# ENTROPY ANALYSIS IN A CHANNEL FLOW WITH TEMPERATURE DEPENDENT VISCOSITY

BY

# **CYNTHIA REITUMETSE NDABA**[B.SC., B.SC HONS]

Submitted in fulfillment of the requirements for the degree of MASTER OF SCIENCE IN APPLIED MATHEMATICS, School of Computational and Mathematical Sciences, University of Limpopo, Private Bag X1106, Sovenga 0727, South Africa

**SUPERVISOR: Professor O. D. MAKINDE** 

**SEPTEMBER 2007** 

#### **DECLARATION**

I declare that the dissertation hereby submitted to the University of Limpopo for the degree of Master of Science has not been submitted by me at this university or any other university and that this is my own work in design and in execution and all material contained therein has been duly acknowledged.

Signed:_						
<i>C</i> –						
Date						

### **CERTIFICATION**

I certify that this work was carried out by **CYNTHIA REITUMETSE NDABA** under my supervision in the Applied Mathematics, University of Limpopo, Private Bag X1106, Sovenga 0727, South Africa.

Professor O. D. Makinde

Supervisor

Ш

# **DEDICATIONS**

To my Mother, my Husband, my little boy Thato, my relatives and all my friends, I love you all.

#### **ACKNOWLEDGEMENT**

I would like to place on record my sincere gratitude to the following people who have contributed towards the successful completion of this research project: Prof. O.D Makinde for his supervision, immense support, patience and motivation. My mother Annah Ndaba and my husband Amulet Makhalemele for always being there for me during difficult times. Dr. P Mhone for his assistance especially with Maple problems. My friends at Turfloop for their patience and understanding and finally Dr. J Moitsheki for his support.

# TABLE OF CONTENTS

Declaration	II	
Certification	III	
Dedications	IV	
Acknowledgements	${f v}$	
List of figures	VIII	
Abstract	XI	
CHAPTER 1	1	
1.1. Introduction	1	
1.1.1.The First Law of Thermodynamics	1	
1.1.2. The Second Law of Thermodynamics	1	
1.1.3.Entropy	3	
1.1.4.Entropy Aspects	6	
1.1.5.Significance of Entropy	8	
1.1.6. Literature review	10	
1.1.7. Viscosity	14	
1.1.8.Channel Flow	16	
1.2. Aim of the Study	17	
1.3.Specific Objectives of the study		
CHAPTER 2	18	
2.1.Summary	18	

2.2.Introduction	15
2.3.Mathematical Model	21
2.4.Solution Method	24
2.5.Entropy Generation Rate	25
2.6. Graphical Results and Discussion	27
CHAPTER 3	36
3.1.Summary	36
3.2.Introduction	36
3.3.Mathematical Model	37
3.4.Solution Method	40
3.5.Entropy Generation Rate	41
3.6. Graphical Results and Discussion	43
CHAPTER 4	
CONCLUSION AND FUTURE WORK	52
APPENDIX	54
REFERENCES	60

## **LIST OF FIGURES**

Figure 1. Illustration of entropy increase and decrease		
for cold and hot bodies.		
Fig. 2.3.1: Schematic diagram of the problem	22	
<b>Fig. 2.6.1:</b> Velocity profile: $G=1$ ; $Br=10$ ; $\alpha=0$ ;	28	
ooooo $\alpha = 0.1$ ; ++++ $\alpha = 0.3$ .		
<b>Fig.2.6.2:</b> Velocity profile: $G=1$ ; $\alpha=0.1$ ;B $r=10$ ;	29	
ooooo $Br = 500$ ; ++++ $Br = 1000$ .		
<b>Fig.2.6.3:</b> Velocity profile: $Br=10$ ; $\alpha = 0.1$ ;	29	
G = 1; ooooo $G = 2$ ; ++++ $G = 3$ .		
<b>Fig.2.6.4:</b> Temperature profile: $G=1$ ; $Br=100$ ;	30	
$\alpha = 0$ ; ooooo $\alpha = 0.1$ ; ++++ $\alpha = 0.3$ .		
<b>Fig.2.6.5:</b> Temperature profile: $G=1$ ; $\alpha=0.1$ ;	30	
Br = 10; ooooo $Br = 50$ ; ++++ $Br = 100$ .		
<b>Fig.2.6.6:</b> Temperature profile: $Br=10$ ; $\alpha=0.1$ ;	31	
G = 1; ooooo $G = 2$ ; ++++ $G = 3$ .		
<b>Fig. 2.6.7:</b> Entropy generation rate: $G = 1$ ; Br = 10; $\Omega = 1$ ;	31	
$\alpha = 0$ ; ooooo $\alpha = 0.1$ ; ++++ $\alpha = 0.3$ .		
<b>Fig. 2.6.8:</b> Entropy generation rate: $\alpha$ =0.1; G = 1; $\Omega$ = 1;	32	
Br = 1; ooooo $Br = 5$ ; ++++ $Br = 10$ .		
<b>Fig.2.6.9:</b> Entropy generation rate: $\alpha = 0.1$ ; $Br = 10$ ; $\Omega = 1$ ;	32	
G = 1:00000G = 2:++++G = 3		

```
Fig. 2.6.10: Entropy generation rate: G = 1; \alpha = 0.1;
                                                                                            33
Br = 10; \Omega = 1; \Omega = 1; 00000 \Omega = 5; ++++\Omega = 10.
Fig. 2.6.11: Bejan number : G = 1; Br = 10; \Omega = 1;
                                                                                            33
\alpha = 0; 00000\alpha = 0.1; ++++\alpha = 0.3.
Fig. 2.6.12: Bejan number: \alpha=0.1; G = 1; \Omega= 1;
                                                                                            34
Br = 1; 00000Br = 5; ++++Br = 10.
Fig. 2.6.13: Bejan number: \alpha = 0.1; Br = 10; \Omega = 1;
                                                                                            34
G = 1; oooooG = 2; ++++G = 3
Fig. 2.6.14: Bejan number: G = 1; \alpha = 0.1; Br = 10; \Omega = 1;
                                                                                            35
\Omega = 1; 00000 \Omega = 5; ++++\Omega = 10.
Fig. 3.3.1: Schematic diagram of the problem
                                                                                            38
Fig. 3.6.1: Velocity profile: G=1; Br=10;
                                                                                            44
\alpha = 0; 00000 \alpha = 0.1; ++++ \alpha = 0.3.
Fig.3.6.2: Velocity profile: G=1; \alpha=0.1;
                                                                                            45
Br = 10; ooooo Br = 50; ++++ Br = 100.
Fig.3.6.3: Velocity profile: Br=10; \alpha = 0.1;
                                                                                            45
G = 1; ooooo G = 2; ++++G = 3.
Fig.3.6.4: Temperature profile: G=1; Br=100;
                                                                                            46
\alpha = 0; ooooo \alpha = 0.1; ++++ \alpha = 0.3.
Fig.3.6.5: Temperature profile: G=1; \alpha = 0.1;
                                                                                            46
Br = 10; ooooo Br = 50; ++++ Br = 100.
Fig.3.6.6: Temperature profile: Br=10; \alpha=0.1;
                                                                                            47
G = 1; oooooG = 2; ++++G = 3.
Fig.3.6.7: Entropy generation rate: G = 1; Br = 10; \Omega = 1;
                                                                                            47
```

 $\alpha = 0$ ; 00000 $\alpha = 0.1$ ; ++++ $\alpha = 0.3$ .

```
Fig.3.6.8: Entropy generation rate: \alpha=0.1; G = 1; \Omega= 1;
                                                                                             48
Br = 1; oooooBr = 5; ++++Br = 10.
Fig.3.6.9: Entropy generation rate: \alpha = 0.1; Br = 10; \Omega = 1;
                                                                                             48
G = 0.5; oooooG = 0.8; ++++G = 1.
Fig. 3.6.10: Entropy generation rate: G = 1;
                                                                                             49
\alpha = 0.1; Br = 10; \Omega = 1; ooooo \Omega = 2; ++++\Omega = 5.
Fig. 3.6.11: Bejan number :G = 1; Br = 10; \Omega = 1;
                                                                                             49
\alpha = 0; 00000\alpha = 0.1; ++++\alpha = 0.3.
Fig. 3.6.12: Bejan number: \alpha=0.1; G = 1; \Omega= 1;
                                                                                             50
Br = 1; oooooBr = 5; ++++Br = 10.
Fig. 3.6.13: Bejan number: \alpha = 0.1; Br = 10; \Omega = 1;
                                                                                             50
G = 0.5; oooooG = 0.8; ++++G = 1.
Fig. 3.6.14: Bejan number: G = 1; \alpha = 0.1; Br = 10;
                                                                                             51
\Omega = 1; 00000\Omega = 2; ++++\Omega = 5.
```

#### **ABSTRACT**

The thermodynamic irreversibility in any fluid flow process can be quantified through entropy analysis. The first law of thermodynamics is simply an expression of the conservation of energy principle. The second law of thermodynamics states that all real processes are irreversible. Entropy generation is a measure of the account of irreversibility associated with the real processes. As entropy generation takes place, the quality of energy (i.e. exergy) decreases. In order to preserve the quality of energy in a fluid flow process or at least to reduce the entropy generation, it is important to study the distribution of the entropy generation within the fluid volume. In this dissertation, the inherent irreversibility in the flow of a variable viscosity fluid in both a closed channel and an open channel is investigated. The channel is assumed to be narrow, so that the lubrication approximation may be applied and the fluid viscosity is assumed to vary linearly with temperature. Both the lower and the upper surfaces of the channel are maintained at different temperature. The simplified form of governing equations is obtained and solved analytically using a perturbation technique. Expressions for fluid velocity and temperature are derived which essentially expedite to obtain expressions for volumetric entropy generation numbers, irreversibility distribution ratio and the Bejan number in the flow field.

In chapter 1, a historic background of the study is highlighted. Both closed and open channels problem are investigated in chapters 2 and 3. In chapter 4, generally discussion

on the overall results obtained from the investigation is displayed together with possible areas of future research work.