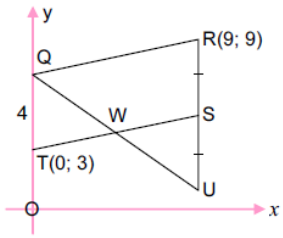
1.4 the equation of the line which is perpendicular to PQ and passes through M, the midpoint of PQ.

2. Determine the angle of inclination of a straight line with the equation $2x + 3y = 5$.
3. Determine the numerical value of p if the straight line defined by $2y = px + 1$ has an angle of inclination of 135°.
4. In the sketch below, lines PQ and MN are shown together with their intercepts with the axes.



Determine the size of θ.

5. In the figure below, QRST is a parallelogram with vertices Q and T lying on the y-axis. The side RS is produced to U such that RS = SU. The length of QT is 4 units and the coordinates of R and T are (9 ; 9) and (0 ; 3) respectively. The line segment QU intersects TS at W.



5.1 Determine the coordinates of Q and U.

5.2 If W is the midpoint of UQ, determine whether W lies on line OR.

5.3 Determine the size of angle $Q\widehat{W}S$.

Figure 6 – 2: Learning activity 2 – assessment for learning focusing on grade 11 analytical geometry

In learning activity two (Figure 6 – 2) the focus was now on Grade 11 analytical geometry. The learners continued to work in their groups in order to achieve the following learning outcome as stated in CAPS:

> Derive and apply:
> - the equation of a line through two given points;
> - the equation of a line through one point and parallel or perpendicular to a given line; and
> - the inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line and $0° \leq \theta \leq 180°$. (DBE, 2011, p. 31).

Bullet 1 and 2 represents a combination of work learned in Grade 9 on linear functions, and in Grade 10 on analytical geometry, in particular the gradient of a straight line, including gradients of parallel and perpendicular lines. Hence, questions addressing bullet 1 and 2 were only of an application nature. Bullet 3, on the other hand, was something new which the learners had to learn, but also involved a combination of knowledge on gradients and trigonometric ratios, which learners had already acquired in previous grades. Hence, the first lesson on this activity started with a whole class discussion in which I took learners through the derivation of an inclination formula, '$m = \tan\theta$', followed by the learners working on application questions, focusing on all three bullets.

The whole class discussion unfolded as a question and answer session, which started with fixed points P and Q and ended with the formula $\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$ as summarised in Figure 5 – 3 below. For the analysis of data, which emerged through learning activity 2, I considered three classroom social interactions. These social interactions took place during this whole class teaching, in learners' small groups and when we later engaged in whole class discussions where learners shared their solutions we considered.

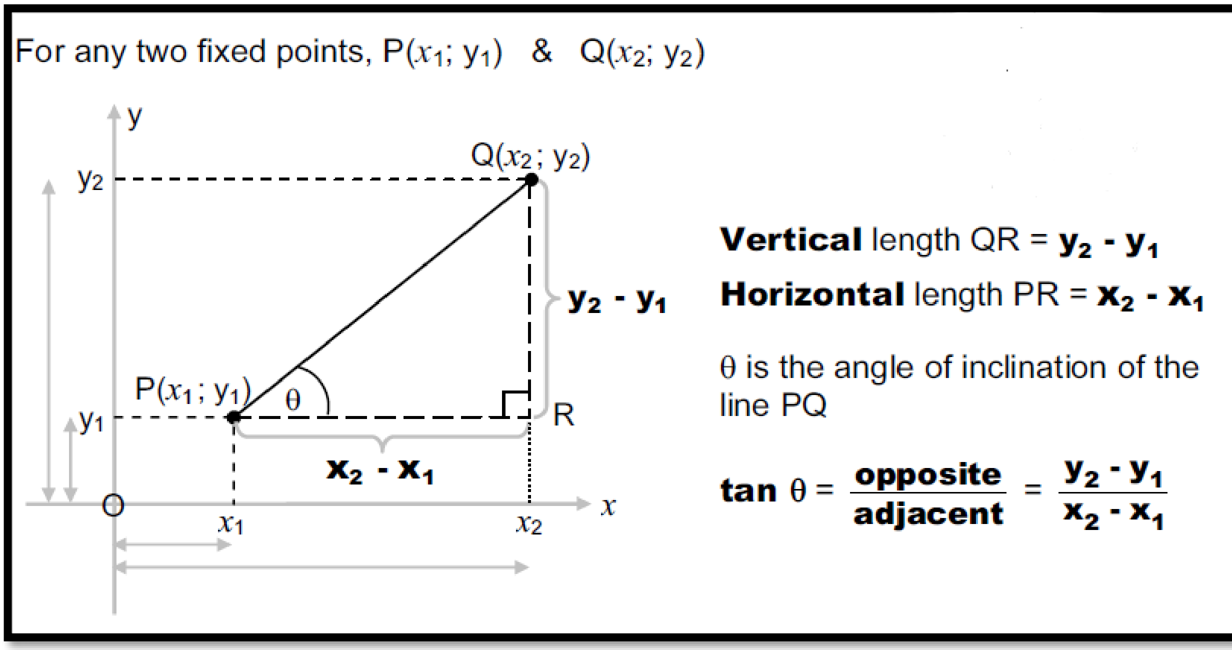


Figure 6 – 3: Summary of chalkboard writings for a discussion on the derivation of a formula for angle of inclination

6.5.1 Learners' existing sociomathematical norms

During the whole-class discussion, learners were able to answer questions which related the coordinates of R, and the length of QR and PR. Finally, the formula $\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$ was derived and accepted by the class. At the end of the discussion, when I requested learners to attempt questions in learning activity 2 (Figure 6 – 2), concerns emerged. Two of the concerns were in the form of questions and the other concern was in the form of a request. Learners asked whether they would be requested to derive the formula in a test and when should they apply the formula. They also requested me to present an already worked out example in which I apply the formula, so that I could show them how the formula is applied.

Since the learners did not ask why we derived the formula, it could mean that they saw the value of the derivation. However, the question "*Can they request* *us to derive this formula in a test*" is an indication that learners regard only those aspects of mathematics which will be examined as important. This question resonates with numerous research findings which indicate that, in most South African classrooms, teaching is mainly examination driven. This is a limitation of the South African mathematics curriculum, as some of the knowledge and skills that are prescribed for teaching and learning are not examined. As a result, most teachers do not teach this knowledge and these skills, which are important for insight understanding of concepts to be developed. The second question "*when must we use this formula*" indicated that the norm for this class is that mathematical formulae must be accompanied by specific conditions under which they are operational.

As noted earlier in excerpt 3, learners tend to associate formulae with certain phrases and end up over generalising and applying the formulae without understanding. The request for a worked out example, on the other hand, is a reflection of the mathematical classroom practice that learners are exposed to. When I responded "*No*" to the first question, some learners asked why did we had derived the formula. Furthermore, I indicated that the formula should be applied whenever it is appropriate to do so, a response which the most learners seemed unhappy with. Similarly, learners were not happy when I indicated that a worked out example was not necessary in this case. It was these interactions which provided me with a window through which I could see the learners' existing classroom practice, together with normative aspects associated with their mathematics classroom discourse.

For me, as guided by a socio-cultural perspective to teaching and learning, I saw it possible for the learners to attempt the activity, as all the questions were within their ZPD. Additionally, deriving the formula for the angle of inclination had provided learners with artefacts (Leung & Frant, 2015) which served as visual mediators (Forman, 2003) that could assist the learners in navigating learning within their ZPD (Christmas et al., 2013). Subsequently, learners attempted to the activity in their small groups. This led to the emergence of a second whole-class discussion, this time based on the learners' solution to question 1.2 of the learning activity. All groups successfully arrived at $\tan\theta = -2$, two groups had $\theta = 63{,}43°$ and one group had $\theta = -63{,}43°$ as their final answers, while the other group provided an inconclusive working out.

When learners presented their solutions to the whole class, the group with an inconclusive attempt asked whether the presented answers made sense as, according to them, θ is an obtuse angle. Learners argued that they applied the formula given, so maybe the diagram is not drawn to scale. When an enquiry was made into the different answers, the group with $\theta = -63{,}43°$ accepted that their answer was not correct as they had used –2, instead of just using 2. When asked why that happened, they indicated that the negative selected a quadrant, and went further to say that, in this case, it meant the second quadrant. This got the group with an inconclusive attempt excited, as it supported their earlier comment that the angle must be obtuse. Through this engagement, the learners agreed that the correct answer was $116{,}57°$. However, they complained that I should have done a worked out example for them, saying that, had I done so, they would not have wasted time. Furthermore, learners attempted to generalise the notion that, when the gradient is negative, it means the angle is obtuse and when it is positive it means the angle is acute.

As reflected earlier in excerpts 1 – 3, learners continued to provide procedural justifications, although this time they did not insist on their justification as the only acceptable justification. They allowed room for other arguments and amended their workings out as they discussed and reached an agreement. In this way the learners portrayed new sociomathematical norms of what counted as mathematically acceptable and different solutions. Here the correctness of the answer lay with the learners and they did not ask me if it was correct or not. Learners' were not persistent regarding the norms which they enacted while working on learning activity 1. They were receptive to the constitution of new norms, an indication that, indeed, classroom culture is not static but, instead, it evolves.

6.5.2 Disrupting learners' existing sociomathematical norms

Since I had already established that learners tend apply the stepped approach when applying formulae and procedures, without an understanding of questions and underlying concepts, in compiling the learning activity, care was taken to include questions, which did not conform to the overuse and application of formulae without understanding, hence, the choice for inclusion of questions 2 – 5, In questions 2 and 3 learners had to apply the formula and regarded the coefficient of x as the gradient, without noticing that the equations were not written in the standard form of $y = mx + c$. Whereas in question 4, learners wrote $m = \tan\theta$ as their entry into the problem and argued whether to use the gradient of PQ or that MN.

The learners ultimately realised that the equations had to be presented in standard form before the gradient could be read and that θ is not even an angle of inclination. These questions allowed learners to develop a comprehension of mathematical concepts and relations. Although the lesson only linked the gradient to the angle of inclination. The activity brought the gradient in the form linear equations. Hence, an opportunity to develop proficiency in mathematics was created and the learners engaged in it. Correspondingly, with regard to procedural fluency two aspects were at the fore now, the ability to carry out procedures both flexibly and appropriately. Flexibility is seen here when the learners were expected to think of a gradient of a line in a variety of ways before applying a formula. Appropriateness, on the other hand, is when the learners have to make sure that, indeed, a particular angle is an angle of inclination, before applying the formula.

Question five, on the other hand, required that learners use reasoning and understanding in order to respond to questions, as opposed doing algorithmic calculations, as noted in excerpts 1 and 2, in order to arrive at the answers. This was a disruption to what was normative for these learners but, since it was at point where learners were open to renegotiation their existing sociomathematical norms, as well open to interactively constituting new norms, there was little resistance. Immediately the learners realised there was insufficient information given for them to apply the formulae. They did not claim that the question could not be answered, as in learning activity 1. Instead, the learners explored ways to respond to the questions and demanded justifications from their group members for any claims made in terms providing answers. It was easy for all groups to come up with the *x* coordinates for both Q and U but, for the corresponding *y* coordinates, a great deal of back and forth discussions unfolded.

This question also allowed learners to engage in mathematics discourse which provided opportunities for developing proficiency in mathematics. Although the intertwined nature of the strands is acknowledged, for this question, strategic competence and adaptive reasoning were the main strands at play, while procedural fluency and conceptual understanding only came in in support of the strategic competence and adaptive reasoning. Learners had to devise their own strategies (DBE, 2018) and provide explanations and justifications for their solutions (Kilpatrick et al., 2001). Question 5.3, in particular, provided learners with the opportunity to engage in their capacity for logical thought and reflection, as well as both their inductive and deductive reasoning.

6.5.3 Constitution of ‘new’ productive sociomathematical norms

Unlike in learning activity 1, where even though learners acknowledged a need for new ways of doing mathematics but they still started by disproving their normative ways, here learners’ were receptive to new sociomathematical norms. When sharing solutions with their peers, the expectation was that they would explain why what was regarded as normative could not be used. During the discussions based on learning activity 2, learners engaged in a discourse which always ended with a taken-as-shared meaning. The class agreed that, if you write an answer only and could show it was obtained through calculation, then a brief explanation must accompany the solutions. For the *x* coordinates of Q and U some explanations were captured as "*Q and U are vertical to T and R respectively"* and, in some cases, as "*QT and RU are vertical lines*". These reasons were provided to support the notion that the *x* coordinate for vertical points is the same.

6.6 SUMMARY

It this chapter I presented and analysis of two learning activities wherein I focused on similar aspects. At the core of the analysis was to identify sociomathematical norms constituted for promoting proficiency in mathematics and how such sociomathematical norms are constituted. The excepts of classroom interactions captured in the chapter evidenced that classroom sociomathematical norms are not rigid and through continuous negotiation they are interactively (re-) constituted. The class initially portrayed that acceptable mathematical explanations and justifications where those provided through step by step algorithms.

Through challenging learners' position and careful selection of mathematical tasks the notion of acceptable mathematical explanations and justification was reconstituted differently. Amongst others, an acceptable explanation must be conceptual and understandable when presented in text. This shift made a significate move towards promoting proficiency in mathematics. It became apparent that I played a vital role in orchestrating the classroom discourse to develop opportunities for disrupting norms which do not promote proficiency in mathematics and instead interactively support the class in constituting sociomathematical norms which promote proficiency in mathematics.

CHAPTER 7: ENACTMENT OF SOCIOMATHEMATICAL NORMS

7.1 INTRODUCTION

This chapter is the second one focusing on analysing data. The set of data analysed here was considered for answering the third and last research question raised in the study. The question as stated in Chapter 1 was posed as “How does the enactment of the constituted sociomathematical norms promote learners’ proficiency in mathematics?” Data in the sections were generated from the start of the seventh week of data collection. This is an indicated that the sense of what was taken-as-shared for the classroom had reached a more stable stage as compared to when I initially started teaching the class at the beginning of the data collection process.

7.2 FOCAL POINT OF THE ANALYSIS

Throughout this analysis, the focus was on how the learners enacted sociomathematical norms and the enactment of the sociomathematical norms promotes learners’ proficiency in mathematics. With regard to the enactment of sociomathematical norms, the three theoretical constructs of the psychological perspective of Cobb and Yackel’s (1996) emergent approach guided the analysis. These are, firstly, learners’ beliefs about their role, role of others and general nature of mathematical activity; secondly, learners’ mathematical beliefs and values and thirdly, the learners’ mathematical conceptions and activity. This approach to analysis does not imply that these constructs are given primacy over their corresponding social perspective constructs, since the two perspectives coexist and develop or evolve concurrently.

Data analysis is also presented under a three headings, which relates to the sociomathematical norms which were constituted in the classroom as they emerged from data analysed in chapter 6. Heading 1 is concerned with the learners' enactment of sociomathematical norms as they engaged with the situations or challenges and justifications (Cobb et al., 1992) during the paired, small group and whole class interaction stages. Heading 2 captures the learners' enactment of the sociomathematical norms of what counts as acceptable mathematical explanations and justifications. Here again, the extent to which this enactment promotes proficiency in mathematics is accounted for. Heading 3 documents and describes how learners enacted the sociomathematical norm of what counts as a mathematically different solution, and how this enactment led to promoting proficiency in mathematics.

As in the analysis of learners' constitution of sociomathematical norms earlier, here mathematical proficiency strands will be considered to account for how enactment promotes the learners' proficiency in mathematics. While the previous analysis mostly looked at the opportunities created, regardless of whether they were used or not, this analysis focused on whether or not the enactment of sociomathematical norms enabled the learners to achieve the strands of mathematical proficiency which, for this study, is regarded as indicative of the learners' mathematical development. The data here are still presented according to the activities in order to preserve the coherence of the classroom social interactions which took place as the learners attempted the learning activities.

7.3 PROLOGUE OF ACTIVITIES AND EXCERPTS

The data presented and analysed here emanate from learning activity 3 (Annexure F), which was more of a learning unit and consisted of a series of four activities. This set of activities was presented in a unit on solving problems in the 2-dimensional plane using trigonometry. Activity 1 was started on the Monday of the seventh week of data collection, and the whole learning activity was completed in week 10.

This was time when I had already developed rapport with the learners, and the majority of class was in harmony in how I facilitated learning and teaching. Even though there were instances were learners wanted to revert to their old ways of doing things but this was seldom. The learning activity was chosen because of its power to portray a picture of the extent to which the sociomathematical norms had shifted over time. The chosen excerpts were taken from classroom interactions in which the concepts to learned were developed.

In particular, excepts of interactions are focusing on the opening section of the learning activity 3, where learners revise the prior knowledge of trigonometric ratios and these lead to them seeing their limitation in finding solutions of triangles. Correspondingly, this is followed by the derivation and application of the sine rule, even though this took long but learning of the cosine and area rule was much quicker. As learners had noticed from sine rule, that they must derive it, know conditions under which it is applied and then use it for determining heights, distances and angles. Therefore, it is needless to say the excerpts chosen were more data rich in that they clearly portray how the enactment of the sociomathematical norms played out, whereas in lessons which follows the enactment was more sophisticated.

7.4 ANALYSIS PART 3: LEARNING ACTIVITY 3

The learning outcomes for learning activity three are stated in CAPS as:

- Prove and apply the sine, cosine and area rules.
- Solve problems in two dimensions using the sine, cosine and area rules.
  (DBE, 2011, p. 37).

7.4.1 *Engagement in situation for challenges and justifications*

During the seventh week of data collection, the classes indicated that they preferred to work in pairs (Figure 7 – 1) and not in groups as we had been doing. Without asking them why, I accepted this as the preferred system of the community's classroom collective activity (Turpen & Finkelstein, 2010). Furthermore, working in pairs would still create opportunities for them to engage in the social interactions which form the basis for the emergence of classroom social norms (Cobb et al., 1992; Much & Shweder, 1978). Subsequently, sociomathematical norms would be enacted as the learners, in their pairs, engaged in the situation for challenges and justifications, as outlined in Chapter 3. In particular, as the learners engaged in mathematical activity, the learners would share, challenge and justify their solutions.

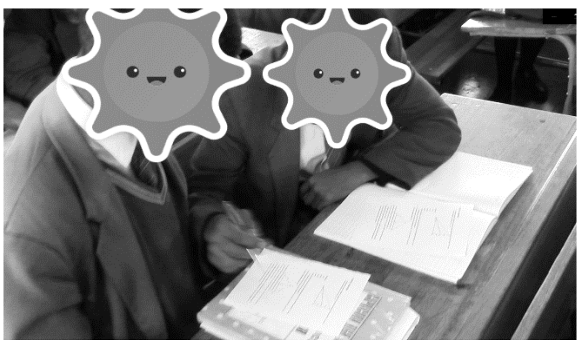


Figure 7 – 1: Learning working in pairs

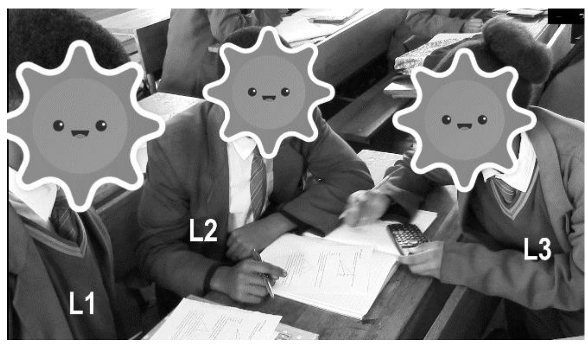
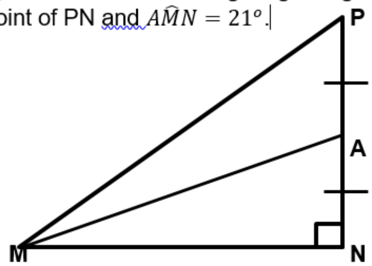


Figure 7 – 2: Sharing solutions with other pairs

Even though the class had agreed to working in pairs, some learners still engaged in social interactions with pairs closer to them. For example, L3 in Figure 7 – 2 turned back to hold a discussion with the pair behind her. This is indicative of the enactment of the social norm of sharing solutions. Excerpt 4 below captures how the discussion unfolded and it is followed by an analysis and interpretation of these learners' beliefs about their roles, the roles of others, the general nature of mathematical activity in the classroom, mathematical conceptions and activity, as well as their mathematical beliefs and values. The latter can be accounted for as the theoretical framework adopted as discussed in Chapter 4 clearly outlined how the sociological and psychological perspectives of the framework coexisted and had as reflexive relationship hence one would not exist without the other. The discussion in Excerpt 4 was based on question 1 of activity 1 in learning activity 3, and is captured in Figure 7 – 3 below.

Learning Activity 3
Activity 1

1. In the sketch below, ΔMNP is drawn having a right angle at N and MN = 15 units. A is the midpoint of PN and $A\hat{M}N = 21°$.



Calculate

1.1 AN

1.2 AM

1.3 $P\hat{M}N$

1.4 MP

1.5 If the ΔMNP was not a right angled triangle would you still respond to 1.1 and 1.2 above as you did? Provide a reason.

Figure 7 – 3: Question 1 of activity 1 in Learning activity 3

Excerpt **4**

4.1	L3	Can I ask.
Inaudible		
4.2	T	I want to hear the question
4.3	L3	When calculating AN, I just take it line since it is the sine rule, and then I can use sine. As in line use the angle that is given 21° and I use the 15 units length. Then I will calculate $15 \sin 21°$, will that give the angle of AN.
4.4	L2	No, you mean the length of AN
4.5	L3	Yes, the units for AN. Will I be right?
Silence		
4.6	L3	I am asking?

4.7 L2 That is what we were still talking about, we will get back to you.

4.8 L3 All I am saying is they gave us the angle 21º and 15 units (writing on the diagram). I am asking if can you use these two (referring to 21º and 15 units) to find out AN?

4.9 L2 Yes, you can use them.

4.10 L1 That is what you are supposed to do.

4.11 L3 Oh okay, thank you!

4.12 T So what do you intend to do?

4.13 L3 Use the 21º, the 15 units and sine to

4.14 L1 To get the length of AN.

4.15 T I want to know, why you decided to use sine.

4.16 L2 Because it has opposite and adjacent (pointing the unknown side AN and given side with length 15 units).

4.17 L3 Ah, you are lying, you are lying.

4.18 L1 We decided to use tan (With L2 saying that as well). So we going to have tan theta is equal to opposite over adjacent $\left(\text{writing } \tan 21° = \frac{AN}{15}\right)$.

4.19 L3 Oh yeah, you are right.

4.20 L1 So sir, even if the topic is sine rule it doesn't mean we going to use sine rule?

The manner in which the discussion in Excerpt 4 started reflects two beliefs held by L3. Firstly, that it is her role to seek for help, clarity or confirmation (4.3 – 4.4) when she is not certain about the attempt to a question and, secondly, that her classmates, and not necessarily me, should play the role of providing her with whatever assistance she required. The second belief is supported by L3's insistence (4.8) that they should respond to her query, even after they had indicated to her that they would do so at a later stage (4.7). This instance, itself, reflects her third belief about the general nature of mathematical activity in the classroom. L3 enacted sociomathematical norms of what counted as acceptable and significantly different solutions. As Cobb and Yackel (1996) noted that "the sociomathematical norm of mathematical difference appeared to emerge in the course of joint activity" (p. 179). It is for this reason that L3 insisted on jointly working on the question with the other two learners. This she did by sharing her understanding of what asked and the approach she thought of using (4.8). However, the joint enactment of this norm could not be realised as the other pair did not have their own solution to compare with that of L3.

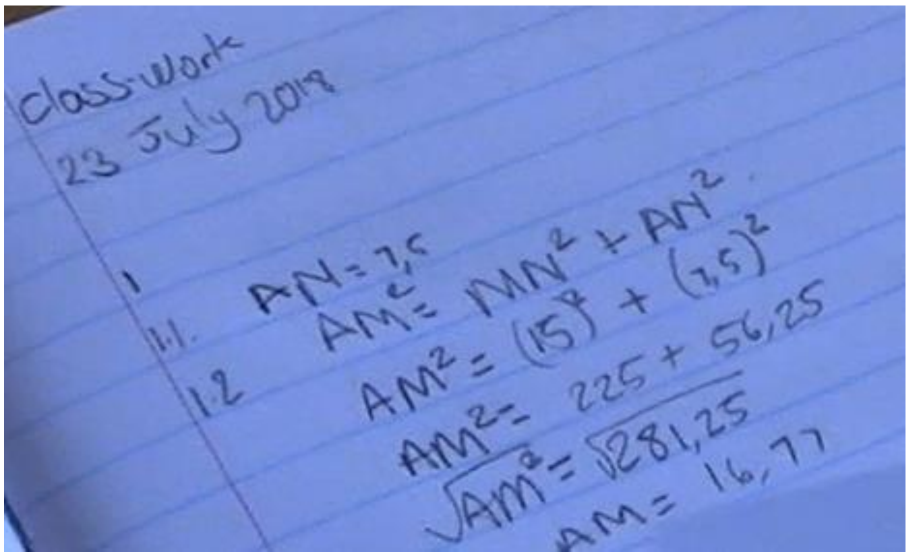
L1 and L2 were very quick to agree with her faulty approach, as they had not yet applied their minds on the question. This signifies the importance of learner-learner interactions being proceeded by learner-material interactions. When I, ultimately, re-invited the learners into the discussion (4.12) that they had concluded (4.11), they managed to amend their solution and came up with a correct one through collaboration. Interestingly, when I asked L3 why she decided to use the sine ratio (4.15), L2 responded instead. This was indicative of the fact that what they had agreed upon now was taken-as-shared for these three learners, However, L3 did not accept L2's justification. This, in turn, led to the learners coming up with a correct solution, as L1 indicated that tan should be used instead (4.18). L1's question (4.20) implied that, since the topic under discussion was sine rule, they thought that they must use sine. This is indicative of the fact that learners do not think of mathematical concepts as integrated but, instead, they think of mathematical concepts as separate entities. This separation of concepts is referred to as *task propensity* (Gravemeijer, Bruin-Muurling, Kraemer, & van Stiphout, 2016).

Similar to L3 in Excerpt 4, L4 in Excerpt 5 also believed that it was his role to ensure that his attempts are correct and acceptable, but he expected me to confirm his attempts. Although the class appeared to be collaborating, and deciding on their own if their attempts were correct and acceptable, L4 shows that not everyone endorsed this norm. Nonetheless, the discussion which occurred at L4's desk is captured in Excerpt 5 below, following by my analysis and interpretation of this discussion.

Excerpt **5**

5.1 T Yes sir!

5.2 L4 Sir, I was asking if I have done the correct things here (referring to question 1.1 and 1.2)



5.3 T Can you explain for me 1.1, is that 7,5?

5.4 L4 We said since this line (pointing MN) is 15 so they are equal.

5.5 T Which lines?

5.6 L4 MN is 15, so we said PN is also 15 because they are equal.

5.7 T How do you know that they are equal?

5.8 L5 Two sides in a triangle (with L4 saying right angle triangle) are equal. Yes, two sides in this kind of a triangle are equal.

5.9 T Okay, give me the types of triangles that you know.

5.10 L5 Ah, right angled, isosceles and …

5.11 T Okay let's talk about the isosceles, what are its properties?

5.12 L5 The base angles in the triangle are equal.

5.13 T Mmmh, then the scalene?

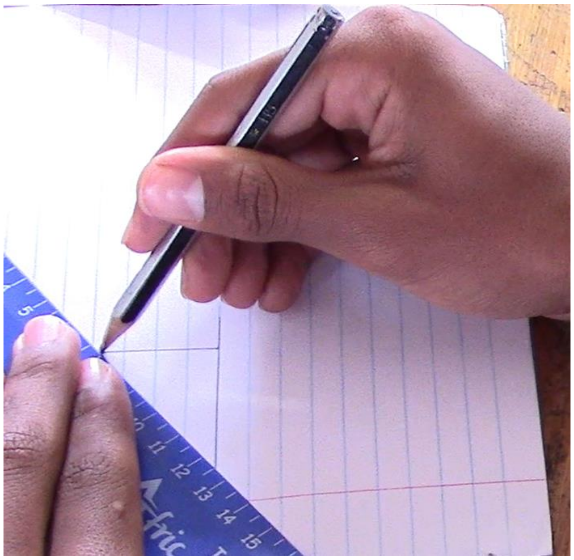
5.14 L4 I am lost.

5.15 T Do you know a scalene triangle, or equilateral triangle?

5.16 L4 No, I only know isosceles and right-angled triangle.

5.17 T Okay, take a ruler, draw a straight line 4cm, then other one vertical 3cm,

		must be at right angle to the 4cm one. Then a line joining the two lines.
5.18	L4	(Drawing as per instructions)



5.19	T	Now measure the line, how long is it?
5.20	L4	(After measuring) It is 5 cm.
5.21	T	Now let us talk about the right angle triangle that we have, does it have any equal sides?
5.22	L4	No
5.23	T	I need you to give me reasons why you saying the two sides (MN and PN) are equal.
5.24	L5	They are not equal sir.

Excerpt 5 best illustrates the student-teacher interaction stage where learners' ideas were interrogated in order to reduce weakness in learners' mathematical understanding and reasoning (Masha, 2004). Even though learner-learner interactions were preferred, learner-teacher interactions provided me with opportunities to engender specific norms. This was done within a sociocultural classroom by mediating learners' learning and by providing them with scaffolds. Furthermore, care needs to be taken to avoid constraining opportunities for mathematical development which would lead to promoting proficiency in mathematics by the learners. Moreover, this excerpt showcase the notion that learners do not only engage in discourse that leads to proficiency in mathematics upon providing correct responses.

L4 concluded that two sides of the triangle were equal (5.6) and, when asked why, L5 offered a justification (5.8). Although the both the conclusion and justification were not correct, this interaction showed that the two learners had collaborated in responding to the question and what was presented was taken-as-shared for them. These learners fell short on knowledge of the types of triangles and their properties and, as a result, applied properties of an isosceles triangle to a right-angled triangle (5.8 – 5.16). In order to disprove the learners' conclusion, without directly telling them that a right-angled triangle is not necessarily an isosceles triangle, learners were given a task (5.17) which, upon execution (5.18), led them to decide for themselves that their earlier conclusion was not correct (5.24).

The mediation provided to learners here (5.8 – 5.21) demanded that they engage in adaptive reasoning. Learners had to precisely reflect on their responses and arguments as they engaged in the discourse and, ultimately, were left with the responsibility to identify or devise an appropriate strategy to enable then to provide a correct solution to question 1.1 of the activity. This responsibility also demanded that the learners engage in strategic competence. Lessons learned here include the fact that inadequate disciplinary knowledge of mathematics leads to inability to enact sociomathematical norms. However, mediation leads to engagement in processes which leads mathematically proficiency of the learners.

The opportunity for challenges and justifications (Cobb et al., 1992) takes place when the learners share their solutions, justify their solutions and challenges each other's solutions. For this class this was largely enacted in cases where the learners worked in pairs or in small groups. However, this opportunity for challenges and justificaions ended up being enacted during the whole class discussion in which learners shared their responses with the whole class. For example, L3 (Figure 7 – 4) and L7 (Figure 7 – 5) shared their attempts at questions 1.1 and 1.3 respectively.

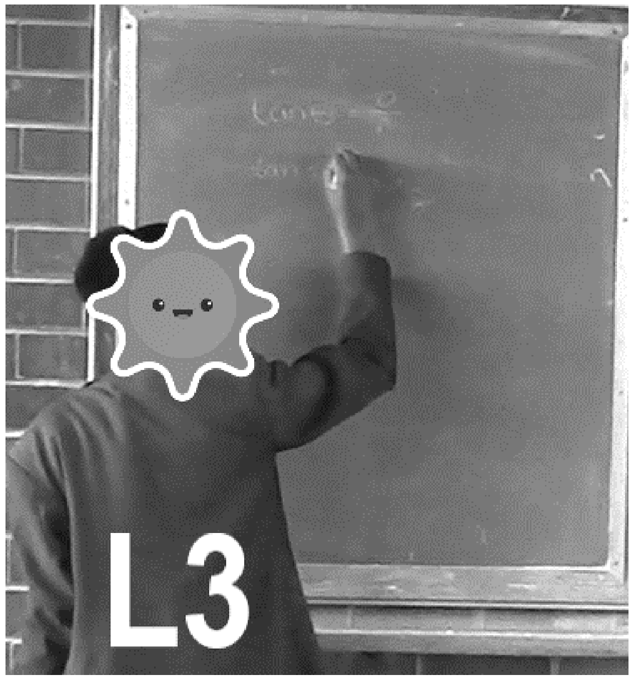


Figure 7 – 4: L3 sharing her solution to question 1.1 with the whole class

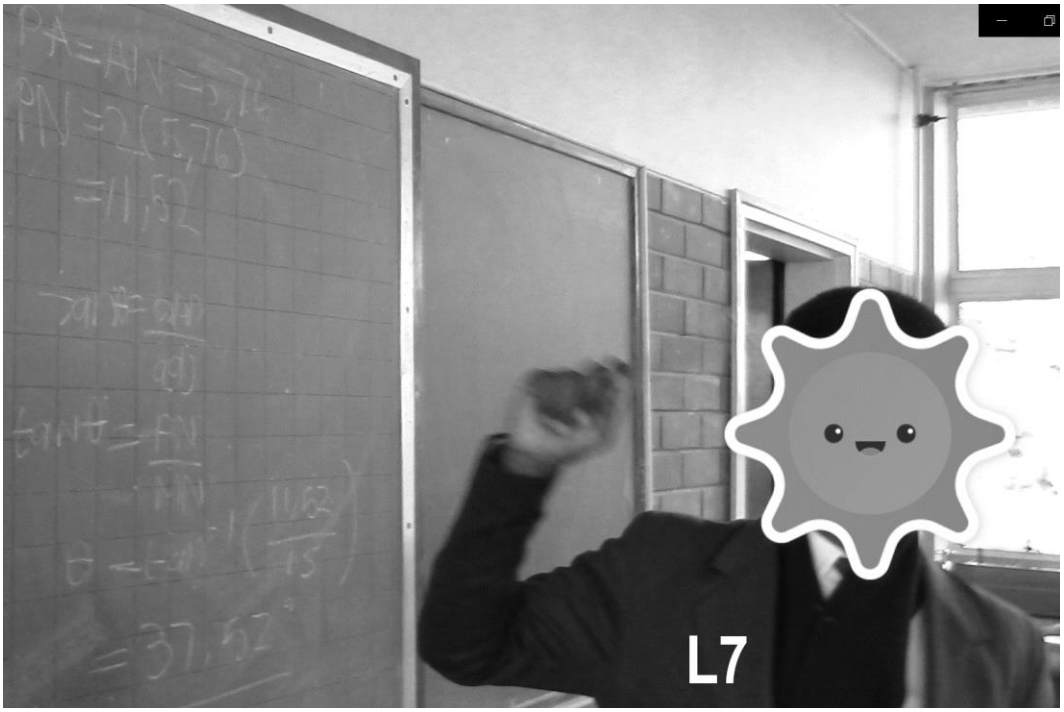


Figure 7 – 5: L7 sharing his solution to question 1.3 with the whole class

It was during the whole class interaction stage that learners continuously engaged in a process of reflection. As L3 and L7 shared their solutions, and explained them in order to clarify them and not merely to justify them, the other learners had to interpret these solutions and compare them with their own. Hence, sociomathematical norms concerned with what counts as mathematically correct, acceptable and different answers, were enacted. Additionally, learners challenged presented solutions by asking questions where they did not understand and by challenging arguments to indicate disagreement. This lead to the enactment of the sociomathematical norms concerned with learners having to take responsibility of interpreting, questioning and challenging the responses of others. Finally, learners whose solutions were challenged attempted to understand the reasoning of the other learners and to provide justifications for their solutions. Throughout the discussions, when consensus was reached and a taken-as-shared meaning was developed and agreed upon, then enacted sociomathematical norms were endorsed.

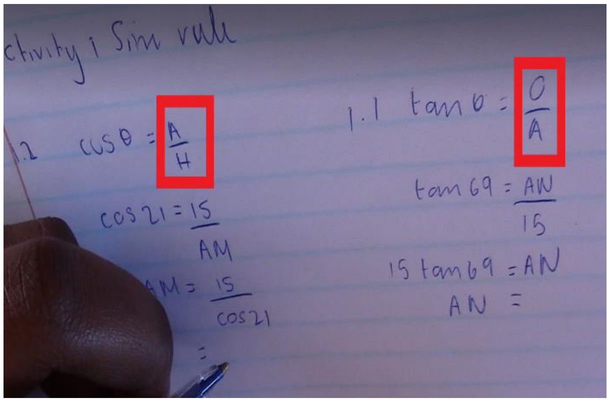
7.4.2 Sociomathematical norms for explanations and justifications

Excerpts 6 – 8 presented here were also based on question 1 of activity 1 in learning activity 3, as captured in Figure 7 – 3 earlier. In this excerpt the analysis focused on the learners' enactment of sociomathematical norms of what counts as acceptable mathematical explanations and justifications. Mathematical explanations differ from mathematical justifications in that explanations are provided to clarify aspects of mathematical thinking, which might not be readily apparent to others (Yackel, 2004). Mathematical justifications, on the other hand, are provided in response to challenges to apparent violations of normative mathematical activity (Cobb et al., 1992). In line with this, as enacted in the classroom where this study was conducted, the mathematical explanations were provided when learners clarified what they did, whereas the mathematical justifications were provided when learners justified why they did what they had done and explained. Hence, excerpts 6 - 8 which follow, showcase these incidences as they occurred in the natural setting of a mathematics classroom. Similar to earlier an analysis, here an interpretation of the extent to which the enactment of sociomathematical norms of what counts as acceptable mathematical explanations and justifications promotes learners' proficiency in mathematics.

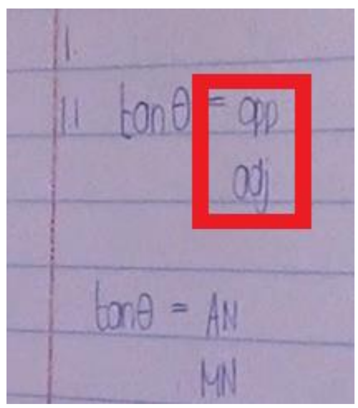
Excerpt 6

6.1 T … what is $\frac{A}{H}$ and $\frac{O}{A}$

6.2 L6 $\frac{A}{H}$ is Adjacent over hypotenuse and $\frac{O}{A}$ is opposite over adjacent



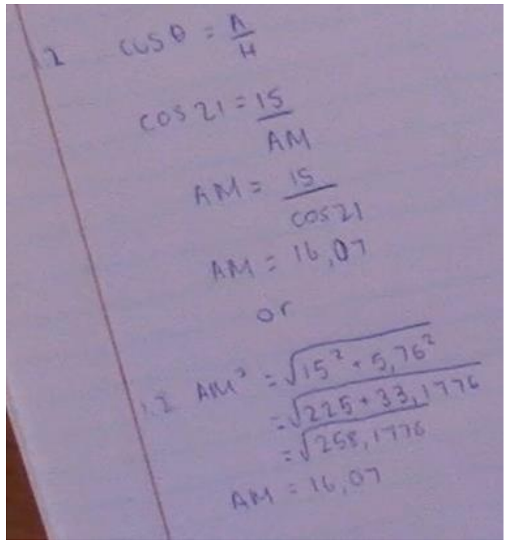
6.3	T	When you write $\frac{A}{H}$ which Δ are you looking at?
6.4	L6	A right-angled Δ
6.5	T	…any or this 1 (refering to the 1 given in question 1)
6.6	L6	This 1 given here
6.7	T	…what is the adjacent side?
6.8	L6	… MN
6.9	T	… what is the hypotenuse?
6.10	L6	… AM
6.11	T	… is it necessary to write $\frac{A}{H}$ instead of writing the actual adjacent and hypotenuse sides for this Δ
6.12	L6	it is very necessary because I need to know why I wrote AM, otherwise I will get confused if I just write the letters. So if I first wrote opposite over hypotenuse, then in the next step I will go to the triangle and check the actual sides.



6.13	T	Is that the only way to do 1.2?
6.14	L6	I am still checking.

Later

6.15 L6



The discussion in Excerpt 6 started on the basis of me trying to understand the learners' representations of trigonometric ratios (6.2). This way of representation was prevalent in the classroom, although the focus of the analysis was now on enactment of newly renegotiated and endorsed sociomathematical norms. However, it is inevitable that learners' existing sociomathematical and mathematical norms keep surfacing, more so because the discussions where based on different mathematical content areas. My interaction with L6 required a justification instead of an explanation, since she had transgressed a mathematical norm. However, even after I recommended an acceptable way of representing the ratio (6.11), she still insisted on her representation being important (6.12) and indicated that not writing it would lead to confusion.

For L6, writing $\cos\theta = \frac{A}{H}$ was justified as a thought process to assist her to remember what she had memorised. Throughout my teaching experience I have come across many teachers who used the mnemonic SOHCAHTOA to help learners to memorise the definitions of trigonometric ratios. It would seem that, even in the case of this class, the learners memorised the ratios and disregarded what was acceptable as a standard mathematical way of representing the ratios. In order to support L6's argument, the presentation by L7 (6.12) was acceptable. Guven and

Dede (2017) cautioned against this overall practice of not taking care of the language of the discipline when teaching. They asserted that it was imperative that care be taken with the correct use of disciplinary language in order for appropriate norms to be constituted and enacted.

Since this way of representing trigonometric ratios was prevalent in this classroom, I addressed it in a whole class discussion. When providing mathematical explanations and justifications, learners engage in adaptive reasoning. Through reflective thought (Kilpatrick et al., 2001), learners evaluate their own work and adapt it (Van De Walle et al., 2010). This strand involved learners being able to explain and justify their mathematical ideas, and being able to communicate their mathematical ideas through appropriate mathematical language and symbols (DBE, 2018). Ultimately, a consensus was reached.

Enactment of sociomathematical norms is also reflected in the classroom's participation structures. As Turpen and Finkelstein (2010) noted, norms are established and enacted through repeated engagement in classroom social practices. Excerpts 7 and 8 follow, and best illustrate the norms which were established and enacted for the classroom in which data were generated. Through the analysis of these excerpts, I also accounted for learners' mathematical development against the achievement of the mathematical proficiency strand.

Excerpt 7

7.1	T	Which angle do they want?
7.2	L2	We are looking for angle P, because now we have M and N so…

7.3	T	Look at question 1.3 ($P\hat{M}N$), where is the 'cap'?
7.4	L2	On M.
7.5	T	So which angle do they want?
7.6	L6	Oh! Angle M, and I told you (referring to L2)
7.7	L1	And we already found it.
7.8	T	But why are you saying it is 21?
7.9	L2	We calculated it.
7.10	L1	Sir, 21 + 90 + A = 180, so A = 69, since AN and AP are equal, then P = 69 because AN // AP.
7.11	T	Why are you saying they are parallel?
7.12	L1	Because of the lines marked on them.
7.13	T	Don't those lines mean they are equal?
7.14	L1	I meant they are equal and not parallel.
7.15	T	Okay since we don't have parallel lines we can't talk about equal angles then.
7.16	L1	Why? If the lines are equal then the angles are also equal.
7.17	L6	No, it means if one line is 15 units, then the other one is also 15 units.
7.18	T	Let me give a hint. The angle M that you want to calculate is in which triangle?
7.19	L6	ΔPMN
7.20	L1	The big one.
7.21	T	So think about the big triangle then. What information do you have about it?
7.22	L1/L2	We have the length of MN and PN.
7.23	L1	And we also have 90º
7.24	T	Now lets talk, what is the relatioship between PN and angle M?

Silence

7.25	T	Would you say PN is opposite, would you say it is adjacent, would you say it is…?
7.26	L2	It is opposite.
7.27	T	Its opposite. So what is the relationship between angle M and MN?
7.28	L1	Adjacent
7.29	L2	Oh yah now I see (leaving the discussion to continue writing)

After I noted that the learners where actually calculating the angle, which they were not required to, I needed to be certain whether they were aware of this or not. When I asked which angle they were requested to calculate (7.1), they indicated angle P. Straight away I realised that the problem was with learners' knowledge of mathematical representation. Interestingly, the learners also provided a justification for their argument (7.2), without being asked why. This situation where learners provide justifications for their arguments was again observed later in the discussion (7.10). These two incidences are indicative of what was becoming normative for this classroom with regard to providing mathematical explanations and justification. In particular, it can be concluded that, for this class, mathematical explanations and arguments should be justified with reasons. That would, in turn, provide grounds for the class to develop taken-as-shared meanings regarding the types of mathematical explanations and justifications which are acceptable.

The learners' uncertainty regarding the required angle was confirmed by L6 (7.6). Immediately L1 claimed that angle M was known and provided a reason for that (7.10). Both the argument and reason were wrong, due to insufficient mathematical knowledge of the implication of equal lengths on the same line and equal sides of a triangle to certain angles. The discussion which unfolded (7.11 – 7.17) is a typical enactment of what counts as acceptable mathematical justifications which yield two views. Firstly, acceptable mathematical justifications should be based on accurate and correct mathematical knowledge about the concept under discussion or accurate knowledge of properties of the shape under consideration. Secondly, the learners should regard the justification as efficient (Chuene, 2011).

When learners grappled with appropriate mathematical content, enactment of sociomathematical norms occurred at a time I mediated learning and provided learners with scaffolds (7.18 – 7.27). During the discussion in Excerpt 7, mathematics proficiency was promoted as learners engaged in a discourse that allowed them to develop strategic competence through my assistance. The strategy was made possible as learners also engaged in adaptive reasoning as we interacted during mediation of learning. In particular, I called on learners to consider information known (7.21). Through reflection, logical and deductive reasoning the learners managed to make mention of all this information, beyond my expectation (7.22 – 7.23). The question which followed (7.24) was intended to provide learners with a direction for them to employ an appropriate strategy. Following a moment of silence, it was clear that the question did not prove to be helpful to learners as intended, only upon rephrasing it (7.25) were the learners able to respond and provide a solution to the question, which was initially asked in the learning activity.

Excerpts 6 and 7 captured the interactions which took place predominantly at learners' desks, either when learners worked in pairs or in a small group with a maximum of four members. Excerpt 8 below, on the other hand, captures a discussion which took place during a whole class discussion and showcases the enactment of the sociomathematical norms of acceptable mathematical explanations and justifications. The purpose of question 1 of activity 1 (Figure 7 – 3) was to bring to the fore the learners' prior knowledge of the solutions of triangles and also to show the limitations of such knowledge. The limitation was evidenced in that the learners could only find solutions to right-angled triangles, as they had knowledge of trigonometric ratios and the Pythagoras theorem only. Hence, this provided a perfect attention focus activity for the learners to engage in, in order to see the need for strategies to find solutions in non-right-angled triangles. These strategies were the sine rule, cosine rule and area rule for learning activity 3. Therefore, the purpose of the whole class discussion (Excerpt 8) was in twofold. Firstly, the purpose of this discussion was to consolidate that part of the activity which focused on learners' prior knowledge. Secondly, the purpose was to introduce part of the activity in which the sine rule was to be derived and applied in order to find solutions to non-right-angled triangles.

Excerpt 8

8.1	T	Let us talk about question 1.5. If the ∆MNP was not a right angled triangle would you still respond to questions 1.1 and 1.2 above as you did?
8.2	L9	Yes (aloud)
8.3	T	Tell us why you saying yes
8.4	L9	In my calculations for 1.1 and 1.2 I used the given information, like 21^o and 15 units, there is no where I used N (which is 90^o). That is why I am saying it does not make a difference.
8.5	T	…L6 is shaking her head
8.6	L6	Sir because question 1.1 we need to use the trig ratios, without the 90^o as when we use trig ratios we need a right-angled ∆ without it we wouldn't find AN, because we just given the length of MN and angle M… we couldn't solve it without the right angle.
8.7	T	…L9, when do we use trigonometric ratios?
8.8	L9	When do we use what?
8.9	T	In question 1.1 you use a trignometric ratio. When do we use trigonometric ratios?
8.10	L9	When you are given the angles and not the sides.
8.11	T	In what type of a ∆ do we use trigonometric ratios?
8.12	L7	In a right-angled ∆ which means we wouldn't be able to answer question 1.1 and 1.2 without it.
8.13	L9	In a right-angle ∆ (giggling)
8.14	T	How did you respond to question 1.2, what did use?
8.15	L9	Theorem of pythagorus
8.16	T	In what type of a ∆ do we use theorem of pythagorus?
8.17	L9	Right-angled ∆
8.18	T	So if the ∆ was not right angled, would you still be able to respond to question 1.1 and 1.2 as you did?
8.19	L7	No (quickly)
8.20	L9	No

L9's ideas became the centre of the whole class discussion as she responded to the question asked (8.1) aloud (8.2). While she gave her justification (8.4), it become apparent that it was not acceptable to majority of the class. Some learners started mumbling (e.g. L7), while others shook their heads (e.g. L6) in order to indicate disagreement. The is an indication that the learners enacted the responsibility of interpreting solutions offered by others and/or comparing them with their own solutions (Cobb et al., 1992) in order to reject the argument which was not efficient to them. L9, in her explanation and justification (8.4). appeared to be arguing that it did not matter if the triangle was right-angled or not, as she did not use the right-angle (90^o) anywhere in her calculations. Even after L6 provided a counter explanation (8.8) as to why the questions could only be attempted if the triangle was right angled, L9 seemed to be confused by this argument.

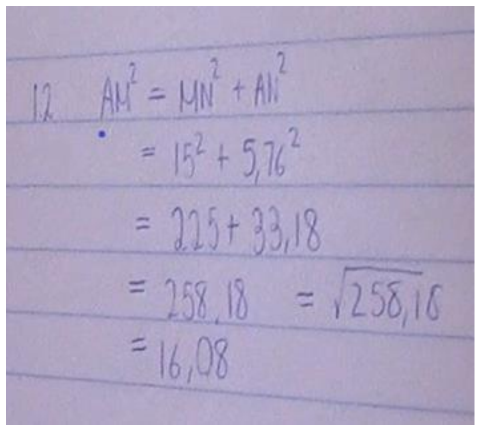
My attempt to mediate learning for L9, in order for her to develop a shared understanding with the whole class, confirmed that, indeed, she was confused by L6's argument. Throughout the mediation process, the scaffolding question, which was posed (8.7 & 8.9), had to be rephrased (8.11) in order for her to understand where the argument lay. As she tried to internalise the discussion as it unfolded, L7 provided yet another explanation (8.12) which, in a way, clarified the one provided by L6 earlier (8.6), in order to indicate to L9 that her explanation (8.4) was not acceptable. Here, L6 and L7 collaborated in sharing with L9 their counter explanations and expected that she understood their reasoning. When L9 started giggling (8.13), this was an indication that she started to see contradictions in how she was responding to questions, and her earlier response to the question 1.5 of the activity 1.

7.4.3 Sociomathematical norm for mathematically different solutions

Excerpts 9 presented here was also based on question 1 of activity 1 in learning activity 3, as captured in Figure 7 – 3 earlier. In this excerpt the analysis focused on the learners' enactment of the sociomathematical norm of acceptable mathematically different solutions. Similar to earlier an analysis, here an interpretation of the extent to which the enactment of sociomathematical norm of acceptable mathematically different solutions promotes learners' proficiency in mathematics.

Except 9

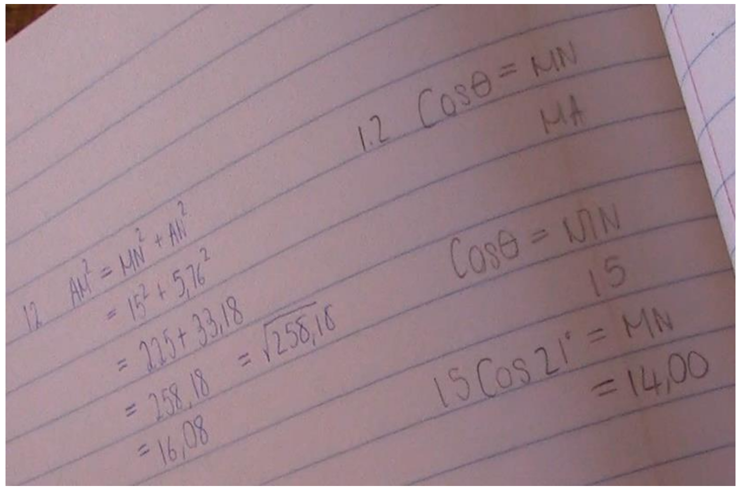
9.1 L7



9.2	T	Is there a different way to respond to question 1.2?
9.3	L7	I don't know, is there another way?
9.4	T	What do you think?
9.5	L8	…we could use one of the trig functions
9.6	T	Which one in particular?
9.7	L8	Since we have two sides, but if you want to use the given one (15 units) … you will use cosine.
9.8	T	why cosine?
9.9	L7	Because we have adjacent side and we want to find the hypotenuse side.

Later

9.10 L7



9.11 T Why are your answers different?

Silence

9.12 L7 Oh sir I did not substitute correctly, 15 is for MN and not for MA as I did, let me correct it.

9.13 T Okay, besides using cosine, is there any other different way of responding to question 1.2?

9.14 L7 Obvious, I can use sine since the opposite side is known, as calculated in question 1.1.

9.15 L8 What if your answer to 1.1. is wrong? Then your answer to 1.2 is also going to be wrong. It is better to use information originally given.

9.16 L7 You see then that will help verify if my answer for question 1.1 is correct.

9.17 L8 But using sine isn't different from using cosine, both are trigonometric ratios

The sociomathematical norm of acceptable mathematically different solutions was not widely endorsed in the classroom. In most cases learners waited for me to ask if there is a different way in which they could respond to the same question (9.2 and 9.13). Beside this it was interesting to note that learners mostly use the same approach as they avoid using previously calculated answers and prefer to use original information provided to them (9.15). As learners engage in a discussion in which they were trying to justify mathematically different solutions (9.13 – 9.17), they also considered the benefits and demerits of presenting different answers. It appears that L8 is receptive of the idea of different answers but expects them to be different concenptually (9.17). However, L8 further causions that care be taken in using answers from previous questions as they would lead to incorrect solutions. L7 on the other hand, views this causion as an opportunity, he asserts that it will help verify if the previous answers are indeed correct (9.16).

L7 proposed to regard any solutions which differs procedurally as mathematically different (9.14). This proposal was rejected by L8 (9.17) with reasons that if the same mathematical knowledge is used in both solutions, then they are not mathematically different. This view was later contested in the whole class discussion and was accepted by the class. What learners did not realise was that I did not expect them to necessarily respond to the same question in many different ways. Instead, I wanted them to consider different ways of responding to the same question in cases where their group members present different solutions. However, for the activities at hand, learners seemed to use the same approach and hence I asked them if there was a different way.

Learners' enactment of the sociomathematical norm of mathematically different solutions promoted their proficiency in mathematics. It allowed learners to use different concepts to respond to the same questions hence promoting conceptual understanfding. Conceptual understanding has to do with learners' comprehension of mathematical concepts (Kilpatrick et al., 2001). Learners must be able to justify the use of identified concepts, hence they will engage in adaptive reasoining which has to do with a capacity for logical thought, reflection and justification (Kilpatrick et al., 2001). As learners consider the different concepts, they have to present their answers in accordance with the concepts. This presentation of answers engages learners in strategic competence and since the solutions needs to be workout, learners will further engage in procedural fluency. Therefore, learners engage with all the mathematical proficiency strands identified for this study as they engage in the enactment of the sociomathematical norm of acceptable mathematically different solutions.

7.5 SUMMARY

In this chapter I presented, analysed and interpreted data, in order to address the question “How does the enactment of the constituted sociomathematical norms promote learners’ proficiency in mathematics?” In doing so focus was first on how the class engaged in the situation for challenges and justifications. Second, on sociomathematical norms for acceptable explanations and justifications. Third and last on sociomathematical norms for mathematically different solutions.

During the analysis I also accounted for ways in which the constitution and/or enactment of sociomathematical norms promoted learners’ proficiency in mathematics. As acknowledged while introducing this study, each and every classrooms has sociomathematical norms, however not all sociomathematical norms enacted in the different classrooms promoted proficiency in mathematics. Therefore, the conditions created by an interaction for learners to achieve mathematical proficiency was used as basis for identifying norms regarded as productive in this study.

CHAPTER 8: CONCLUSION AND RECOMMENDATIONS

8.1 INTRODUCTION

In the previous two chapters, I gave an analysis of data generated through the data collection methods outlined in Chapter 5. The theoretical framework discussed in Chapter 2 guided the analysis itself. This chapter is the concluding section of this thesis and, therefore, the research questions that guided the study in order to achieve its purpose are answered here. The limitations of the study will then be outlined and, finally, recommendations of areas for further research, focusing on sociomathematical norms in mathematics classrooms, are suggested.

The three research questions which guided this study were:

- What are the sociomathematical norms constituted for promoting learners' proficiency in mathematics?

- How are the sociomathematical norms for promoting learners' proficiency in mathematics constituted and enacted in the classroom?

- How does the enactment of the constituted sociomathematical norms promote learners' proficiency in mathematics?

In presenting the conclusion, the traditional way of answering research questions separately is not followed. Instead, two sub-headings aligned to the first and the second research questions are used. Answers to the third research question are weaved together with those of the first and the second research questions. In essence, as I conclude on what sociomathematical norms were constituted, I also account for how the constitution of such sociomathematical norms promotes the learners' proficiency in mathematics. Similarly, when concluding how sociomathematical norms were constituted and enacted, I also accounted for how the constitution and enactment of sociomathematical norms promotes learners' proficiency in mathematics. This was the case as the constitution of the sociomathematical norms cannot be separated from their enactment during classroom discourse.

8.2 CONCLUSION

In drawing conclusions, first, it should be noted that I recognise that all classrooms have norms which are fostered by teachers and endorsed by the classroom community (Cobb et al., 2001; Voigt, 1995) and such norms influence the learning that takes place within the classrooms (McClain & Cobb, 2001; Yackel et al., 2000). As a result, it is important for teachers to be aware of, and understand, sociomathematical norms (Kang & Kim, 2016; Zembat & Yasa, 2015) and be able to foster these norms during teaching and learning (Partanen & Kaasila, 2015; Yackel et al., 2000). Additionally, McClain and Cobb (2001) posit that the establishment of sociomathematical norms directly influences our mathematical agenda. The mathematical agenda refers to what we teach mathematics for and why we teach it. Therefore, if teachers are not aware and do not understand the sociomathematical norms which they are fostering during teaching and learning, they may foster norms which are not aligned to their mathematical agenda.

In my case, my mathematical agenda is to teach my learners for relational understanding, so that they develop appropriate mathematical process skills and become mathematically proficient. This resonates with South African mathematics classrooms, whose mathematical agenda (what we teach mathematics for and why we teach it) is captured in the South African Framework of Teaching Mathematics for Understanding (DBE, 2018). I summarise this mathematical agenda to mean that we teach for *proficiency in mathematics* (Kilpatrick et al., 2001). This is the same agenda which guided teaching and learning for the classroom in which data were generated and, hence, the conclusions drawn here should be understood in this context. Furthermore, teachers should be aware of, and understand, their philosophy of teaching, as it guides the instructional decisions taken by teachers. Even though the sociomathematical norms are constituted by the class and largely enacted by learners, teachers play a pioneering role in ensuring that productive sociomathematical norms are orchestrated. Productive sociomathematical norms in the case of this study are those norms which directly support and promotes learners' proficiency in mathematics.

8.2.1 Constituted Sociomathematical Norms

The sociomathematical norms constituted during the study were concerned with acceptable mathematical explanations, mathematical justifications and mathematically different solutions. These sociomathematical norms were not constituted separately from each other but, instead, were constituted concurrently alongside each other. The sociomathematical norm of acceptable mathematically different solutions was interactively constituted when learners negotiated and endorsed what they regarded as accepted to show that solutions are mathematically different. While working out the sides of triangles, it became acceptable that answers were mathematically different when some learners used trigonometric ratios, while others used the Pythagoras theorem. However, it was not acceptable that solutions were mathematically different in cases where, for example, they were requested to determine the length of the hypotenuse side of the triangle and some learners used the trigonometric ratio of sine while others used the trigonometric ratio of cosine. In this case, both groups of learners used the same concepts of trigonometric ratios, the same factual knowledge in defining the trigonometric ratios and followed the same procedure in carrying out the calculation.

Therefore, mathematical solutions are regarded as different if the factual, conceptual and/or procedural knowledge of mathematics used is different but the answers are (and must be) the same. For example, in determining whether a triangle is right-angled or not (Question 3 of Learning Activity 1, Figure 6 – 1), some learners used the concept of gradient of a line together with the fact that the product of the gradients of perpendicular lines is negative. Whereas, other learners used the concept of distance and the fact that the Pythagoras theorem holds true for a right angle triangle. For the two approaches cited here, the final answer will be the same but the learners would have used different factual and conceptual knowledge of mathematics in order to get to this answer. In supporting the constitution of the sociomathematical norm of acceptable mathematically different solutions, activities should be designed such that they contain questions which can be attempted in more than one way.

The latter was the case with all activities used in this study. For examples with Learning Activity 1 (Figure 6 – 1), question items 1.1, 3.1, 4.1, 4.3 and 5 were approached in different ways which all supported the constitution of a criteria for what counted as mathematically different solutions. Similarly, question items 4 and 5 of Learning Activity 2 (Figure 6 – 2) and question item 1.1 – 1.5 of Learning Activity 3 (Figure 7 – 3) also supported the constitution of the criteria of what counted as mathematically different solutions. Furthermore, the constitution of the sociomathematical norm of mathematical difference can be fostered if during the classroom discourse teachers continuously ask learners if there is a different way in which the same question could be attempted. Eventually, learners would consider different ways in which they could attempt the same question without being asked.

Sociomathematical norms of acceptable mathematical explanations and acceptable mathematical justifications were constituted during classroom discourse as learners engaged in the situation of *challenges* and *justifications* (Much & Shweder, 1978). Through back and forth negotiations, ultimately, taken-as-shared meanings regarding acceptable mathematical explanations and acceptable mathematical justifications were endorsed. Although these sociomathematical norms particularly depended on the mathematical activity at hand, generally acceptable mathematical explanations and acceptable mathematical justifications are those that make correct reference to factual and conceptual mathematical knowledge and may include the use of appropriate and efficient knowledge of mathematical procedures.

The constitution of the sociomathematical norms of acceptable mathematically different solutions, acceptable mathematical explanations and acceptable mathematical justifications promoted learners' proficiency in mathematics. In particular, all the strands of mathematical proficiency pioneered by Kilpatrick et al. (2001) reflected in the South African Framework of Teaching Mathematics for Understanding (DBE, 2018) are achieved together with the constitution of sociomathematical norms. These strands are *conceptual understanding*, *procedural fluency*, *strategic competence* and *adaptive reasoning* (DBE, 2018; Kilpatrick et a., 2001). Mathematical proficiency strands are interwoven (Kilpatrick et al., 2001). As a result, they are interdependent on each other.

Conceptual understanding has to do with the learners' comprehension of mathematical concepts, operations and relations (Kilpatrick et al., 2001) and the ability to apply ideas and justify their thinking (DBE, 2018). In this study, conceptual understanding was achieved when the sociomathematical norm of acceptable mathematically different solutions, acceptable mathematical explanations and acceptable mathematical justifications were constituted.

Starting with the very first activity whose main purpose was to tease out sociomathematical norms which existed already but also to use any opportunity created to (re-)negotiate the sociomathematical norms, conceptual understanding was achieved. When the explanation of how the answer $x = 9$ was obtained from $\frac{1}{2} = \frac{1}{x-7}$ without any additional working out, the discourse led to learners demonstrating an understanding of the equals sign as a concept of equivalence, in order to get to a correct answer. When this norm was constituted, learners applied their conceptual mathematical knowledge. However, the mathematically different solutions require acceptable mathematical explanations and acceptable mathematical justifications, hence, I earlier indicated that the sociomathematical norms are not constituted separately from each other. Therefore, in the same way that learners constituted sociomathematical norms of acceptable mathematical explanations and justifications, conceptual understanding is achieved.

As learners achieved conceptual understanding, other strands were also achieved. For instance, strategic competence which is concerned with the learners' ability to formulate, represent and solve mathematical problems (Kilpatrick, et al., 2001) was achieved as learners provided mathematical explanations and justifications. One example of such is illustrated by the following quote, wherein the learners attempted…

> So what we did was to try plot the points M and N on the Cartesian plane. It was easy to plot M(7 ; 1) because we know the exact point. So if we draw a line passing through M such that it is perpendicular to the -axis, it is a like a vertical line (drawing on the board). On this line the value of at any point is 7, so even where *y* is 2, the value is 7 (L9, 3.24 & 3.26, p. 106).

Through navigating with multiple representation the learners devised their own strategies in order to solve the mathematical problem at hand. Some of them obtained their answers using diagrams instead of having a step by step calculation as one would expect. While engaged in this process, they also achieved adaptive reasoning as they were justifying their solutions, proving and disproving other common strategies which are normally used to respond to similar kind of questions.

Adaptive reasoning is concerned with the learners' ability to reflect and think logically and to, then, provide explanations and justifications (Kilpatrick et al., 2001) as illustrated by the quote below:

> We cannot use the formula for gradients of perpendicular lines because we are given information about only one line. Then, in our group we draw the Cartesian plane (drawing it on the board), look at the -axis and realised it has a gradient of zero (0), see why we cannot use the formula? Check if you substitute by zero on $m_1 \times m_2 = -1$, then you will be left with 0 = – 1(L9, 3.24 & 3.26, p. 106).

In this case learners were disproving the appropriateness of other common strategies, thus demonstrated skills of evaluating and adapting their own and others' work and then communicating mathematical ideas through appropriate mathematical language and symbols. Hence, they put forward explanations like "write an explanation that a line perpendicular to the x-axis is a vertical line and has a constant x value throughout" (L12, 3.30, p. 106). This was brought as a justification and way of presenting a solution which did not have a step by step calculation.

To illustrate the interdependence of the strands, as learners engaged in learning activity 2 (Figure 6 – 2), strategic competence and adaptive reasoning were at the forefront particularly in two instances. First, when the angle learners were required to calculate was not an angle of inclination, but lied between two given straight lines. Second, when the gradient nor points on the line were not given but an equation of the straight line was given but not in standard form of $y = mx + c$. Learners engaged in proving and disproving different strategies until they ended up devising strategies which assisted in solving the questions.

Furthermore, strategic competence and adaptive reasoning were also achieved even when the sociomathematical norm of mathematically different solutions was constituted. During the lessons learners were asked if there is a different way to attempt any of the questions they had attempted already. Then this led to learners thinking of and coming up with different ways of attempting the same question. They engaged in representing and solving the same question more than ones.

Finally, procedural fluency was achieved together with all the other three strands. Procedural fluency is carrying out mathematical procedures flexibly, accurately, efficiently and appropriately (Kilpatrick et al, 2001) and knowing when to use a particular procedure (DBE, 2018). Flexibility was seen for example as learners engaged in learning activity 2 (Figure 6 – 2), in calculating the angle of inclination. Although they used a formula derived during the whole class discussion they had to think about a gradient of a line in a variety of ways before applying the formula. Appropriateness, on the other hand, was seen when learners had to make sure that indeed a particular angle is an angle of inclination before applying the formula.

Furthermore, accuracy also depicts the achievement of this strand. One example of this was seen when learners compared their responses which were obtained through different methods. The comparison of the answers and approaches always led them to agree on an approach which was efficient, hence utterances like:

> "What if your answer to 1.1. is wrong? Then your answer to 1.2 is also going to be wrong. It is better to use information originally given." (L8, 9.15, p. 138)

These utterances were made to justify an approach that will be both accurate and efficient.

It is apparent from this study that constitution of productive sociomathematical norms promoted learners' proficiency in mathematics, which was the main goal for teaching and learning of mathematics. The arguments about how the constitution of sociomathematical norms promoted learners' proficiency in mathematics also addressed Widjaja's (2012) call for studies which examine sociomathematical norms as one of the aspects of mathematical classroom discourse that promotes mathematics learning in classrooms. As a result, these earlier arguments show the importance of teachers' ability to facilitate the interactive constitution of productive sociomathematical norms in their classrooms. For these reasons, I propose a model for fostering the constitution of sociomathematical norms as summarised by Figure 8 – 1 in the sections that follows below. The model also addresses Güven and Dede's (2017) call for studies on how sociomathematical norms should be established.

8.2.2 Enactment of Sociomathematical Norms

The sociomathematical norms which were enacted during the study included acceptable; mathematical explanations, mathematical justifications and mathematically different solutions. In this section, focus is on how the constituted sociomathematical norms were enacted, and how the enactment of these norms promoted learners' proficiency in mathematics. In concluding on the enactment of sociomathematical norms I focus on how this takes place in the natural setting of a mathematics classroom and how teachers can foster the enactment of sociomathematical norms. In this study, to enact refers to putting into practice. Therefore, as I draw conclusion I also reflect on how I created opportunities for learners to enact sociomathematical norms. The latter in turn reveals the expected roles a teacher has to assume for teaching that supports engendering of sociomathematical norms.

Learners' enactment of sociomathematical norms was fostered when I played four different roles during teaching and learning. The four roles included; continually offering learners scaffolds and mediating their learning during the classroom discourse, as well as creating opportunities for learners to collaborate and engage in whole class discussions. In the writing of Wood et al. (1976) "scaffolding is a process that enables a child or a novice to solve a task or achieve a goal that would be beyond his unassisted learning" (p. 70). However, during this study scaffolds were not only offered for learners to complete tasks, but also to provide explanations and justifications that contributed to the endorsed criteria for what counted as acceptable mathematical explanations, mathematical justifications and mathematically different solutions. Scaffolds were offered in the form of leading questions, which led to learners engaging in mathematical thinking and thus achieving proficiency in mathematics. The latter supported learners in cases where they lacked mathematical knowledge to complete the task but most importantly created opportunities for learners to enact sociomathematical norms and thus achieved proficiency in mathematics.

Example of scaffolding offered for learners to complete a task can be seen when learners could not respond to question 1.4 of learning activity 1. After learners tried two strategies they concluded that the information provided was inadequate for them to complete the question (3.1 & 3.3). I asked if there was another different way they could attempt the question without using a formula. Thereafter, I allowed learners time to reconsider the question with my hints (3.10 – 3.15). Realising that learners fell back to their initial strategies, I told them the question cannot be answered using a formula. It was after this hint that learners devised a new strategy and ultimately completed the task appropriately.

Throughout the engagements cited in the paragraph above, strategic competence and adaptive reasoning were mostly at play, with conceptual understanding serving as a springboard for ensuring that acceptable factual and conceptual knowledge of mathematics was used. Scaffolds that enabled learners to contribute towards a criteria for acceptable mathematical explanations and justifications were provided through ‘counter’ or contradictory questions. For example, when L4 asked if numerators and denominators can always be equated when solving equations involving fractions as a follow up to the explanation of how was obtained from . I asked him if he would equate them if was given? Another example was when L4 and L5 in Excerpt 5 insisted that the given triangle was isosceles because it was a right angled triangle. I gave them adjacent sides with lengths four and five units respectively and requested them to construct a right angled triangle. These engagements contributed towards a criteria for acceptable mathematical explanations and justifications.

Mediating learning refers to offering learners with artefacts, tools and mediators which will assist in moving their learning forward (Dixon-Krauss, 1996; Forman, 2003). In this study, an example of this can be seen in Excerpt 7 when learners grappled with appropriate mathematical content. It was through mediating learners' learning (7.18 – 7.27) that an opportunity for enacting sociomathematical norms was created. Additionally, mathematics proficiency was promoted as learners engaged in a discourse that allowed them to develop strategic competence through my assistance. The strategy was made possible as learners also engaged in adaptive reasoning as we interacted. In particular, I drew learners' attention to information known (7.21). Through reflection, logical and deductive reasoning the learners managed to mention of all the given information (7.22 – 7.23). The question that followed (7.24) was intended to provide learners with a direction to employ an appropriate strategy. But it only became clear after I rephrased it (7.25). Thus, the learners were able to respond and provide a solution to the question, which was initially asked in the learning activity.

Another example of mediation of learning was played out when other learners became mediators of learning in Excerpt 8. For L9 to ultimately have a shared understanding with the whole class, her learning was mediated through arguments by L6 and L7. Throughout the mediation process, the scaffolding question, which was posed (8.7 & 8.9), had to be rephrased (8.11) in order for her to understand what the argument was about. As she tried to internalise the discussion, L7 provided another explanation (8.12), which clarified the one initially provided by L6 (8.6). The mediation that we offered allowed L9 to engage in adaptive reasoning through logical thought and reflection.

Multiple opportunities for learners to collaborate and engage in whole class discussions were created. These opportunities allowed for enactment of sociomathematical norms which led to promoting proficiency in mathematics. Collaboration mostly happened during small groups or pair discussions when learners engaged in learner-learner interactions. One such example can be seen in Excerpt 4 when I asked L3 why she decided to use the sine ratio (4.15), but L2 responded instead. This was indicative of the fact that what they had agreed upon the justification and it was taken-as-shared between this pair of learners even though their answer was not acceptable. However, L3 did not accept L2's justification; this led to learners engaging in collaboration again in order to come up with a shared justification.

Ideas that had potential in contributing to the criteria for what counted as acceptable mathematical explanations, justification and different solutions emanated from learners' collaborations. These were mostly enacted in the groups or pairs of learners until endorsed, but did not represent what was taken as shared for the whole class. Hence, engaging learners in a whole class discussion allowed these norms to be ultimately negotiated and endorsed by the whole class. One such example can be seen in Excerpt 8 when L6 and L7 collaborated in sharing with L9 their counter explanations and expected that she understand their reasoning. These collaborations and whole class discussions always involved learners' use of mathematical facts, concepts and procedures. Hence, when consensus was reached, learners' achieved mathematical proficiency strands. In this instance, my role was to encourage collaborations as learners worked in small groups or in pairs and thus ensured sustained whole class discussions.

8.2.3 A Model for Fostering Constitution of Socioathematical Norms

At the centre of the model lie four interdependent goals which I propose for constituting sociomathematical norms that promote proficiency in mathematics. Along these goals are the expected teaching and learning responsibilities which teachers should engage in and carry out to foster constitution of productive sociomathematical norms.

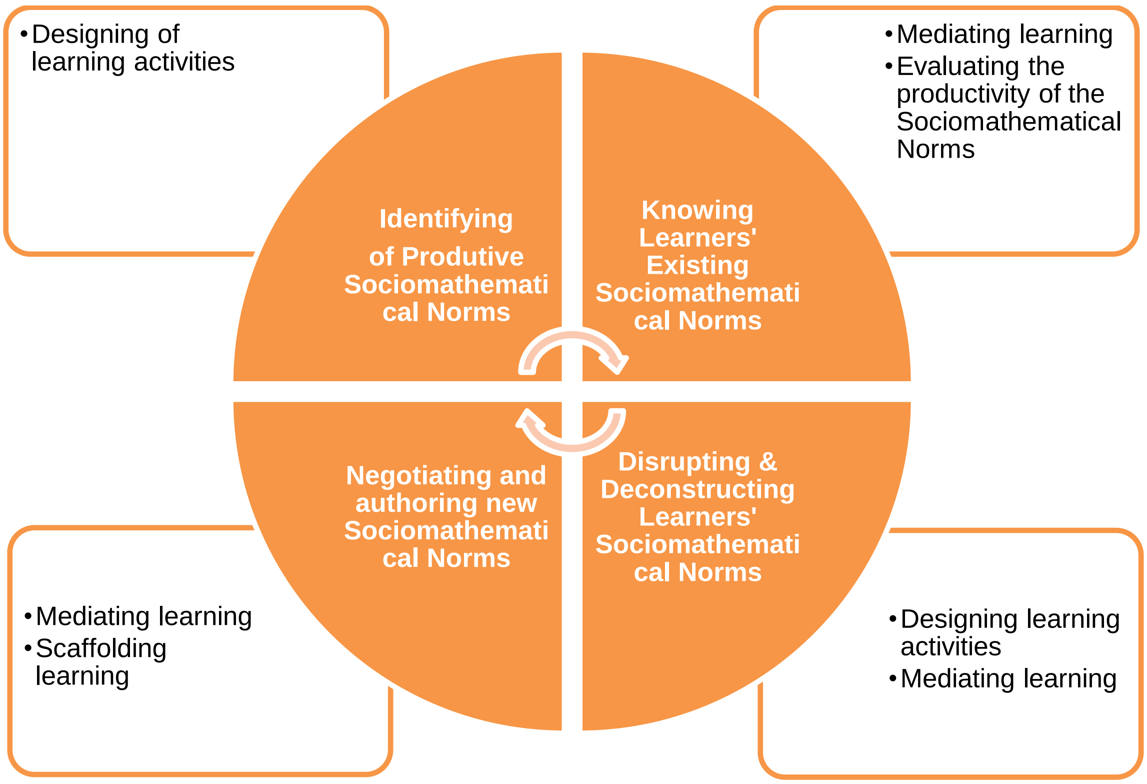


Figure 8 – 1: A proposed model for fostering constitution of sociomathematical norms

Constitution of sociomathematical norms begins with a teacher identifying productive sociomathematical norms that should be fostered during teaching and learning. These are sociomathematical norms which support teachers' mathematical agenda. The identification of such sociomathematical norms requires teachers' awareness and understanding of these norms (Kang & Kim, 2016; Zembat & Yasa, 2015) that will be engendered during teaching and learning. A teacher thus has a responsibility to develop appropriate learning activities which will require learners to enact the identified norms. In pursuance of sociomathematical norms of mathematical explanations and justifications, a teacher should consider what is important for learners to include while explaining and justifying their mathematical solutions. This requires a teacher to design learning activities in which learners are explicitly expected to explain and justify their solutions. If, for example, acceptable mathematical explanations are to reflect multiple representations (word, algebraic, graphical and pictorial/diagrammatical) then the design of the activities should explicitly require learners to show such representations.

Sociomathematical norms are not explicitly taught by teachers to learners, instead they are learned implicitly as learners engage in mathematical activity with their peers and a teacher (Bowers et al., 1999; McClain & Cobb, 2001; Yackel et al., 2000). As a result, learners in secondary mathematics classrooms come with certain sociomathematical norms, which were established in earlier schooling grades. As such, it is also imperative for teachers to know the learners' existing sociomathematical norms. These existing sociomathematical norms should be brought to the fore through the facilitation of learning activities designed for learners to enact teachers' envisaged productive sociomathematical norms. Hence, during the facilitation of learning activities that require learners to enact productive sociomathematical norms identified by teachers, they should mediate mathematical classroom discourse in such a manner that learners' existing sociomathematical norms come to the fore. Therefore, during classroom discourse teachers should consistently pose questions which call on learners to explain and justify their solutions. As learners enact their existing sociomathematical norms, teachers should evaluate the enacted norms.

Evaluation of the existing sociomathematical norms should be made against those identified by a teacher for the class. If learners' enacted sociomathematical norms are different from those identified by a teacher, then a teacher should evaluate the productivity of such norms with respect to their mathematical agenda. Sociomathematical norms are negotiated and collectively agreed upon (Turpen & Finkelstein, 2010). As a result, a teacher should be receptive of and endorse productive sociomathematical norms negotiated by learners which were not identified by a teacher. However, learners' existing sociomathematical norms which are not productive to teachers' mathematical agenda should be disrupted and deconstructed. The responsibility of "initiating and guiding the renegotiation of social [mathematical] norms" (Cobb & Yackel, 1996, p. 177) lies with teachers.

Learners' existing unproductive sociomathematical norms can be disrupted by a careful selection and design of learning activities which do not conform to these unproductive sociomathematical norms and they should be deconstructed by providing counter arguments during classroom discourse. For example, if learners' acceptable mathematical explanation is the one that consists of a step-by-step algorithm, this can be disrupted by giving learners activities that will not require any algorithm in presenting or explaining their mathematical solutions. Deconstruction of sociomathematical norms through raising counter arguments can take place, for example, when arguments contrary to learners' sociomathematical norms are raised. If learners argue that long algorithms are applied in order to ensure that full marks are awarded, then showing learners which parts of their algorithm will be awarded marks will deconstruct the learners' argument. Disrupting and deconstructing learners' existing sociomathematical norms, in turn creates opportunities for learners to author and teachers to negotiate new sociomathematical norms, which will be endorsed by the whole class.

When what was accepted as standard in doing mathematics for a particular classroom has been disrupted or deconstructed, learners will come up with new ways of doing mathematics. These new ways will be interactively negotiated, reconstructed, and renegotiated with the whole class until they are taken-as-shared. Hence, they will be endorsed as normative ways of doing mathematics for the class. If learners fail to author new sociomathematical norms through scaffolding and mediation, then a teacher should put forward new norms. If these norms are accepted and endorsed by the whole class then they will be regarded as normative.

Sociomathematical norms establish concurrently with mathematical knowledge. Mathematical knowledge exists in three dimensions, which are factual, conceptual and procedural (Anderson & Krathwohl, 2001). Correspondingly, when teachers teach the facts, concepts and procedures of mathematics, the learners concurrently constitute sociomathematical norms in relation to the mathematical facts, concepts and procedures taught. For example, if mathematical facts are taught with reasons, then it will become normative for learners that facts must be accompanied by reasons. Similarly, if facts are taught as rules without reasons, it will become normative for learners to use facts as rules. If the fact that 30 = 1 (three exponent zero is equal to one) is learned as a rule without reasons given, learners will accept other facts such as 0! = 1 (zero factorial is equal to one) without pondering why. Therefore, establishment of sociomathematical norms relates to how mathematics is taught and what mathematics is taught for.

How mathematics is taught and what mathematics is taught for are not explicitly taught. Instead, they are learned implicitly as learners engage with learning material designed by either teachers or presented in text books. Additionally, they are learned as teachers speak and write mathematics during teaching and learning. As such, learners develop their own perceptions about how mathematics is taught and what mathematics is taught for. Some of the learners' perceived ideas may be different from those intended by a teacher. As a result, knowing learners' existing sociomathematical norms is imperative for endorsement, disruption, deconstruction, renegotiation and re-endorsement of these norms.

8.3 LIMITATIONS AND RECOMMENDATIONS

Undoubtedly, the purpose of this case study, which was to explore the constitution of sociomathematical norms in a class where teaching was undertaken to promote learners' proficiency in mathematics, was addressed. This thesis responded the need to undertake studies that focus on how sociomathematical norms are established (Guven & Dede, 2017) and examination of the relationship between sociomathematical norms and opportunities for mathematics learning in classrooms (Widjaja, 2012).

However, there may be limitations that relate to transferability of this study. I clearly had a philosophy of teaching and learning, that is closely linked to the theory that framed this study. This is not typical of average teachers in public schools. Therefore, even with schools classes similar to the one in the study, the findings of this study may be transferable to limited number of schools. A similar argument may be raised based on the sample for the study. The participants of this study were familiar with engaging in mathematical task. As such, they might have been more receptive to what I tried to achieve with this study. With participants that have different traits, it is possible that there might have been resistance and possibly different results.

On the other hand, I faced a challenge with juxtaposing this study with local literature as this study may be among the first of its kind. I have not come across studies that address a purpose similar to that of this study and that involved secondary school learners. While this may be a limitation, it also highlights the necessity of studies that engender specific classroom interactions to promote learners' proficiency in mathematics. This is vital in the context of the prescribed framework that guides teaching and learning in South Africa. Otherwise teaching informed by the framework may fail because of limited theory that informs its implementation. Therefore, it is still necessary to create knowledge on normative aspects of mathematics classes at schools in South Africa as was suggested by Chuene (2011).

8.4 SUMMARY

In this chapter, I responded to the research questions raised at the beginning of this thesis. Additionally, I proposed a pragmatic model which teachers can use to foster the constitution of productive sociomathematicl norms which promotes learners' proficiency in mathematics. Finally, I discussed the limitations of the study and highlighted areas recommended for further research.

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Zembat, I. O., & Yasa, S. A. (2015). Using Classroom Scenarios to Reveal Mathematics Teachers' Understanding of Sociomathematical Norms. *International Journal of Education in Mathematics, Science and Technology*, *3*(3), 242-261.

ANNEXURES

ANNEXURE A: LETTER FOR REQUESTING PERMISSION TO CONDUCT RESEARCH

Enquiries
Mr Lekwa Mokwana

02 May 2017

Limpopo Provincial Department of Education
Private Bag X9489
Polokwane
0700
Attention: Head of Department

Request for permission to conduct research

1. The matter above bears reference.
2. I am a Mathematics Education, doctor of philosophy (PhD) student at the University of Limpopo – Turfloop Campus. As part of the requirements for the fulfilment of the degree I need to conduct a research and produce a thesis.
3. This letter serves to request for permission to conduct research at in Pietersburg Circuit of the **Capricorn Education District**.
4. The topic of the proposed research is **"An exploration of learners' sociomathematical norms: A case of promoting proficiency in mathematics"**. Even though the research will be classroom based, my research agenda will not affect the day to day academic activities of the school. Relevant research ethics will be adhered to.
5. Before I can commence with the research the University's ethical committee will issue a certificate for ethical clearance for the proposed research. In order for such certificate to be issued, I need to submit a permission letter from the department of education.
6. I will be glad if the department can grant me the permission to research.

Kind regards!

Mokwana LL (Mr)

ANNEXURE B: PERMISSION LETTER FROM LIMPOPO DEPARTMENT OF EDUCATION

LIMPOPO

PROVINCIAL GOVERNMENT

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF

EDUCATION

Ref: 2/2/2 Enq: MC Makola PhD Tel No: 015 290 9448 E-mail:MakolaMC@edu.limpopo.gov.za

Mokwana LL

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.

The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"AN EXPLORATION OF LEARNERS SOCIO-MATHEMATICAL NORMS : A CASE OF PROMOTING PROFICIENCY IN MATHEMATICS".**

2. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the schools concerned.

3.3 The conduct of research should not anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

REQUEST FOR PERMISSION TO CONDUCT RESEARCH:MOKWANA LL

CONFIDENTIAL

Cnr. 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X9489, POLOKWANE, 0700
Tel: 015 290 7600, Fax: 015 297 6920/4220/4494

The heartland of southern Africa - development is about people!

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

23/05/17

Ms NB Mutheiwana
Head of Department

Date

REQUEST FOR PERMISSION TO CONDUCT RESEARCH:MOKWANA LL

CONFIDENTIAL

ANNEXURE C: PERMISSION LETTER FROM THE SCHOOL

Mr L L Mokwana
POLOKWANE

By e-mail:

Sir

REQUEST TO CONDUCT RESEARCH & DATA COLLECTION

PROJECT NUMBER	**: TREC/368/2017: PG**
SUBJECT	**: MATHEMATICS**
GRADES	**: 11 & 12**
DATES	**: 7 MAY 2018 TO 8 JUNE 2018**
DATES	**: 16 JULY 2018 TO 31 AUGUST 2018**

2018 -04- 16
LIMPOPO DEPT. OF EDUCATION
CAPRICORN DISTRICT

1. Your letter dated the 11th of April 2018, has reference;
2. Your request to conduct research has been approved;
3. School hours are as follows: Monday-Friday, 7:00-14:00
4. The HOD for Mathematics you will be working with is

We look forward to welcoming you at our school on **7 May 2018** and **16 July 2018**.

Kind regards

Copy to: University of Limpopo
Department of Research Administration & Development
TURFLOOP

E-mail address:

ANNEXURE D: RESEARCH ETHICAL CLEARANCE CERTIFICATE

University of Limpopo

Department of Research Administration and Development

Private Bag X1106, Sovenga, 0727, South Africa

Tel: (015) 268 3935, Fax: (015) 268 2306, Email: anastasia.ngobe@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE

ETHICS CLEARANCE CERTIFICATE

MEETING: 16 September 2020

PROJECT NUMBER: TREC/368/2017: PG - Amended

PROJECT:

Title:	Sociomathematical Norms Constituted for Promoting Learners' Proficiency in Mathematics.
Researcher:	LL Mokwana
Supervisor:	Prof RS Maoto
Co-Supervisor:	Prof KM Chuene
School:	School of Education
Degree:	PhD in Mathematics, Science and Technology Education

PROF P MASOKO

CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE

The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: REC-0310111-031

Note:

i) This Ethics Clearance Certificate will be valid for one (1) year, as from the abovementioned date. Application for annual renewal (or annual review) need to be received by TREC one month before lapse of this period.

ii) Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee, together with the Application for Amendment form.

iii) PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.

Finding solutions for Africa

University of Limpopo
Department of Research Administration and Development
Private Bag X1106, Sovenga, 0727, South Africa
Tel: (015) 268 4029, Fax: (015) 268 2306, Email:Abdul.Maluleke@ul.ac.za

TURFLOOP RESEARCH ETHICS COMMITTEE CLEARANCE CERTIFICATE

MEETING: **02 November 2017**

PROJECT NUMBER: **TREC/368/2017: PG**

PROJECT:

Title:	An exploration of learners' enactment of sociomathematical norms: A case of promoting proficiency in mathematics
Researcher:	LL Mokwana
Supervisor:	Prof RS Maoto
Co-Supervisor:	Dr KM Chuene
School:	School of Education
Degree:	PhD in Mathematics, Science and Technology Education

PROF TAB MASHEGO
CHAIRPERSON: TURFLOOP RESEARCH ETHICS COMMITTEE

The Turfloop Research Ethics Committee (TREC) is registered with the National Health Research Ethics Council, Registration Number: **REC-0310111-031**

Note:

i) **Should any departure be contemplated from the research procedure as approved, the researcher(s) must re-submit the protocol to the committee.**

ii) **The budget for the research will be considered separately from the protocol.**
PLEASE QUOTE THE PROTOCOL NUMBER IN ALL ENQUIRIES.

Finding solutions for Africa

ANNEXURE E: CONFIRMATION LETTER FROM LANGUAGE EDITOR

The Computer Room

Desktop Publishing • Web Design • Proof Reading • Editing

Your one stop document handling service

Plot 48, Palmietfontein, Polokwane, 0699
Postnet Suite 226 • Private Bag X9307 • Polokwane • 0700
Tel: 076 079 0214 • Fax: 086 216 7380

Date: 4 November 2019

To Whom it May Concern

I hereby confirm that I have proof-read the document entitled: "An Exploration of Learners' Enactment of Sociomathematical Norms: A Case of Promoting Proficiency in Mathematics" authored by Mr Lekwa Mokwana, and have suggested a number of changes which the author may or may not accept, at his discretion.

Each of us has our own unique voice as far as both spoken and written language is concerned. In my role as proof-reader I try not to let my own "written voice" overshadow the voice of the author, while at the same time attempting to ensure a readable document.

Please refer any queries to me.

Andrew Scholtz

ANNEXURE F: Learning Activities

Learning Activity 1

1. L$(-1\,;3)$, M$(7\,;1)$ and N$(x\,;2)$ are points in a Cartesian plane.

   Calculate x if:

   1.1 the gradient of MN is $\frac{1}{2}$.

   1.2 N is the midpoint of LM.

   1.3 the length of MN is $\sqrt{2}$ units.

   1.4 line MN is perpendicular to the x-axis.

   1.5 L, M and N lies on the same straight line.

2. Determine the values of x and y if M$(2\,;\,-3)$ is the midpoint of P$(3\,;8)$ and Q$(x\,;y)$.

3. E$(-4\,;\,-1)$, F$(2\,;3)$ and G$(6\,;\,-3)$ are vertices of a triangle.

   Prove that:

   3.1 ∆EFG is right-angled.

   3.2 $F\hat{E}G = 45°$

4. W$(0\,;\,4)$, X$(5\,;3)$ and V$(2\,;\,1)$ are vertices of a triangle.

   4.1 Prove that $\hat{V} = 90°$

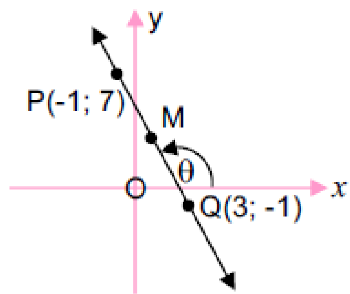
   4.2 Determine the coordinates of Y and Z, if WXYZ is a rhombus with diagonals WY and XZ intersecting at V.

   4.3 Prove that WXYZ is a square.

5. A$(0\,;7)$, B$(9\,;9)$, C$(9\,;5)$ and D$(0\,;3)$ are vertices of a quadrilateral. Prove that ABCD is a parallelogram.

Learning Activity 2

1. P(- 1 ; 7) and Q(3 ; -1) are two points on a straight line in a Cartesian plane as shown in the diagram below.



Determine:

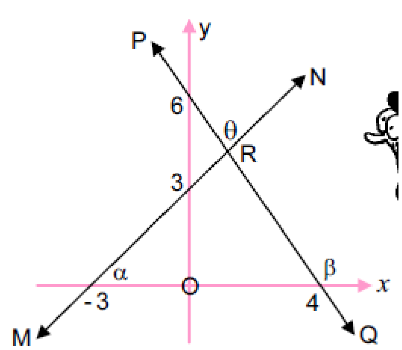
1.1 the equation of line PQ.

1.2 the size of θ.

1.3 the equation of the line which is parallel to PQ and passes through the point (-5 ; 1).

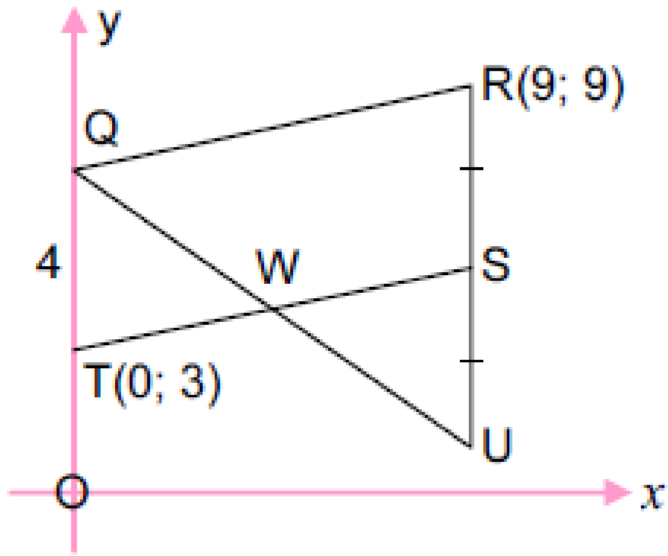
1.4 the equation of the line which is perpendicular to PQ and passes through M, the midpoint of PQ.

2. Determine the angle of inclination of a straight line with the equation $2x + 3y = 5$.
3. Determine the numerical value of p if the straight line defined by $2y = \mathrm{p}x + 1$ has an angle of inclination of 135º.
4. In the sketch below, lines PQ and MN are shown together with their intercepts with the axes.



Determine the size of θ.

5. In the figure below, QRST is a parallelogram with vertices Q and T lying on the y-axis. The side RS is produced to U such that RS = SU. The length of QT is 4 units and the coordinates of R and T are (9 ; 9) and (0 ; 3) respectively. The line segment QU intersects TS at W.



5.1 Determine the coordinates of Q and U.

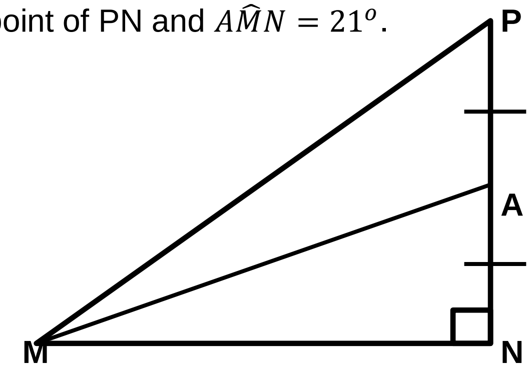
5.2 If W is the midpoint of UQ, determine whether W lies on line OR.

5.3 Determine the size of angle $Q\hat{W}S$.

Learning Activity 3

ACTIVITY 1: Sine Rule

1. In the sketch below, ∆MNP is drawn having a right angle at N and MN = 15 units. A is the midpoint of PN and $A\hat{M}N = 21^o$.



Calculate

1.1 AN

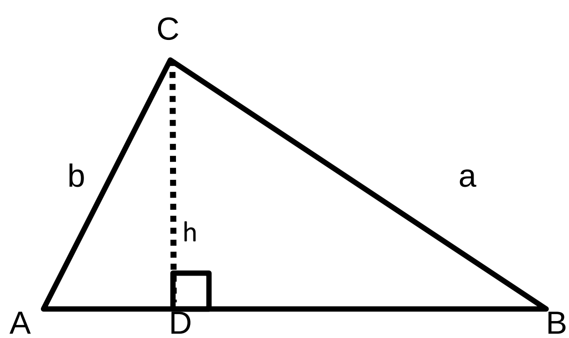
1.2 AM

1.3 $P\hat{M}N$

1.4 MP

1.5 If the ∆MNP was not a right angled triangle would you still respond to 1.1 and 1.2 above as you did? Provide a reason.

2. In the sketch below, ∆ABC is drawn having AB = c, BC = a and AC = b, line CD = h and it is perpendicular to AC.



2.1 Refer to ∆ACD and write down the ratio of $\sin A$

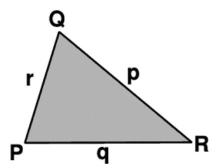
2.2 Refer to ∆BCD and write down the ratio of $\sin B$

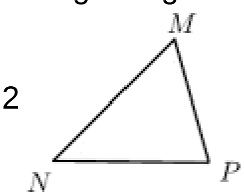
2.3 For the ratios you have in 2.1 and 2.2 above make h the subject the equation, explain what do you notice.

2.4 Equate the two equations formed in 2.3 above then simply by ensuring that As are on one side of the equation and Bs are on the other side. (**Note**: Your answer here is the SINE rule formula)

Activity 2: Sine Rule

1. Write down the sine rule formula for the following triangles.

1.1 

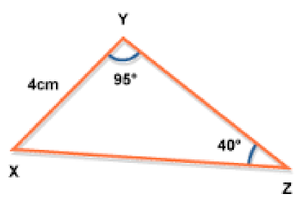
1.2 

1.3 ΔXYZ

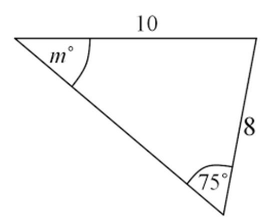
1.4 ΔEFG

2. For each of the following triangles calculate the sizes of the unknown sides and/or angles specified.

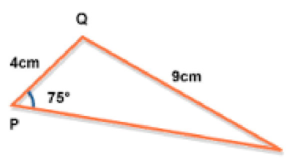
2.1 XZ



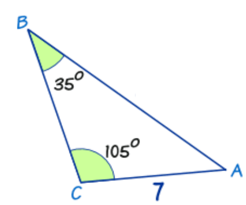
2.2 m



2.3 $\hat{Q}$

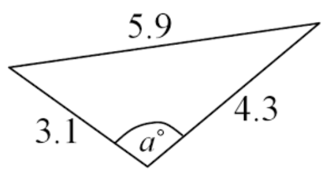


2.4 BC

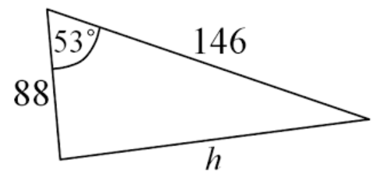


3. Consider the following triangles, and calculate the numerical values of a and h respectively. Provide a detailed explanation for your answer.

3.1

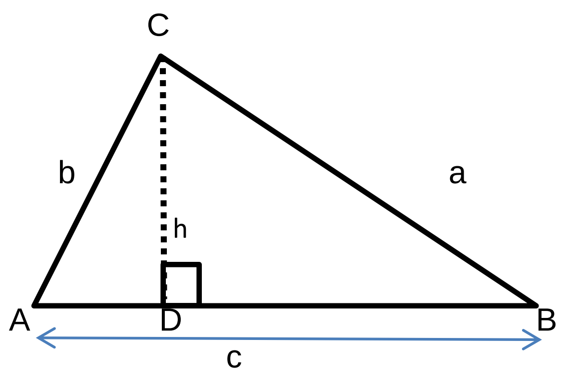


3.2



Activity 3: Cosine Rule

1. In the sketch below, ∆ABC is drawn having AB = c, BC = a and AC = b, line CD = h, AD = x and it is perpendicular to AC.



1.1 Write down the length of DB in terms of **c** and **x**.

1.2 Refer to ∆BCD and write down Pythagoras theorem equation, simplify it and label it equation 1.

1.3 Refer to ∆ACD and write down Pythagoras theorem equation, make h^2 the subject of the formula and label it equation 2.

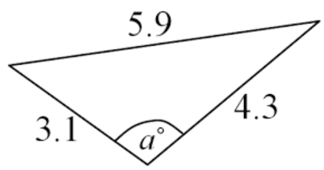
1.4 Refer to ∆ACD and write down the ratio for $\cos A$, then make x the subject of the formula, label it equation 3.

1.5 Substitute equation 2 into equation 1 then simplify.

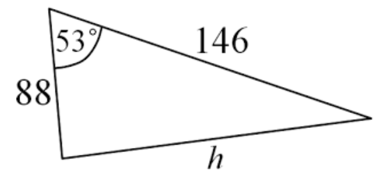
1.6 Substitute equation 3 into your answer in 1.5 above. (Note: Your answer here is the COSINE rule).

2. Consider the following triangles, and calculate the numerical values of a and h respectively. Provide a detailed explanation for your answer.

2.1



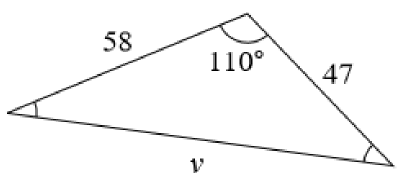
2.2



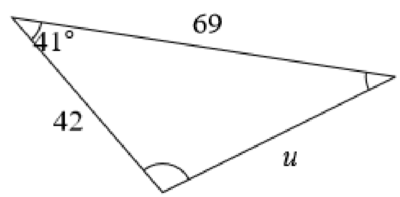
3.

1: Find the unknown quantity (correct to 1 decimal place):

a)

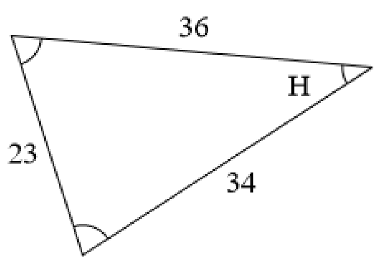


b)

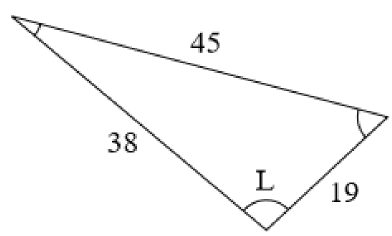


2: Find the unknown quantity (correct to 1 decimal place):

a)



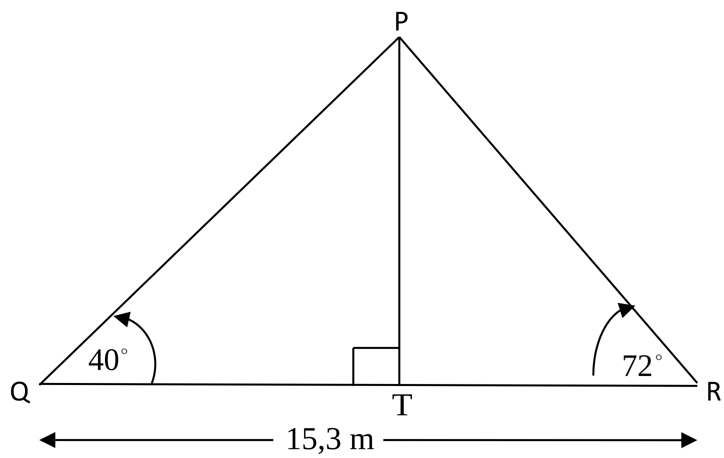
b)



Activity 4: Sine, cosine and area rule

QUESTION 1

1.1 From two points Q and R in the same horizontal plane as T, the foot of a pylon PT and on the opposite side of it, the angles of elevation of the top of the pylon P are 40° and 72° respectively. The distance between the two points Q and R is 15,3 m.

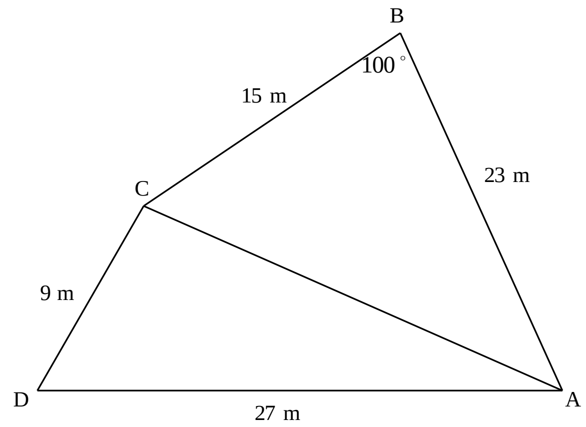


Find:

1.1.1 PQ

1.1.2 PT

1.2 A piece of land has the form of a quadrilateral ABCD with AB = 23m, BC = 15m, CD = 9m and AD = 27m. $\hat{B} = 100^\circ$. The owner decides to divide the land into two plots by erecting a fence from A to C.



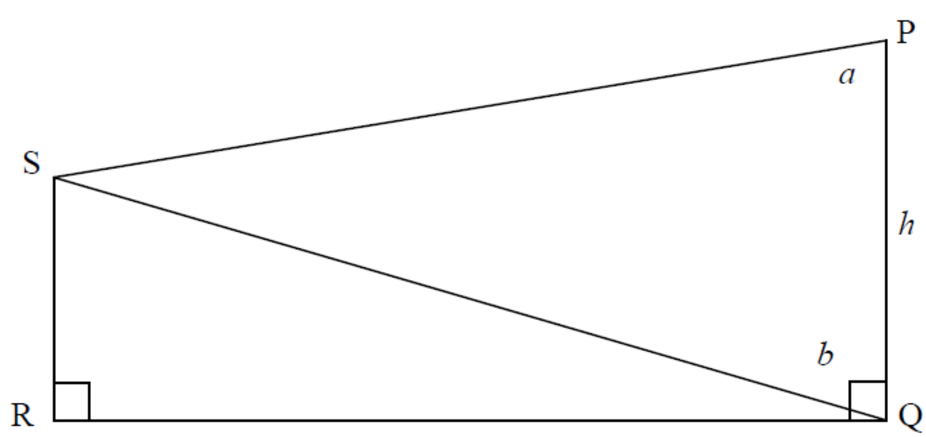
1.2.1 Calculate the length of the fence AC.

1.2.2 Calculate the size of $\hat{D}$.

1.2.3 Calculate the area of the entire piece of land ABCD

QUESTION 2

2.1 In the figure below, $S\hat{P}Q = a$, $P\hat{Q}S = b$ and $PQ = h$. PQ and SR are perpendicular to RQ.

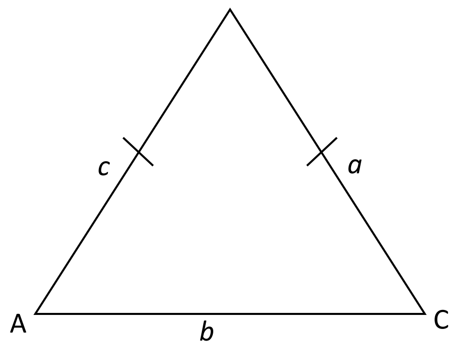


2.1.1 Determine the distance SQ in terms of *a*, *b* and *h*.

2.1.2 Hence show that $RS = \dfrac{h\sin a\cos b}{\sin(a+b)}$.

2.2 In the diagram below, ΔABC is isosceles with AB = BC.

B

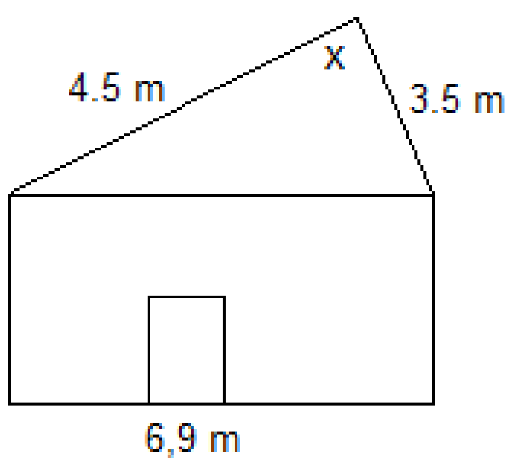


Prove that $\cos B = 1 - \frac{b^2}{2a^2}$.

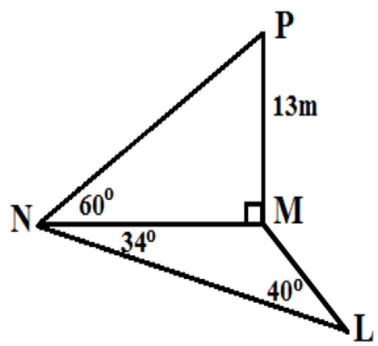
QUESTION 3

3.1 Given that: $a^2 = b^2 + c^2 - 2bc\cos A$, hence prove that:
$1 + \cos A = \frac{(b+c+a)(b+c-a)}{2bc}$

3.2 The diagram shows the side view of a house with sloping roof. Calculate the size of angle x, between the two sloping sides of the roof.



3.3 In the diagram, PM is perpendicular to the horizontal plane LMN. If PM=13metres, $P\hat{N}L = 60°, M\hat{N}L = 34°$ and $M\hat{L}N = 40°$

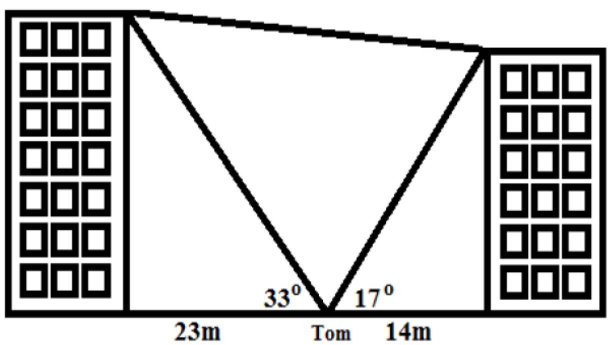


Calculate:

3.3.1 the length of LN

3.3.2 the area of ΔLMN

3.4 Tom stands between two buildings of different heights. He is 23m away from the building on his left and the angle of elevation between him and the building is 33°. The building on the right is 14m away and is at an angle of elevation is 17°.



Calculate:

3.4.1 the length of the building on the right

3.4.2 the distance from Tom to the top of the building on the left

3.4.3 the length of a piece of rope which connects the two buildings at the top.

ANNEXURE G: Communication with Prof Erna Yackel

Mokwana, Lekwa

Request for expert point of view

Mokwana, Lekwa · Mon, Jun 1, 2020 at 9:34 AM To:

Greetings Professor

I hope you are doing well in this trying time of a global Pandemic.

I am humbly requesting your point of view as a proponent of sociomathematical norms. In studying learners' classroom sociomathematical norms is it always the case that a research question focusing on the role of the teacher must be included?
I have attached a summary of a study focusing on learners' sociomathematical norms as problematised, however, colleagues are of the view that there should be atleast one research question focusing on the teacher's role. In making this suggestion they also acknowledge that they are not experts of sociomathematical norms.

Kindly assist as your critic or point of view will be highly

appreciated. Kind regards,

Lekwa

Synopsis.docx
15K

Erna Yackel · Thu, Jun 4, 2020 at 3:23 AM
To: "Mokwana, Lekwa" ·

I am attaching two documents. One is my reply to your question. The second is the plenary paper I refer to in my reply.

Good luck with your study. The notion of sociomathematical norms is a very subtle one and one that many misunderstand so I applaud you in seeking additional information.

Erna Yackel

Received 04 June 2020

You have asked me a very straight-forward question, "In studying learners' classroom sociomathematical norms is it always the case that a research question focusing on the role of the teacher must be included?" My answer is anything but straight-forward but I hope that it will provide sufficient clarification of the notion of classroom sociomathematical norms that you will have the answer to your question.

We (Paul Cobb and myself) developed the notion of sociomathematical norms over the period of several years as we were attempting to make sense of the mathematical learning in several classroom researh design experiments. (At that time we used the language of classroom "teaching experiments.") These took place in elementary school classrooms, several of which were grade 2 classrooms. I am not sure what your background sources are for the notion of sociomathematical norms so I am suggesting some papers that are central to understanding the notion itself. I will list these and then I will explain some of my thinking relative to your study.

The main publication that introduced the notion of sociomathematical norms is

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and automony in mathematics. *Journal for Research in Mathematics Education, 27*, 458-477.

If you have not read this paper I strongly encourage you to do so because it explains in detail how the notion was developed and also explains its theoretical underpinnings. Another paper that includes much of the same information is

Cobb, P., & Yackel, E. (1996) Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31* (3/4), 175-190.

This paper is much more theoretically oriented and, unless you want to delve deeply into that, you may wish to bypass reading this paper. Both of these papers are readily available through your library sources.

Later, I presented a plenary address at PME International in 2001 in which I attempted to lay bare the theoretical underpinnings of sociomathematical norms in a concise yet comprehensible way and included extensive examples from a university-level differential equations class for illustrative purposes. I am assuming that it may not be easy to access the proceedings that contain this paper so I am attaching it for your convenience. I recommend that you read this paper.

Based on the references you include in your problem statement, I assume that you are familiar with the paper

Yackel, E., Rasmussen, C., & King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *Journal of Mathematical Behavior, 19,* 275-287.

This paper is based on a university-level differential equations class.

Now to answer your question.

As we began our study of classrooms we quickly realized that in addition to taking a psychological perspective, it was essential to take a sociological perspective. In other words, we could not account for what was happening solely in terms of individual students' learning and activity. Instead we had to take account of the interactions that were taking place in the classroom. That requires a sociological perspective. The notion of norms became central to our developing understandings. Initially we focused on social norms and later developed the notion of sociomathematical norms, as discussed in the papers listed above.

While the psychological perspective that underpinned our investigations was constructivism, the underpinning sociological perspective was *symbolic interactionism*. It is impossible to understand the notion of sociomathematical norms that we introduced without understanding symbolic interactionism. The best reference for that is

Blumer, Herbert. (1969). *Symbolic interactionism.* Englewood Cliffs, NJ: Prentice-Hall.

I strongly recommend reading at least the first section of chapter 1, that is, pages 1-12. That will most likely be sufficient background for your purposes.

It is important to understand that norms are *regularities in interaction patterns*. As such, the way one documents norms is to document these regularities in the (classroom) interaction patterns. That involves figuring out how to show which interaction patterns are operative—or we could say which interaction patterns characterize the classroom. That typically involves showing how they come to be in the classroom. Further, while interaction patterns are regularities in interaction, symbolic interactionism holds that interaction patterns are continually constituted and reconstituted with each interaction. That is why we speak of norms being *constituted*. Moreover, because of this continual constitution and reconstitution, it is possible for the patterns of interaction to shift and morph over time, which is precisely the reason it is possible for specific norms to develop within classrooms as the school terms progresses. (And is why you would be able to talk about the norms that "evolve" over the school term.)

Now to make two specific points:

- Because norms are regularities in interaction patterns, norms are not an individualist notion. That is, we cannot talk about a student's norms or the norms of students. We can only talk about norms of the collective, the group, i.e., the norms that characterize the *classroom*.

- Second, every classroom has norms, both social and sociomathematical. It isn't that some classrooms have norms and others don't. They ALL do. However, classrooms differ in the *nature of these norms*. For example, in traditional mathematics classrooms, it is normative that explanations are acceptable if they describe rules and procedures, whereas in inquiry mathematics classrooms, to be acceptable an explanation has to be about mathematics objects that are experientially real to the students. (This is an example of the difference between the sociomathematical norm relative to acceptable explanation in two different types of instruction.) Likewise, in traditional mathematics instruction it is normative that students rely on the teacher as the mathematical authority in the classroom, whereas in inquiry mathematics instruction mathematical authority resides in the reasoning and argumentation of the students. (This is an example of the difference between the social norms in two different types of instruction.)

These two points speak both to your problem statement and to the way you have phrased the research questions. The implication in the first sentence of your problem statement is that there are some classrooms that do not have sociomathematical norms. Well, all classrooms do. But certain types of sociomathematical norms "contribute to conditions that make meaningful learning of mathematic possible." So it is incumbent on you to be specific about the *nature of the norms* that make meaningful mathematics learning possible. This point applies to the entire first paragraph of the problem statement. In fact you make this same point yourself in the second paragraph of your problem statement where you say, "… argue for studies that focus on determining which sociomathematical norms should be established and how they should be established." (I would say *constituted* rather than *established*, though that is a minor point.)

I'm sure that you can now see that the research questions need to be reformulated to reflect the fact that sociomathematical norms are characteristics of the collective, i.e., the classroom as a whole (which includes both the students and the teacher) and are not characteristics of the individuals that comprise the collective.

Specifically regarding the teacher, the papers I have referenced above give a lot of attention to the way norms are interactively constituted, including the unique role of the teacher in this process of interactive constitution. Therefore, while it is not necessary to have a specific research question directed at the teacher, when attempting to understand, analyze, and explain the norms that characterize the classroom, you have ample opportunity, if you wish, to discuss ways in which the teacher can influence (though not control) the nature of the (social AND sociomathematical) norms that are constituted.

This may be more than you bargained for when you asked me your question but I have to admit that over the years I have reviewed many papers addressing issues related to sociomathematical norms. The most frequent issue I have had to address is that many authors fail to understand this notion of interactive constitution, i.e. the symbolic interactionist underpinnings of the entire notion of sociomathematical norms.

Good luck with your study.